

Home Work Set Chapter 7 (part 1)

1) A quantum rotator can be described by $H = A L^2 + B L_z$. $A = 5 \text{ eV} / \hbar$ and $B = 1 \text{ eV} / \hbar$. Draw an energy level diagram for the first 3 energy levels, $\ell = 1, 2, 3$

2) Find the degeneracy in the 1st 4 Energy levels of the Particle in a 3d box.

3) The Laguerre Polynomials are an Orthonormal Set of eigenfunctions (like i,j,k unit vectors).

$$\text{Show that } \int_0^\infty R_{10} R_{10} r^2 dr = 1 \text{ and } \int_0^\infty R_{10} R_{20} r^2 dr = 0$$

4) An electron is in the 2p atomic state $\psi_{2p} = A(\psi_{200} + i\psi_{210} - \sqrt{2}\psi_{21-1} + \sqrt{2}\psi_{21+1})$

(a) Determine the normalization constant A . (hint: $\psi_{2p}^* \psi_{2p} = 1$)

(b) What is the average energy of this state

$$E = \langle \psi_{2p}^* H \psi_{2p} \rangle \quad \text{e.g. } \langle \psi_{100}^* H \psi_{100} \rangle = -13.6 \text{ eV} ?$$

(c) What is the average $\langle L^2 \rangle$ and $\langle L_z \rangle$ of this state?

5) See the Reduced Mass attachment. Make the substitution and derive the Lagrangian ($L = T - V$) in terms of the reduced mass for this two body to one body transform. (similar for Hamiltonian $H = T + V$)

6) The form of the radial wave function for two orthonormal states are

$R_{10}(r) = A e^{-r/a}$ and $R_{20}(r) = B e^{-r/a} (1 - br) e^{-r/2a}$. Can we determine A, B, and b? Give the concise argument.