Relativistic gravity, compact objects and strong-field gravity

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Abstract

This is a semi-technical presentation of the succession of events that led Einstein to formulate his theory of general relativity. We present an overview of the field of relativistic astrophysics and motivate the study of compact objects (such as neutrons stars and black holes) as strong-field gravity probes for testing possible modifications to general relativity.

1 A PANORAMA OF PHYSICS BEFORE 1915

General relativity is the theory of space, time and gravity, formulated in final form by Albert Einstein in 1915. The theory superseded the gravitational theory of Isaac Newton, formulated in the mid-17th century, which had so accurately explained gravitational phenomena on a vast scale, from the falling objects on Earth to the motion of celestial bodies in the sky. Newtonian gravity is based on the idea that any two bodies in the Universe attract each other with a force whose magnitude is proportional to the product of their masses and inversely proportional to their distance squared. Together with Newton's three laws of motion and the equality between inertial and passive gravitational masses[1], Newtonian gravity developed greatly through 17th and 19th centuries [24] culminating in Laplace's *Mecánique Céleste* (published between 1798 and 1827), the prediction of a new trans-uranic planet by Le Verrier, and the subsequent observational confirmation of what became known as Neptune by Galle in 1846.

By the end of the 19th century, physics found itself in a privileged position among sciences, for it was thought that all of its fundamental aspects had been understood. At this time, the motion and dynamics of bodies could be formulated in the Newtonian mechanics framework. Electromagnetic phenomena (including the description of light) could be understood through Maxwell's electrodynamics, and the study of heat and temperature could be formulated within the theory of thermodynamics and kinetic theory of gases. This set of theories is what we now call classical physics [22]. Nevertheless, in 1900 Lord Kelvin pointed out that *"the beauty and clearness of the dynamical theory, which asserts heat and light to be modes of motion, is at present obscured by two clouds [...]"* [11]. The two clouds were (*i*) the failure in detecting the luminiferous ether, the hypothetical supporting medium for light propagation, and (*ii*) the correct description of the blackbody radiation which thermodynamical and electromagnetic considerations failed to explain. As we now know, the blackbody radiation led Planck (1900) to introduce the idea that energy is *quantized*, which paved the road for the development of quantum mechanics. The absence of the ether is tightly related with Einstein's 1905 special theory of relativity. If an ether existed, in principle it should be possible to detect small changes in the travel time of light using an interferometer as Earth orbited the Sun. This detection never happened. The failure in measuring any such modifications in the experiments by Michelson and Morley provided evidence for the constancy of the speed of light, as *postulated* in special relativity. The special theory relativity has as a second postulate: the equivalence of the laws of physics for any two inertial reference frames. This notion of a *covariance* of the equations of physics demanded a relativistic extension of Newtonian mechanics to a relativistic mechanics[2], which must be used to correctly describe the dynamics of bodies at velocities comparable to the speed of light.

[1] The inertial mass of an object quantifies its resistance to be accelerate when acted upon by a force. The passive gravitational mass quantifies the gravitational force felt by an object in presence of another. The equality between these two masses is known as the *weak equivalence principle* and has been verified experimentally with very high precision.

[2] Maxwell's electromagnetism was *already* invariant under the set of transformations relating to inertial observers known as the *Lorentz transformations*. The laws of thermodynamics are also invariant. The application of thermodynamics to relativistic systems was developed by Einstein and Planck in 1907-1908 [25]. This is *not* a straightfoward task however, see [4].

The special theory of relativity caused a paradigm shift in our understanding of space and time. In classical physics, space and time are absolute notions, in the sense that, for instance, any two observers, irrespectively of their motion, would measure the flow of time in the same manner. In the new theory of special relativity, the postulates force us to give up the idea of an absolute time, and notions as fundamental as simultaneity and distances become reference dependent. It was soon realized in the works by Einstein and Minkowski that special relativity is much simplified if described using a single notion of *spacetime*. In this setting it is not the distance between two points or the time interval between two events which are independent of the state of motion of an observer with respect to another, but the *spacetime* interval which it is[3].

2 General relativity and beyond

In the years following 1905, Einstein embarked on an effort to formulate a relativistic theory of gravity. As early as 1907, he had already introduced the core idea behind what would later become general relativity. In a 1907 paper, in addition to assuming the weak equivalence principle, Einstein argues that given two system, one in a static uniform gravitational field and one in uniform accelerated motion with respect to the first, one *cannot* use the laws of physics to distinguish them. He writes that *"we shall therefore assume complete physical equivalence between the gravitational field and the corresponding acceleration of the reference system."*[4]. From this assumption it follows that time flows faster in a in gravitational potential. This effect is known as gravitational redshift. Morever, light would be deflected by the gravitational field of a massive body. During this period Einstein's attention was much more focused on the fast development of quantum mechanics. It would not be until 1911 that he would turn his efforts back to the development of a relativistic theory of gravity. This led up to a series of four papers [14] in November 1915. The last of these paper contains the final formulation of general relativity.

An extension of Einstein's 1907 insights on the connection between accelerated motion and gravitational fields can be stated in modern form as follows [17]: no measurement, gravitational or not, carried out in a sufficiently small freely-falling laboratory (i.e in the absence of any non-gravitational force) in a gravitational field can reveal the existence of gravity *locally*, within the confines of the laboratory. A crucial consequence of this *Einstein equivalence principle*[5] is that a gravitational field can locally be turned off and all laws of physics, as described by special relativity, must hold. We are inevitably led to conclude that the inclusion of gravity tells us that the inertial reference frame of special relativity is that which is freely falling in a gravitational field.

Consider now that we have two experimentalists inside sufficiently small laboratories in free fall in the Earth's gravitational field. Within the limits of their laboratories Einstein's equivalence principle holds, and they won't be able to detect the gravitational field. Now, if one considers the relative distance between the two free-falling laboratories, one can show that the separation between them is proportional to the gravitational field, to its *derivatives*. The outcome of this is that while we can locally turn off gravity invoking Einstein's equivalence principle, globally, the non-homogeneities of the gravitational field cannot be turned off.

The connection between Einstein's equivalence principle and special relativity together with the situation described above has far-reaching consequences. Since the crucial element in the Einstein-Minkowski formulation of special relativity is the existence of a metric tensor $\eta_{\mu\nu}$, which tells us how to measure distances is spacetime, we can formulate gravity in terms of a general metric tensor $g_{\mu\nu}$, which for sufficiently small scales reduces to $\eta_{\mu\nu}$. In other words, relativistic gravity must be described by a metric theory. Because gravity manifests itself globally, through its non-homogeneities, to describe gravity we must allow $g_{\mu\nu}$ to vary on sufficiently large scales. Since $g_{\mu\nu}$ tells us how to measure distances in spacetime, we conclude that gravity is a theory of *curved* spacetime. The notion of free-fall a gravitational field is elegantly generalized in general relativity by saying that free-falling observers follow *geodesics* in a curved spacetime.

From special relativity we learned that mass - the source of gravity in the Newtonian theory - and energy are equivalent (as stated in the emblematic $E = mc^2$). Therefore, in general relativity, we must allow energy to generate gravitational fields as well. The connection between curved spacetime and matter-energy is summarized

[3]Technically we define this interval as $\Delta s^2 = \eta_{\mu\nu}\Delta x^\mu \Delta x^\nu$, where $\eta_{\mu\nu}$ is known as Minkowski's metric and Δx^μ are displacements in the four-dimensional spacetime. This relation can be thought as a Pythagorean theorem for spacetime.

[4]We must be careful at this point. Einstein's argument cannot be applied to the real world because in Newtonian theory, a uniform gravitational field can only be produced by an unphysical infinite flat sheet of uniformly distributed matter. See discussion in [17]. The spacetime describing a uniformly accelerating observer in special relativity is known as Rindler spacetime.

[5]Relaxing one or other ingredients of the principle lead weaker versions of it. All relativistic theories of gravity *do* obey at least the weak equivalence principle, already stated.

in *Einstein's field equations*

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}, \tag{1}$$

where $g_{\mu\nu}$ is the spacetime metric, which, again, can locally be set equal to $\eta_{\mu\nu}$. On the other hand, $R_{\mu\nu}$ and R - known as the Ricci tensor and scalar, respectively - carry information about spacetime *curvature* (indeed these quantities are defined in terms of derivatives of $g_{\mu\nu}$). The information about the matter content present in the spacetime is encoded in $T_{\mu\nu}$, the "energy-momentum tensor". The constants G and c are respectively Newton's universal gravitational constant and c the speed of light in vacuum.

General relativity had two immediate triumphs. The first was in providing an explanation of the precession of the perihelion of Mercury. Despite the successes of Newtonian gravity in describing the motion of astronomical bodies, there was accumulating evidence of an unexplained anomaly in the motion of Mercury. In Newtonian gravity, the bound motion of point particles is described by ellipses[6]. The influence of the other planets however, makes the location of the perihelion precess at a slow rate. Even when the influence of all other planet was taken into account, there was a residual precession rate of 42.7 arc s/century which remained unexplained. Einstein showed that approximating his field equations for weak gravitational fields (as the Sun's) and slow-velocity motions (as that of Mercury's orbit), general relativity predicted *exactly* the same mysterious precession rate. A second success confirmation was the of the deflection of light by massive bodies. The deflection of light causes an apparent shift in the position of stars as they fall behind the Sun. This effect was observed by Eddington and his team in the 1919 solar eclipse, using observations made in Sobral (Brazil), and in São Tomé and Princípe, close to the west African coast. The measurement of the gravitational redshift proved to be a much harder task, and it was not achieved until 1960 by the Pound-Rebka experiment.

For a long time it was generally accepted that general relativity would have little or neglible effects in astrophysics. This scenario started to change in the 1960 in a quick succession of events (narrated in [31]). Important for us is: (*i*) the discovery of the first *quasar*, a small region of spacetime sourcing the emission of a very large amount of energy through electromagnetic radiation, and (*ii*) the development of a competing theory to general relativity (Brans-Dicke theory [3]). The first discovery led astrophysicists to consider general relativity to explain quasars. At present, the outpouring of energy from quasars is understood as sourced by the accretion of matter by supermassive black holes. Brans-Dicke theory showed that it was possible to formulate alternatives to general relativity that were compatible (at the time) with available experimental tests of relativistic gravity. Shortly after the discovery of the first quasar, in 1967 Bell and Hewish, discovered the first pulsar, a magnetized rotating neutron star[7]. This class of stars were first proposed by Baade and Zwicky in 1934. Estimates of their maximum mass revealed that these object are very dense. The strong gravitational field of these stars require the use general relativity in their description [16].

It quickly became clear that relativistic gravity *does* play an important role in the description of certain systems in Universe, and the field of *relativistic astrophysics* was born. In this context it is natural to ask whether general relativity is the correct description of relativistic gravity. The following decades witnessed a flourishing of precision tests of relativistic gravity, made possible by technological advances and theoretical progress in the understanding of metric theories of gravity, epitomized by the parametrized post-Newtonian formalism [31, 15]. Over the years we have seen, over and over again, general relativity passing with flying colors all experimental tests [30]. However, all of these tests probe gravity in the weak-field regime [2], with the possible exception of compact binary systems [8] [29].

At present there are compelling reasons to believe that general relativity must be modified in both its low- and large-energy limits [2]. In the infrared (low-energy) limit, cosmology tells us that a large fraction of the energy content of the Universe is due to dark matter (accounting for e.g, the "missing mass" in galactic systems) and dark energy (responsible for the small cosmological constant that makes the Universe expand at an accelerating rate). Attempts have been made to explain these entities in terms of modifications of general relativity. In the ultraviolet

[6] A famous result in classical mechanics, Bertrand's theorem [21], says that the only two forces for which bound orbits exists are either proportional to the distance r (as for Hookian spring) or inversely proportional to r^2, i.e $-1/r^2$, as the Newtonian gravitational force.

[7] The combined effect of magnetic field and rotation causes these neutron stars to emit radiation from their magnetic poles. This signal is detectable by telescopes only when the radiation is pointed towards Earth. Since the star rotates, the signal appears and fades periodically, much like as if we were facing a light-house, hence the name pulsar.

[8] Binary systems consist of a pair of objects (stars and/or black holes) orbiting each other. General relativity, and all metric theories of gravity for that matter, predict the emission of gravitational radiation from these systems. The energy lost in the process translates into a shrinking of the system's orbit, which can be measured with great precision. The study of binary systems provides us with an indirect verification of the existence of gravitational waves. The agreement between the general relativistic predictions and the observational data from PSR B1913+16 (a double neutron star system) resulted in the Nobel Prize won by Hulse and Taylor in 1993.

(high-energy) limit, general relativity is non-renormalizable; this problem can be cured by high-energy corrections to the theory. Additional terms to general relativity appear, for instance, in the low-energy limit of string theory.

3 COMPACT OBJECTS AS STRONG-GRAVITY PROBES

As we have discussed there are classes of objects in our Universe whose description requires relativistic gravity. We elaborate more on neutron stars and black holes next.

3.1 NEUTRON STARS

Neutron stars [7] are the remmants supernovae, the most energetic events in the universe since the Big Bang. They are extremely compact and dense stars, with a typical mass of $M = 1.4\,M_\odot$ ($M_\odot \approx 2 \times 10^{30}$ kg denotes the Sun's mass) packed in a small sphere of merely 10 km. Moreover, neutron stars known as magnetars can harbor ultra-strong magnetic fields [26], about 10^{13} stronger than the Earth's magnetic field. Others, known as pulsars, can spin incredibly fast with periods of revolutions of the order of a dozen miliseconds [13]. In their interior, gravity is so strong and matter has such high densities (supranuclear) that these conditions are not reproducible by any terrestrial experiment. In fact, one of the greatest uncertainties in the description of these object is in the *equation of state* of matter: we do not know with certainty the correct description of matter at the supranuclear densities in the interior of neutron stars. This translates into uncertainties in the bulk properties of neutron stars, such as the mass and radius. On the gravitational side, neutron stars are relativistic objects requiring the use of general relativity in their description. It is therefore natural to ask if neutron stars can carry observable imprints of gravitational theories beyond general relativity.

In summary, these celestial bodies provide us with a unique astrophysical laboratory for testing our understanding of fundamental aspects of physics. Future Earth and space-based experiments such as the SKA [28], NICER [1], LOFT [6] and AXTAR [20] will open a new era of precision measurements of neutron star properties and thereby allow us to test fundamental physics.

3.2 BLACK HOLES

One of the most surprising consequences of general relativity is the prediction of the existence of objects whose gravity is so strong that even light can not escape from their attraction. They are characterized by the presence of an event horizon, a no-return boundary in spacetime. These objects called black holes by Wheeler, are poetically described by Chandrasekhar [5] as "*the most perfect macroscopic objects there are in the Universe*" for "*the only elements in their construction are our concepts of space and time*". Black holes are not only perfect, in the sense described by Chandrasekhar, but they are also the simplest objects, since the uniqueness theorems developed during the 70s – the so-called "Golden Age" of relativity – state that the most general black hole solution in vacuum in general relativity is completely described by only two parameters: their mass M and its angular momentum J. This solution, obtained in an heroic effort [23] by Kerr [12], is expected to describe astrophysical black holes, such as Sagittarius A* at the center of our galaxy. The uniqueness theorem led to what became known as the *no-hair* conjecture, which states that the final outcome of gravitational collapse must be a Kerr black hole.

Black hole are also predicted in many gravitational theories which contain extra scalar degrees of freedom (such as in Brans-Dicke theory [8]) and under certain assumptions, the no-hair conjecture: black hole solutions in these theories are the same as in general relativity. Black hole solutions with some type of hair can be constructed either by relaxing some the theorem's assumptions, or by considering different types of fields [9].

Black holes as predicted in some modified theories of gravity will differ from the Kerr solution and therefore could lead to potentially observable effects in different astrophysical scenarios. It is expected that within the few next years we will be able to study with unprecedent levels (e.g. with the Event Horizon Telescope (EHT)) our galaxy's very own black hole Therefore, it is of great relevance to investigate these objects in modified theories of gravity [27] and to confront their predictions against observations [18, 10, 19].

4 Conclusions

In summary, the next generation of astrophysical observatories will allows us for the first time to probe the strong-field limit of Einstein's theory of general relativity. In some sense, the situation is similar to that of the 60s and

70s, where technology allowed us for the first time to do precision measurements of relativistic effects within the confines of our Solar System.

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