Project Listen Up: Drumhead resonances in circular elastic membranes -- on a budget!

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Project Listen Up Session
**Major Goals**

- Design a simple, inexpensive, appealing, and educational demonstration of resonances in two dimensions.
- Use materials easily obtained by a smaller department or high school teacher with limited resources.
- Nice physical manifestation of the mathematics.
Final Product – top view
Final Product – bottom view

Retask those old Whoopee cushions!

8” speaker
The set up

Whoopie cushion cut in half

9" PVC pipe

6" speaker

function generator

Radio Shack amplifier
Theory

- Circular membrane of radius $a$ and transverse wave speed $c$
- 2D wave equation in polar coordinates.

$$\frac{\partial^2 \Psi}{\partial r^2} + \frac{1}{r} \frac{\partial \Psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \Psi}{\partial \theta^2} = \frac{1}{c^2} \frac{\partial^2 \Psi}{\partial t^2}$$

- Separable PDE: $e^{i\omega t}$ time dependence and spatial solutions:

$$\psi(r, \theta) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} A_{nm} J_n(\frac{\lambda_{nm} r}{a})(B_n \cos n\theta) e^{i\omega t}$$
Modes

mode 0,1  
mode 0,2  
mode 0,3  
mode 1,1  
mode 2,1  
mode 2,2
Animate with Mathematica

Code can be downloaded from Wolfram Demonstrations Project:
http://demonstrations.wolfram.com/NormalModesOfACircularDrumHead/

J.R. Gladden, Dept. of Physics
High Speed Video
mode 1,1
# Cost in blood and treasure

<table>
<thead>
<tr>
<th>Item</th>
<th>Source</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whoopie cushion</td>
<td>Walmart</td>
<td>$1.00</td>
</tr>
<tr>
<td>Hose Clamps</td>
<td>Walmart</td>
<td>$1.00</td>
</tr>
<tr>
<td>PVC pipe</td>
<td>scrap from machine shop</td>
<td>free</td>
</tr>
<tr>
<td>Speaker</td>
<td>Dr. Raspett’s lab</td>
<td>free ($24 @ Radio Shack)</td>
</tr>
<tr>
<td>function generator</td>
<td>lab (common even in high schools)</td>
<td>should already have</td>
</tr>
<tr>
<td>amplifier</td>
<td>lab (not absolutely necessary)</td>
<td>~$45</td>
</tr>
<tr>
<td>time</td>
<td></td>
<td>about 4 hours</td>
</tr>
</tbody>
</table>
Another good simulation

Java based simulation by found Paul Falstad

http://www.falstad.com/circosc/.
Changing boundary conditions

- Placing a hand near (but not touching) the vibrating surface shifts the natural frequency.
- If resonance is linear, removing your hand returns the system to the original state.
Linear Resonance: Mode (0,1)
Non-linear case: Mode(1,1)

- Resonance peak is double valued.
- Once the peak is shifted, the same resonance state may not be restored.
Non-linear Resonance: Mode (1,1)
Follow Up

- These slides, videos, and links to the external sites will be posted on my website in a few days:

  www.phy.olemiss.edu/~jgladden/

  Thanks for your attention!