SCIENTIFIC COMPUTING: LECTURE 11

- Numerical Integration
  - Background
  - Trapezoid
  - Simpson's 1/3 rule
  - Other methods: Monte Carlo

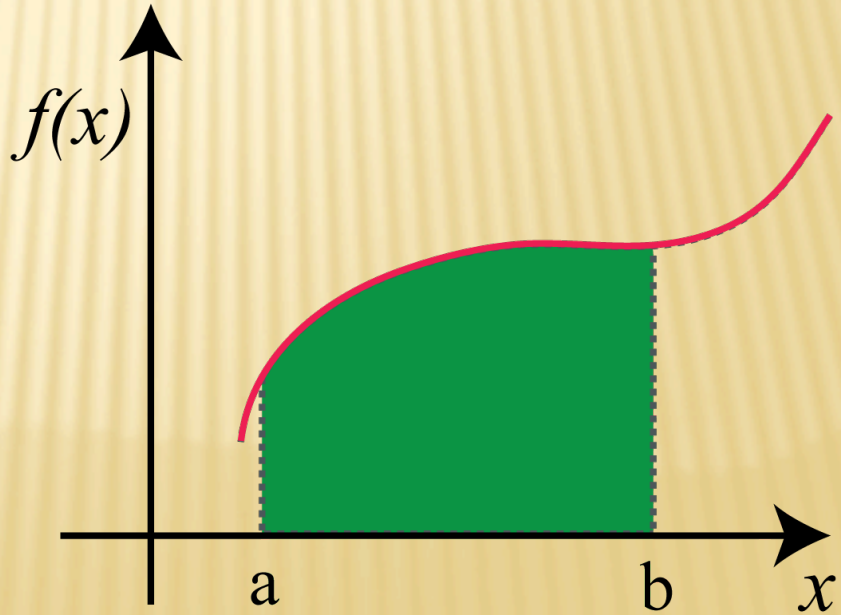
CLASS NOTES

- HW#4 due today.
- You should be reading in Appendix A.3

INTEGRATION – SOME REMINDERS…

- Integration is the inverse operation to differentiation.
- Integral of a function is a measure of the "area" bounded by the function and the horizontal axis.
- Many analytic functions can integrated analytically to get a closed form solution.
- Area above axis is (+), area below axis is (-).

$$\int_a^b f(x)dx = \text{area}$$



SOME EXACT SOLUTIONS

$$\int x^m dx = \frac{1}{m+1} x^{m+1} \qquad \int \frac{1}{x} dx = \ln(x)$$

$$\int sin(x) dx = -cos(x)$$

$$\int_0^{\pi} sin(x) dx = 2.00$$

$$\int_a^b x^m dx = \left[\frac{1}{m+1} x^{m+1} \right]_a^b$$

NUMERICAL APPROXIMATION

- On a computer, integration becomes summation.
- Numerical integration is also called *quadrature*.
- Break integral range into multiple "panels" with a simple shape.
- Add areas of all panels to get total area.
- Newton-Cotes Method: top of each panel defined by a polynomial of order n.
  - n=1: line - Trapeziod method
  - n=2: parabola – Simpson's 1/3
  - n=3: cubic – Simpson's 3/8
- For many applications, Trapezoid method is good enough.

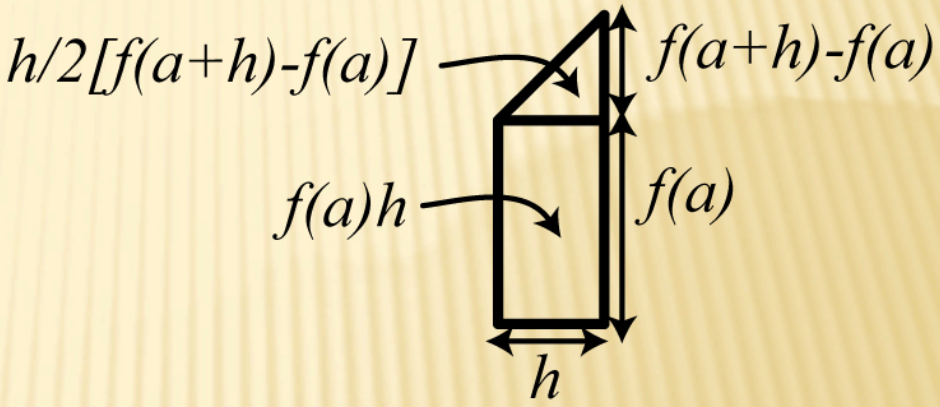
TRAPEZOID METHOD

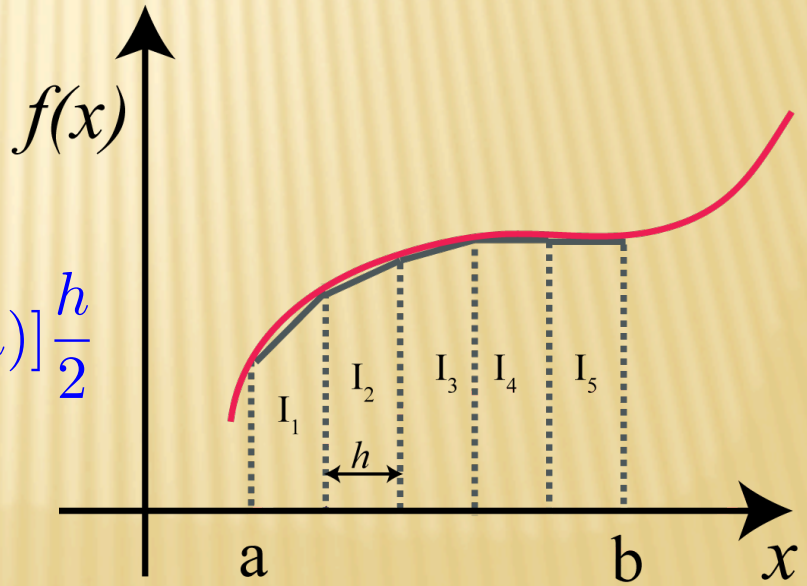
- Geometry of panels is trapezoidal.
- Area of a trapezoid:

$$[f(a+h)+f(a)]\frac{h}{2}$$

- Total area for N panels:

$$\begin{aligned} I &= I_1 + I_2 + I_3 + \ldots + I_N \\ &= \sum_{i=1}^{N-1} [f(a+ih)+f(a+(i+1)h)]\frac{h}{2} \end{aligned}$$





TRAPEZOID METHOD – THE CODE

```
def trap(f,xmin,xmax,numPanels):
        Isum=0.
        h=(xmax-xmin)/float(numPanels)
        for i in range(numPanels):
                x=xmin+i*h
                Isum+=(f(x+h)+f(x))*h/2.
        return Isum
```

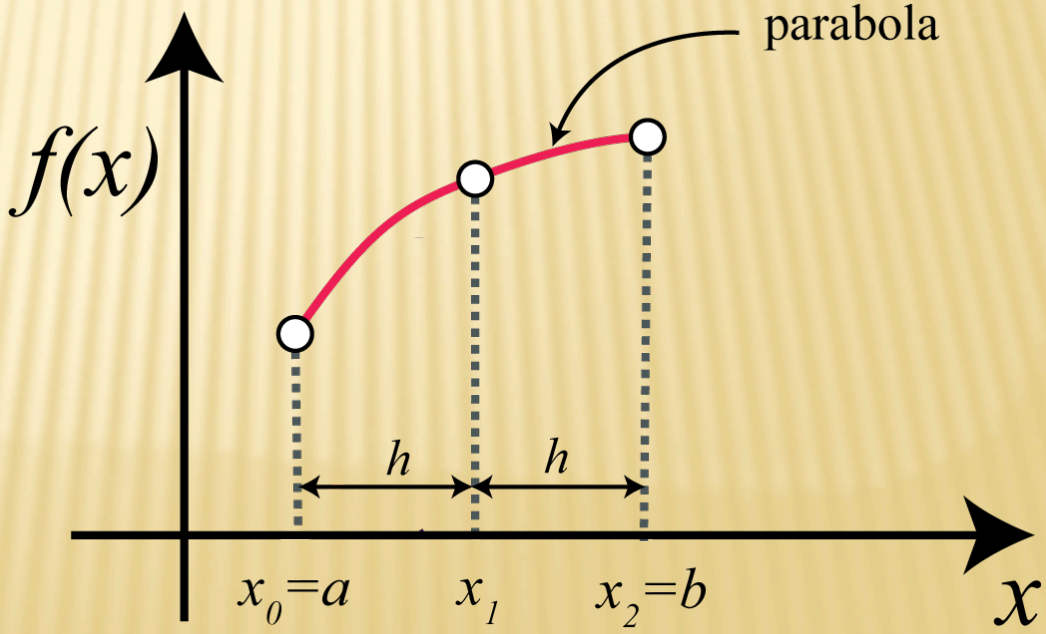
- Trapezoid error is of order $\sim h^2$
- Trapezoid method is can be coupled with Richardson extrapolation to improve error further
  - known as the Romberg method

SIMPSON'S 1/3 RULE

- Panel cap is a quadratic curve rather than a straight line.

$$\int_a^b f(x)dx \simeq \left[f(a) + 4f(\frac{a+b}{2}) + f(b)\right]\frac{h}{3}$$

- Requires evaluation at 3 points rather than 2.
- Error is of order ~h^4.



SIMPSON'S 1/3 CODE

```
def simp13(f,xmin,xmax,numPanels):
        Isum=0.
        h=(xmax-xmin)/float(numPanels)
        for i in range(0,numPanels,2):
                x=xmin+i*h
                Isum+=(f(x)+4*f(x+h)+f(x+2*h))*h/3
        return Isum
```

EXERCISE: TRAPEZOID ERROR

- Use the trapezoid method to estimate the integral:

$$\int_0^1 e^{-x} dx = -[e^{-1} - e^0] = 0.632120558829$$

- Compute and plot the % error vs number of panels.