Project Listen Up: Drumhead resonances in circular elastic membranes
-- on a budget!

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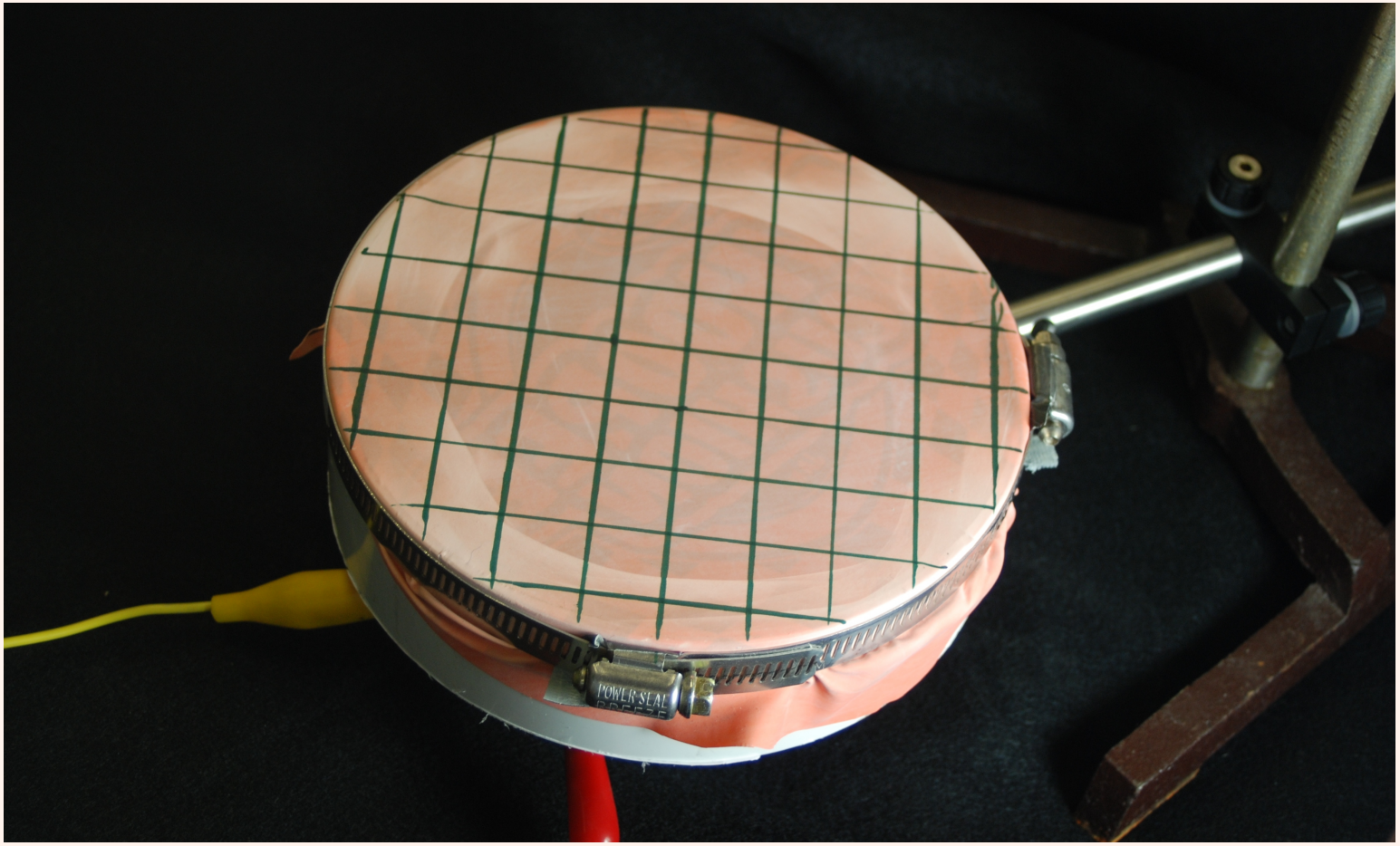
ASA Fall 2009
Project Listen Up Session

Major Goals

- Design a simple, inexpensive, appealing, and educational demonstration of resonances in two dimensions.
- Use materials easily obtained by a smaller department or high school teacher with limited resources.
- Nice physical manifestation of the mathematics.

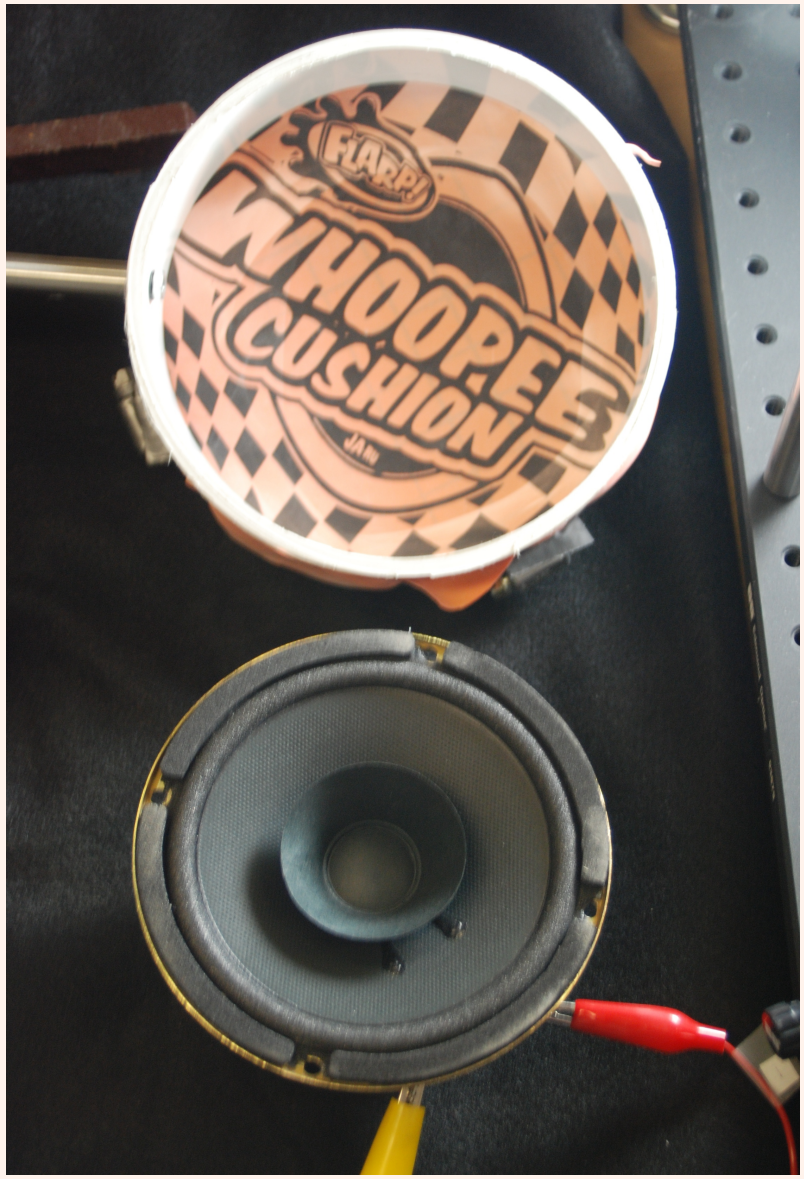
Final Product – top view



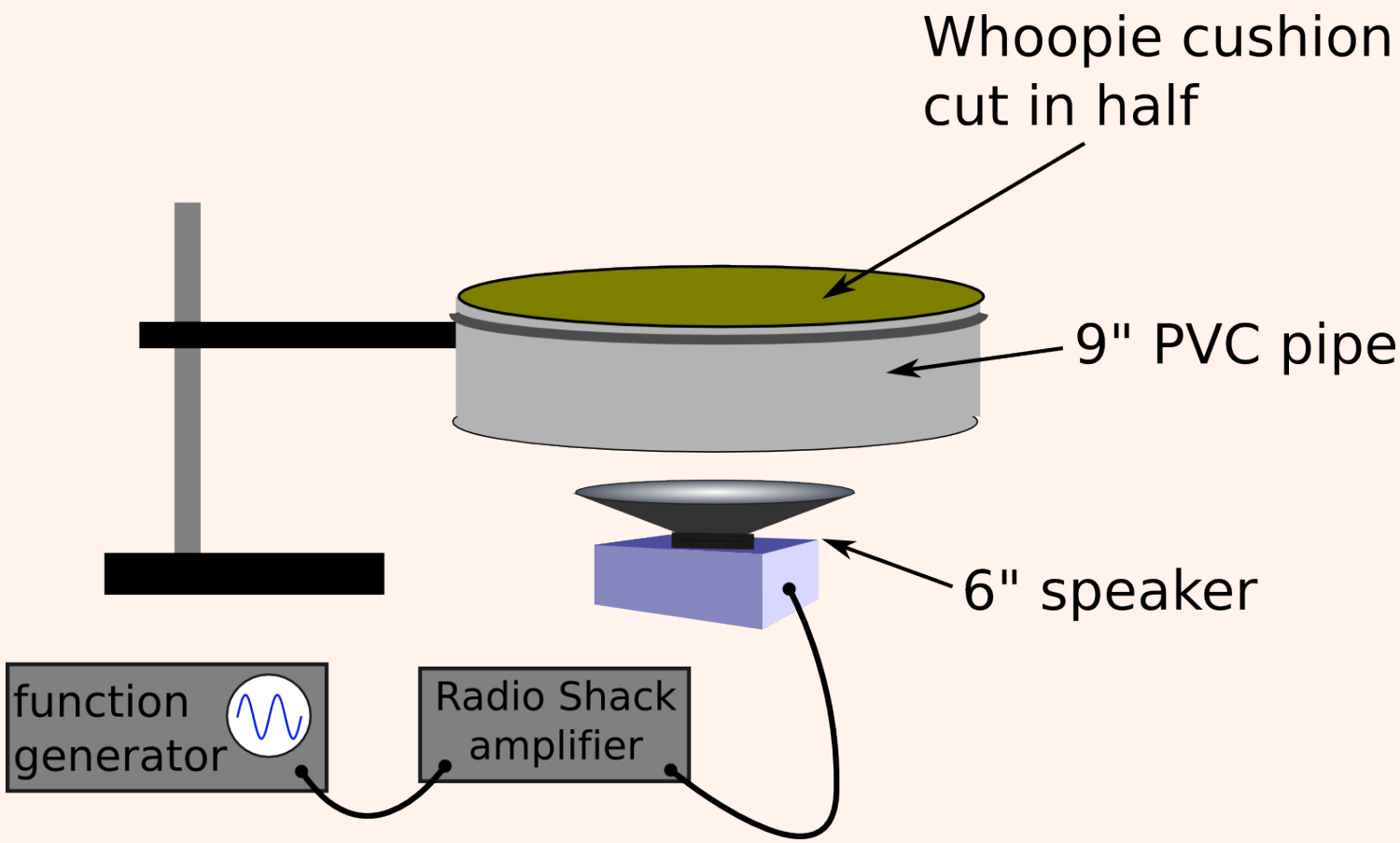
Final Product – bottom view

Retask those old Whoopee cushions!

8″ speaker



The set up



Theory

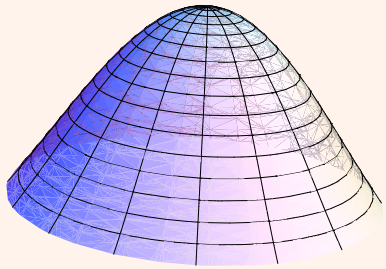
- Circular membrane of radius a and transverse wave speed c
- 2D wave equation in polar coordinates.

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r}\frac{\partial \Psi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Psi}{\partial \theta^2} = \frac{1}{c^2}\frac{\partial^2 \Psi}{\partial t^2}$$

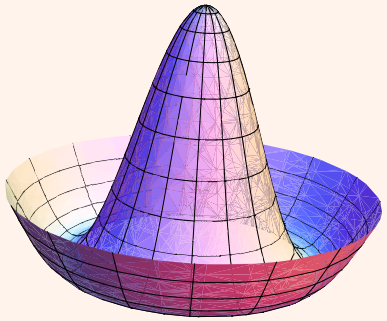
- Separable PDE: $e^{i\omega t}$ time dependence and spatial solutions:

$$\psi(r,\theta) = \sum_{n=0}^{\infty}\sum_{m=1}^{\infty} A_{nm} J_n(\lambda_{nm} r/a)(B_n \cos n\theta)$$

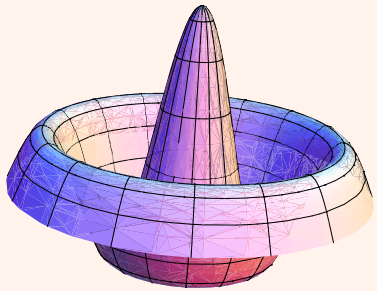
Modes



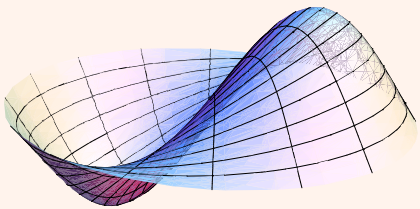
mode 0,1



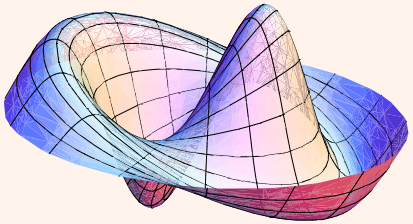
mode 0,2



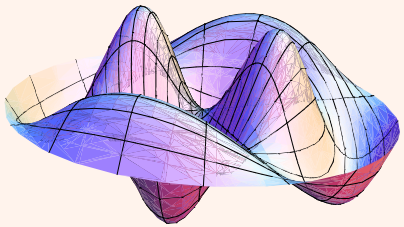
mode 0,3



mode 1,1



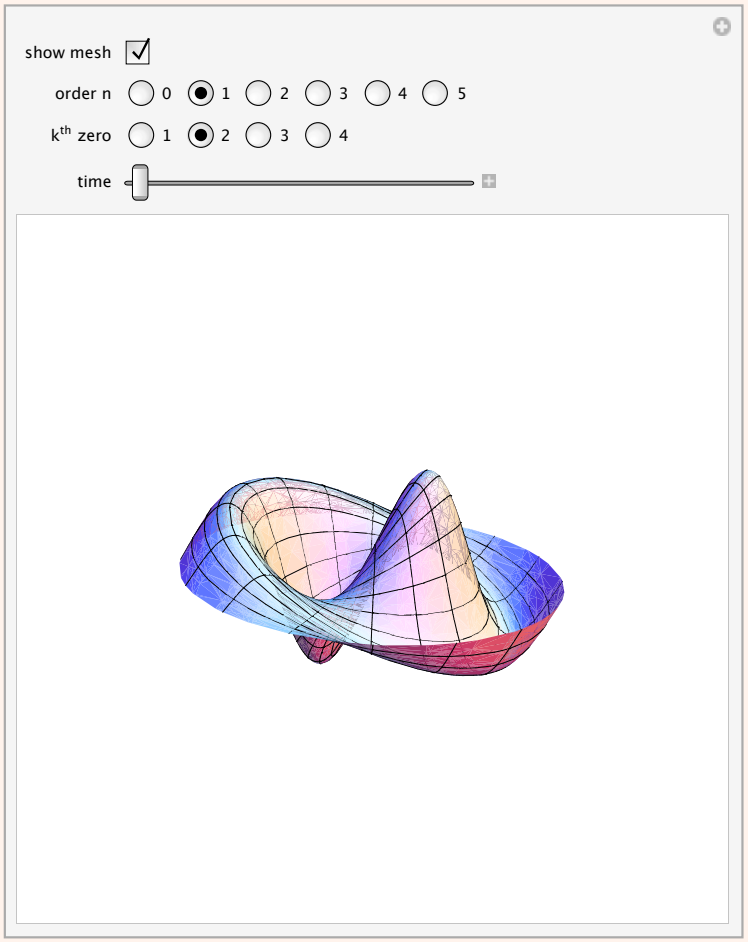
mode 2,1



mode 2,2

Animate with Mathematica

Out[1]=



Code can be downloaded from
Wolfram Demonstrations Project:
http://demonstrations.wolfram.com/NormalModesOfACircularDrumHead/

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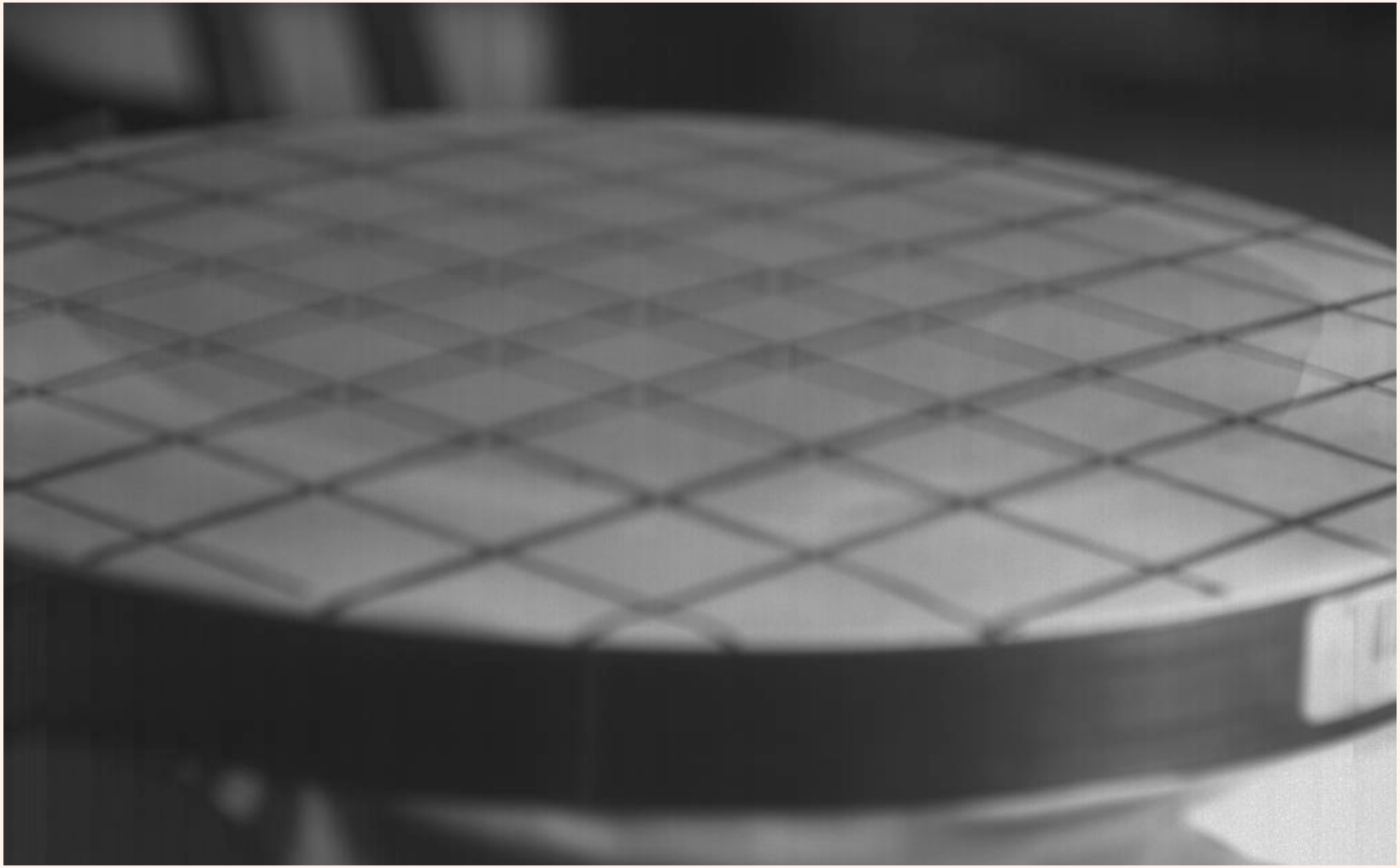
High Speed Video
mode 0,0



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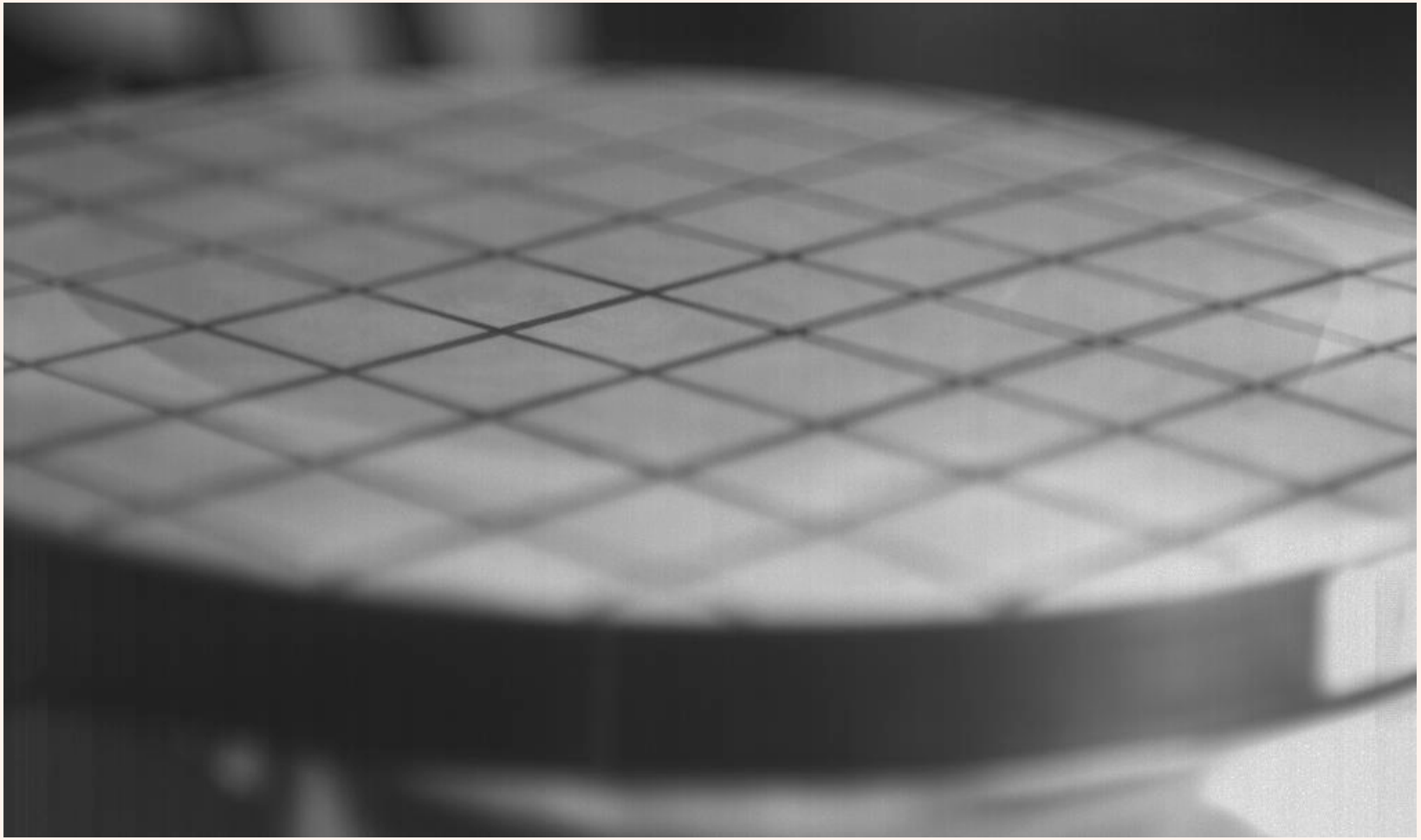
High Speed Video
mode 1,1



The University of Mississippi

Institute for Advanced Education
in Geospatial Science

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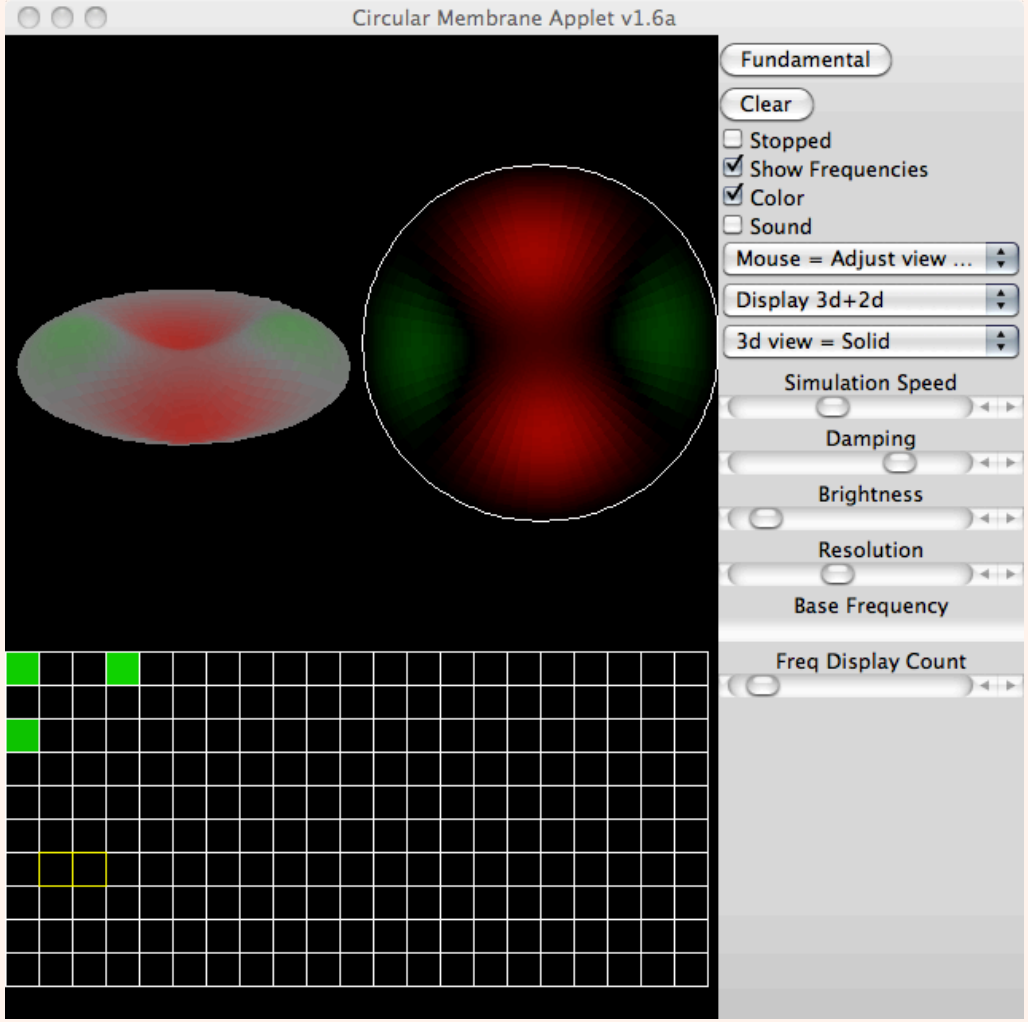
Cost in blood and treasure

Item	Source	Cost
Whoopie cushion	Walmart	$1.00
Hose Clamps	Walmart	$1.00
PVC pipe	scrap from machine shop	free
Speaker	Dr. Raspett's lab	free ($24 @ Radio Shack)
function generator	lab (common even in high schools)	should already have
amplifier	lab (not absolutely necessary)	~$45
time		about 4 hours

Another good simulation

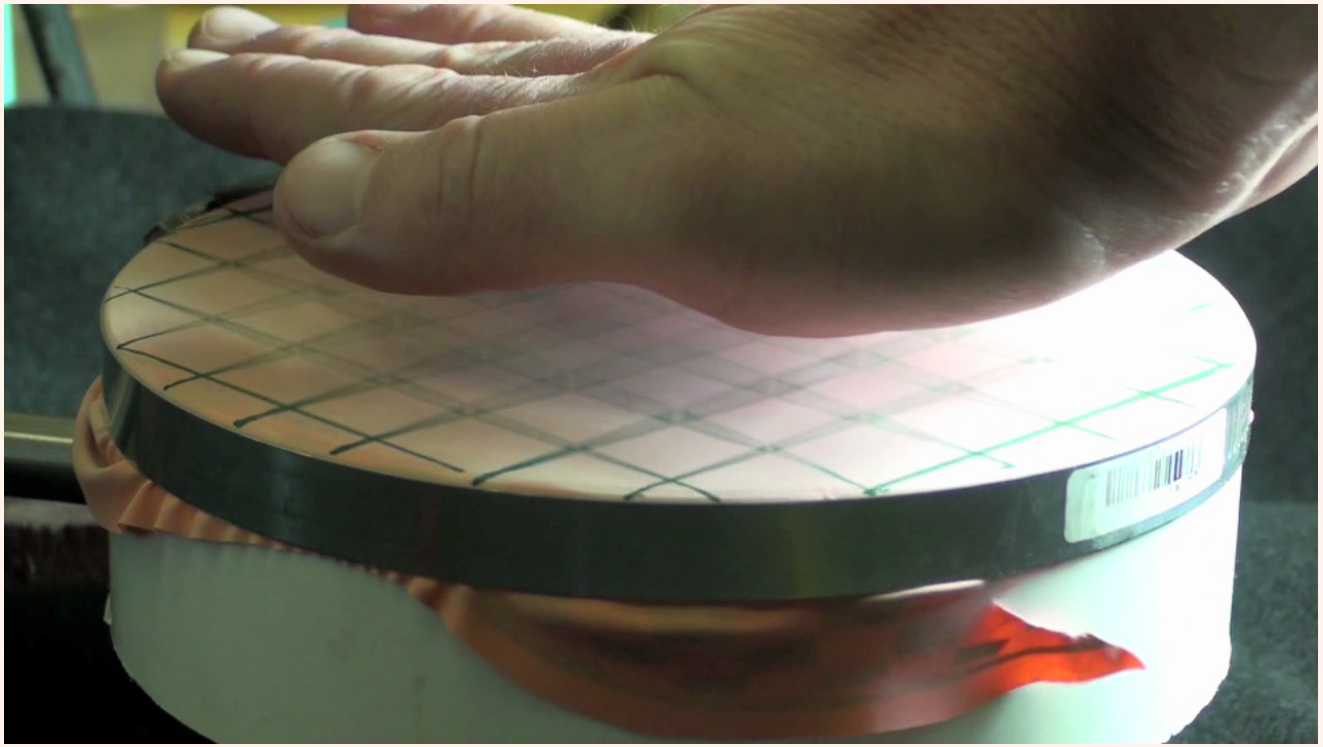


Java based simulation by found Paul Falstad

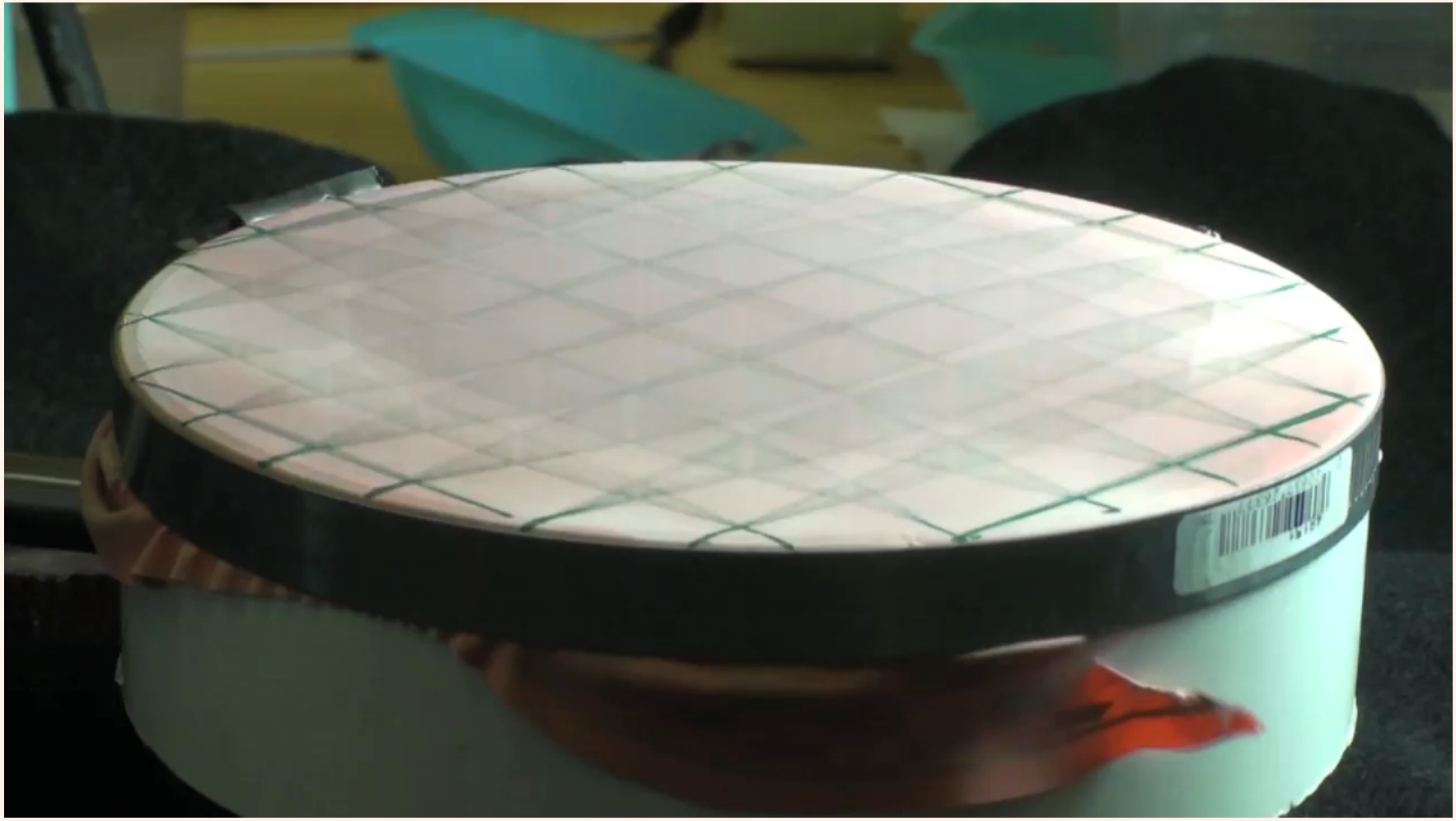
http://www.falstad.com/circosc/.

Changing boundary conditions

- Placing a hand near (but not touching) the vibrating surface shifts the natural frequency.
- If resonance is linear, removing your hand returns the system to the original state.



Linear Resonance: Mode (0,1)



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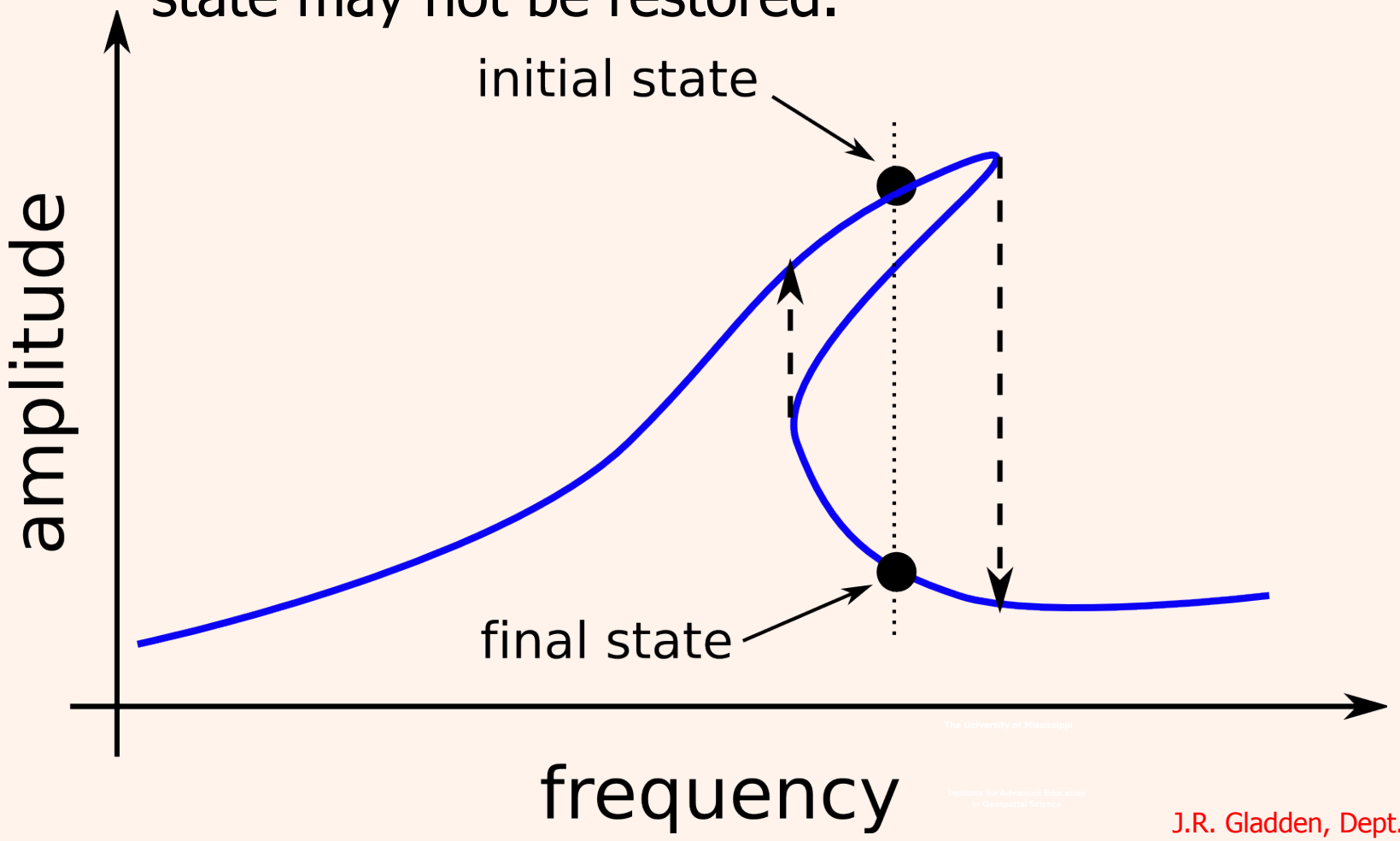
Non-linear case: Mode(1,1)

- Resonance peak is double valued.
- Once the peak is shifted, the same resonance state may not be restored.



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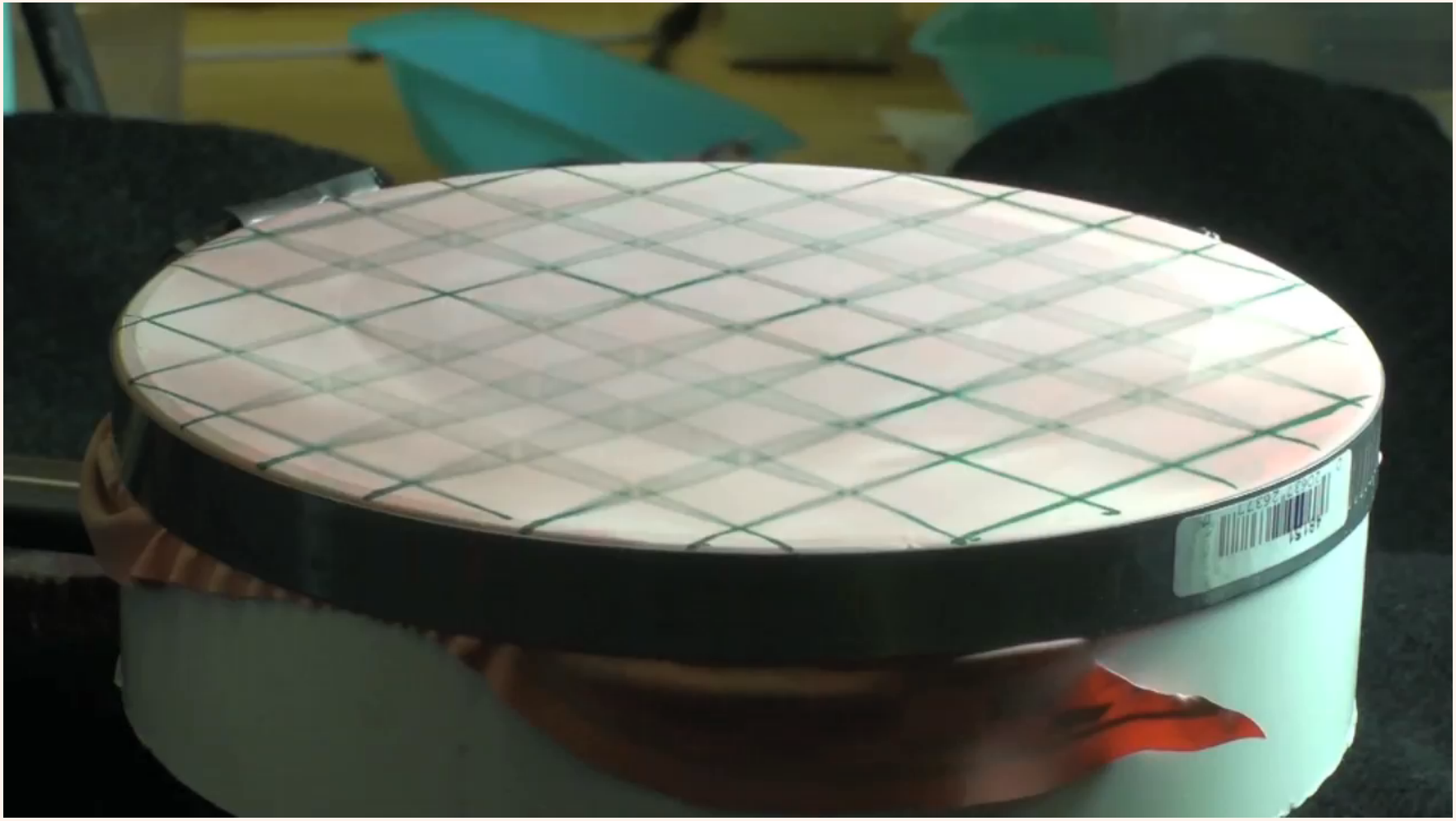
Non-linear Resonance: Mode (1,1)



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Follow Up

- These slides, videos, and links to the external sites will be posted on my website in a few days:

www.phy.olemiss.edu/~jgladden/

Thanks for your attention!