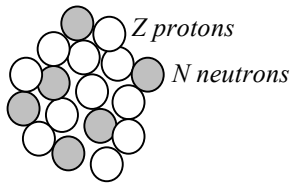
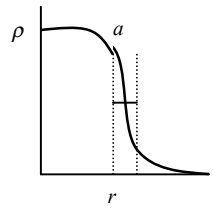
Topics in Nuclear Physics

Nuclear Radius

The atomic nucleus is made of an aggregate of protons and neutron (nucleons) which are bound together by the strong nuclear force. It is generally spherical in shape but can be deformed. The atomic mass number $A=N+Z$ refers to the number of neutrons N and protons Z. The atomic weight of the element is the average weight including isotopic fractions.





$$\rho(r) = \frac{\rho_0}{1+e^{\frac{r-R}{a}}}$$

$\rho_0 = 0.165 N/f^3$ *mean nuclear density*

a = 0.65fm *nuclear skin depth*

$r = 1.25 A^{1/3}\, fm$ *nuclear radius*

If we assume that the nuclear volume $4/3\pi R^3 \sim A$ is proportional to the atomic mass number, the nuclear radius r_A can then be described by the relation $r_0 = 1.3 \times 10^{-13}\ cm$

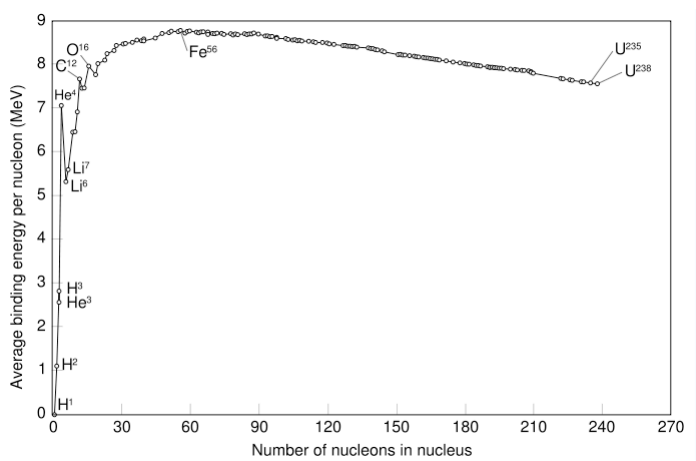
$$r_A = r_0 A^{1/3}$$

Binding Energy

The mass of a nuclei is given by the sum of constituents minus the binding energy.

$$m_X c^2 = Z\ m_p c^2 + N\ m_n c^2\ - E_B$$

The binding energy per nucleus as a function of atomic mass number A is displayed below. The average binding energy per nucleus is about $BE/A \sim 8 MeV/N$.



Fe-56 is the most stable element and unstable nuclei above and below Fe-56 tend to decay decay down to Fe-56 by (1) **fissioning** and (2) **alpha-beta decays chains**.

The **Semi-empirical Mass Formula** is a parameterization of the binding energy curve in terms of a volume, surface, Coulomb, n/p asymmetry, and pairing terms. You can read about these elsewhere. (http://en.wikipedia.org/wiki/Semi-empirical_mass_formula)

$$E_B = \underset{\substack{term\\ 15.75 MeV}}{a_{volume}}\ A - \underset{\substack{term\\ 17.8 MeV}}{a_{surface}}\ A^{2/3} - \underset{\substack{term\\ 0.711 MeV}}{a_{Coulomb}}\ \frac{Z(Z-1)}{A^{1/3}} - \underset{\substack{assymetry\\ term\\ 23.7 MeV}}{a_{n/p}}\ \frac{(A-2Z)^2}{A} + \delta(A.Z)_{\substack{pairing\\ term}}$$

$$\begin{array}{ll} 0, & A\ odd \\ +11.18 MeV & Z,N\ odd\ A\ even \\ -11.18 MeV & Z,N\ even\ A\ even \end{array}$$

Nuclear Potential Well depth

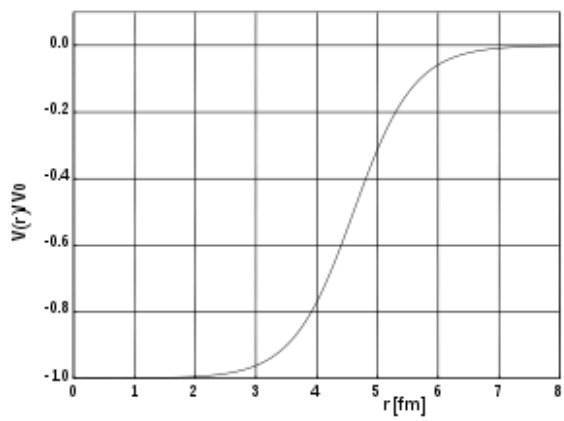
In a mean field approach,(a nucleon is acted upon by all others) we can assume the nuclear potential is proportional to the nuclear density. The Woods-Saxon potential nuclear *V(r)* is given below: https://en.wikipedia.org/wiki/Woods%E2%80%93Saxon_potential

$$\rho(r) = \frac{\rho_0}{1+e^{\frac{r-R}{a}}} \quad \Rightarrow V(r) = \frac{-V_0}{1+e^{\frac{r-R}{a}}}$$

$V_0 \simeq$ -50 *MeV* *mean well depth*

$a \simeq 0.50\,fm$ *nuclear skin depth*

$R = 1.25A^{1/3}\,fm$ *nuclear radius*



Alpha, Beta, Gamma Decay

Nuclei change states by the fundamental processes of α,β,γ decay, where parent nucleus *P* decays to daughter *D*. All decays conserve charge and mass number.

$P_Z^A \rightarrow D_{Z-2}^{A-4} + \alpha_2^4$ alpha decay

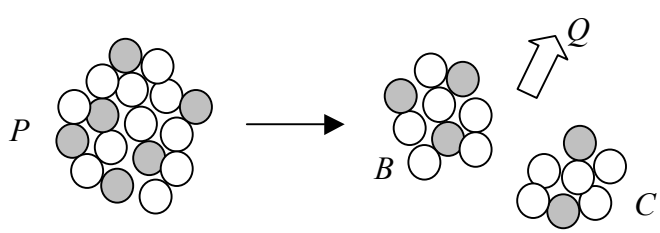
$P_Z^A \rightarrow D_{Z+1}^{A} + e_{-1}^0 + \bar{\nu}_e$ beta decay

$P_Z^{A*} \rightarrow P_Z^A + \gamma_0^0$ gamma

Alpha particles will tunnel through the Coulomb barrier with small but finite tunneling probability $\sim|\Psi out|^2$, where Ψout is the outside wave function. The tunnel rate $R = |\Psi out|^2 \, x \, f$ is enhanced by the enormous trial frequency, *f,* with which the α attempts to escape.

Spontaneous Fission

Many nuclei are unstable. They break in to parts releasing enormous binding energy given by Einstein's equation. *Q* is the energy released in the reaction.



$${}^{A}P_Z \rightarrow {}^{A_1}B_{Z_1} + {}^{A_2}C_{Z_2} \qquad A = A_1 + A_2, \quad Z = Z_1 + Z_2$$

$$M({}^{A}P_Z)c^2 = M({}^{A}B_Z)c^2 \; + \; M({}^{A}C_Z)c^2 + Q$$

$$Q = [M({}^{A}P_Z) - M({}^{A}B_Z) - M({}^{A}C_Z)]c^2$$

Uranium, thorium, neptunium, and plutonium, undergo **spontaneous fission**s involving long decay chains. The decays proceed through α,β,γ emissions.

Decay Chains

Consider Np-237 decaying to Bi-209 $\quad {}^{237}_{93}Np \rightarrow {}^{209}_{83}Bi + n\alpha^{4}_{2} \; + \; m\beta^{0}_{-1}$

Fission will take place through the fundamental processes of alpha, beta, and gamma decays.
How many alpha and beta decays occur? Conservation of atomic mass number A and charge Z give:

$$237 - 209 = 28 = 4n \;\; \Rightarrow \;\; n = 7$$
$$93 - 83 = 10 = \;\; 2n - m \;\; \Rightarrow \;\; m = 4$$

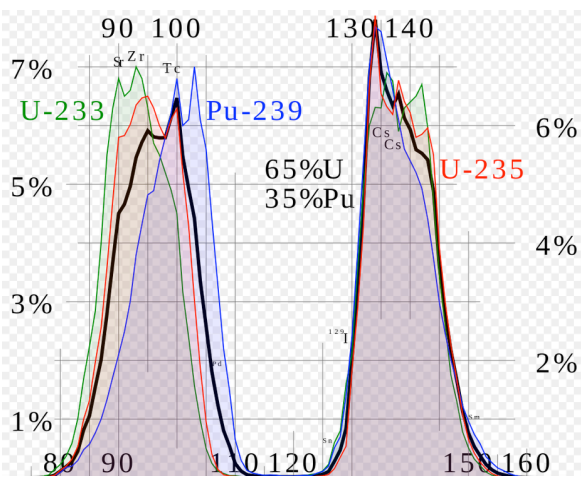
Induced fission is a form of nuclear reaction. Neutron bombardment of U-235 gives

$$n + U^{235} \rightarrow U^{236*} \rightarrow Kr^{92} + Ba^{141} + 3n$$

The $3n$ excess will have some probability of inducing secondary, tertiary, etc. fissions and then a **chain reaction** to produce large amounts of energy per gram of material.

The most common **nuclear fuels** U-235 and Pu-239 are alpha emitters and thus easily stored and handled. They are only dangerous if ingested! After fission the numerous decay products emit more dangerous gamma and beta radiations, creating high level nuclear waste which must be permanently stored away.

https://en.wikipedia.org/wiki/Fission_products_(by_element)#/media/File:ThermalFissionYield.svg



Isotope	Radiation	Half-life	GI absorption	Notes
Strontium-90/yttrium-90	β	28 years	30%	
Cesium-137	β,γ	30 years	100%	
Promethium-147	β	2.6 years	0.01%	
Cerium-144	β,γ	285 days	0.01%	
Ruthenium-106/rhodium-106	β,γ	1.0 years	0.03%	
Zirconium-95	β,γ	65 days	0.01%	
Strontium-89	β	51 days	30%	
Ruthenium-103	β,γ	39.7 days	0.03%	
Niobium-95	β,γ	35 days	0.01%	
Cerium-141	β,γ	33 days	0.01%	
Barium-140/lanthanum-140	β,γ	12.8 days	5%	
Iodine-131	β,γ	8.05 days	100%	
Tritium	β	13 years	100%	[a]

95+/-15 **135+/-15**

Fission product yields by mass for thermal neutron fission of U-235, Pu-239, a combination of the two typical of current nuclear power reactors, and U-233 used in the thorium cycle.

Liquid Drop Model

The model proposes that the nucleus is elastic like a liquid drop held together by the nuclear force. If the drop is agitated and made to vibrate by neutron bombardment, for example, it may split apart in to two parts releasing enormous energy Q. The semi-empirical mass formula was derived from such a model.

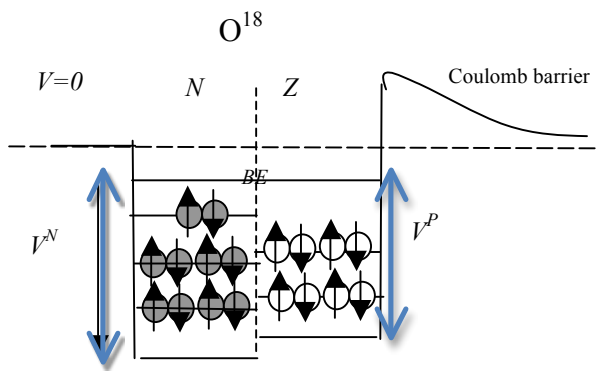
$$E_B = \underset{\substack{term \\ 15.75\,MeV}}{a_{volume}} A - \underset{\substack{term \\ 17.8\,MeV}}{a_{surface}} A^{2/3} - \underset{\substack{term \\ 0.711\,MeV}}{a_{Coulomb}} \frac{Z(Z-1)}{A^{1/3}} - \underset{\substack{assymetry \\ term \\ 23.7\,MeV}}{a_{n/p}} \frac{(A-2Z)^2}{A} + \underset{\substack{pairing \\ term}}{\delta(A.Z)} \begin{array}{ll} 0, & A \;\; odd \\ +11.18\,MeV & Z,N \;\; odd \;\; A \;\; even \\ -11.18\,MeV & Z,N \;\; even \;\; A \;\; even \end{array}$$

Shell Model

In the *Nuclear Shell Model* the nucleons are thought to lie in a potential well of depth *8MeV/A*:

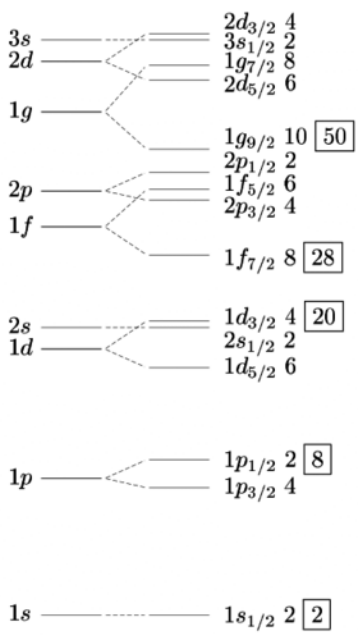
$$V^N(r) = -8MeV \times N \qquad V^P(r) = -8MeV \times Z + \sum_{ij} \frac{kq_i q_j}{r}$$

The model can account for many nuclear properties including nuclear energy levels and binding energy. It forms the basis of our quantitative understanding of the nucleus.



In the model the neutron and proton are paired by a **pairing force**. The protons feel an additional repulsive Coulomb force shifting their levels up**.** **Even-even nuclei** are the most tightly bound due to the nuclear pairing force. **Even-odd** the least stable. The nuclear shells fill up the nuclear well with strong spin-orbit forces engaged due to the close nuclear spacing. A typical shell model wiki-scheme is shown below. The filled shells with **2, 8, 20, 28,50, 82, 126** are called magic numbers and describe particularly stable nuclei with regard to *A* and *N*. He^4_2, Ca^{40}_{20}, Pb^{208}_{82} are good examples of very stable magic nuclei.

$^{208}Pb_{82}$ Z=82 N=126 is doubly magic!



<-- $^{40}Ca_{20}$ Z=20 N=20 magic

Energy released in Alpha, Beta, Gamma Reactions
Shifts in the nuclear structure can cause $\alpha,\ \beta,\ \gamma$ radiation to be emitted from the nucleus.

$$P_Z^A \rightarrow D_{Z-2}^{A-4} + \alpha_2^4 \qquad \text{alpha decay}$$
$$P_Z^A \rightarrow D_{Z\mp1}^{A} + \beta_{\pm1}^0 + \nu_0^0 \qquad \text{beta decay}$$
$$P_Z^{A*} \rightarrow D_Z^A + n\gamma_0^0 \qquad \text{gamma decay}$$

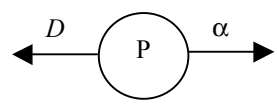
From conservation of energy in the parent P to daughter D decays

$$E_P = E_D + E_x \quad x = \alpha,\beta,\gamma$$
$$T_P + M_P c^2 = T_D + M_D c^2 + T_x + M_x c^2$$
$$Q = \left(M_P - M_D - M_x\right)c^2 = T_D + T_x - \underbrace{T_P}_{\sim 0\ at\ rest} > 0$$

where Q is defined as the free kinetic energy released in the decay. Notice Q is positive definite or the decay can not occur!

Alpha Decay In alpha decay the daughter nucleus and alpha particle share the excess kinetic energy given off in the decay.

$$Q = \left[\ M(P_Z^A) - M(D_{Z-2}^{A-4})\ - M(\alpha_2^4)\right]c^2$$

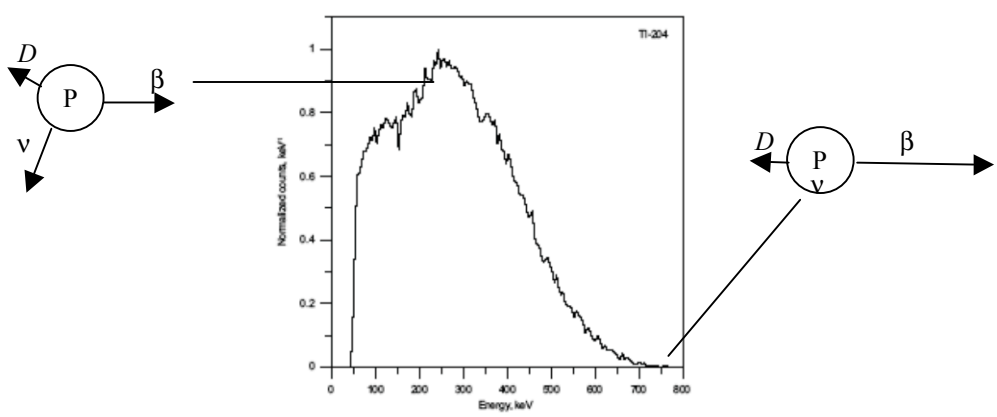


$$Q = KE_\alpha + KE_D \qquad \textit{conservation of energy}$$
$$|\ p_\alpha\ | = |\ p_D\ | \qquad \textit{conservation of momentum}$$
$$p_\alpha^{\ 2} / 2M_\alpha = p_D^{\ 2} / 2M_\alpha$$
$$KE_\alpha = (2M_D / 2M_D)p_D^{\ 2} / 2M_\alpha = \left(M_D / M_\alpha\right)KE_D$$
$$4KE_\alpha = (A-4)KE_D \qquad Q = KE_\alpha(1 + \frac{4}{A-4}) \ \Rightarrow\ KE_\alpha = \frac{A-4}{A}Q \qquad KE_D = \frac{4}{A}Q$$

Q

Beta Decay In beta decay the daughter , β particle, and neutrino share the excess kinetic energy given off in the decay. The β kinetic energy is thus not **monochromatic** (of single energy), but can take on a wide spectrum depending on angle.

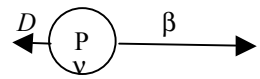
$$Q = \left[\ M(P_Z^A) - M(D_{Z-2}^{A-4}) - M(\beta) - \underbrace{M(\nu)}_{\sim 0}\ \right]c^2$$



1) The average value of the beta energy will be $\boxed{\bar{E}_e \sim Q/3}$.

2) In the special case when the neutrino is ~motionless the beta attains almost all the kinetic energy, we may use the 2-body decay ratio like above

$$Q_{\max} = KE_\beta \sim \left(\frac{M_D}{M_D + M_e}\right) Q \sim Q$$



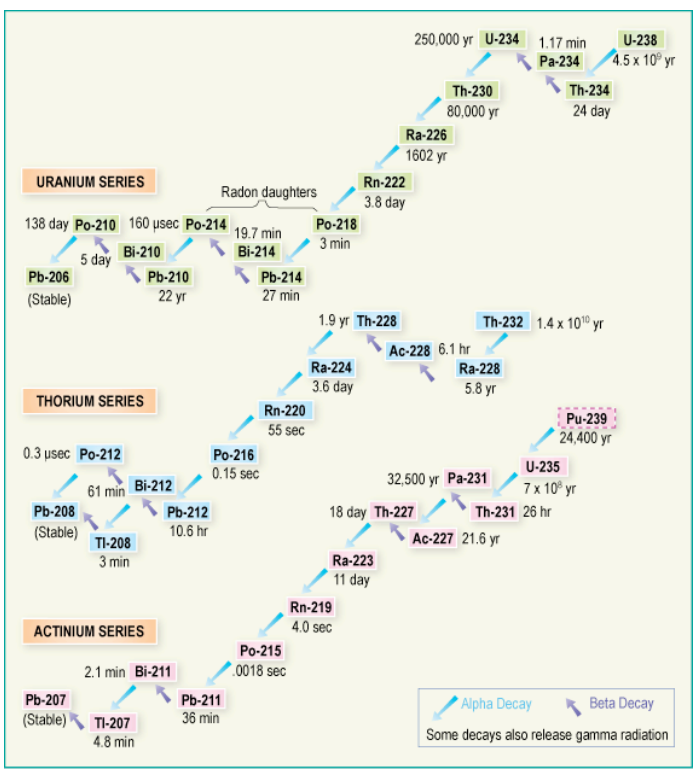
$$\boxed{Q_{\max} \sim Q}$$

Origin of the Elements and Radioisotopes

Light elements H, He, Li, Be were formed during the Big Bang. Elements up to Fe are created in Red and Super Giant Stars and dispersed to planetary nebulae. Heavy elements from Fe to U can be created in supernovae explosions and dispersed in their nebulae.

Radionuclide Decay Chains

Long lived radionuclides Uranium-238, Thorium-232, and Plutonium-239, Neptunium-237, created in violent astrophysical events, accelerators, reactors, nuclear tests, decay to the stable nuclei Lead (Pb) through alpha and beta emission. The prominent U, Th, Ac decay chains are shown below. Radium gas is formed in many of the series, which can be of concern as it seeps to the surface and may be inhaled.



Homework

1- Determine the nuclear radius r_N of the uranium (U-238) nucleus. Express your answer in units of "Fermi". 1f $=10^{-13}$ cm.

2- Use the solution for the square well to determine the first three energy levels of a nucleon in the U-238 atom. (m_N is the nucleon mass and r is the nuclear radius).

$$E_n = \frac{2\hbar^2 v_n^{\ 2}}{m_N (2r_N)^2} \qquad v_n \simeq \frac{n\pi}{2} \text{ deep well approximation}$$

3- In the alpha decay Po-212 -> Pb-208 + He-4 determine the alpha particle kinetic energy in units of MeV. 1 u = m(C-12)/12 = 931.49 MeV

4- In neutron beta decay n-> p + e + v determine the end point energy of the electron in MeV/c^2 . What is the average electron energy in MeV/c^2?