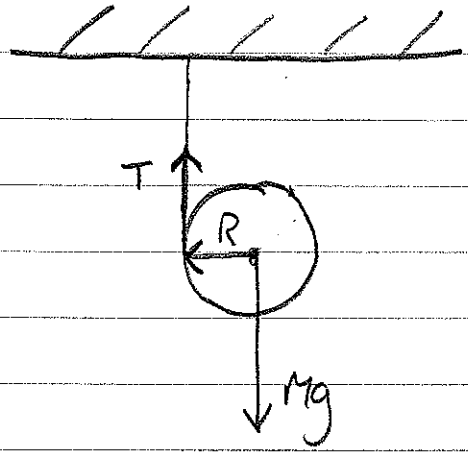
Falling Cylinder



Mass is M
Radius is R
What is acceleration (a) of the center of mass?

① Use Newton's 2nd Law (only y components here)

$$\Sigma F = T - Mg = Ma$$

so

$$a = \frac{T - Mg}{M}$$

, we need T, the tension in the string

② Use Rotational Form of Newton's 2nd to get T

$$\Sigma \tau = I\alpha \quad , \quad \tau = RT\sin\theta \quad , \quad \theta = 90^\circ$$
$$= RT$$

and $\alpha = \frac{a_T}{R}$ and $a_T = -a$ because a_T is ↑ and a is ↓

so

$$RT = -\frac{I}{R}a$$

or $T = -\frac{I}{R^2}a$ → now plug in to ①

to get

$$a = -\frac{I}{MR^2}a - g$$

or $$a\left[\frac{I}{MR^2} + 1\right] = -g$$

and finally

$$a = \frac{-g}{\frac{I}{MR^2} + 1} \rightarrow$$ general result for any geometry (Cylinder, Sphere, hoop, ...)

Now apply to Cylinder

So $I = \frac{1}{2}MR^2$

then

$$a = \frac{g}{\frac{MR^2}{2MR^2} + 1} = -\frac{2}{3}g$$

Negative sign is OK because we chose ↓ to be the negative direction and it is accelerating downward at $\frac{2}{3}$ the rate if the string were not there!