Nuclear Counting Errors

When counting N decays from a radiation source the decay probability will follow Poisson statistics and the error on N is given by

$$\text{err(N)} = \sigma_N = \sqrt{N}$$

.

If we subtract a background B to obtain the best estimate of the true number of signal counts (S) then

$$S = N - B$$

$$\text{err(S)} = \text{err(N - B)} = \sqrt{\sigma_N^2 + \sigma_B^2} = \sqrt{N + B}$$

Thus the background uncertainty adds to the overall error even though we are subtracting the background.

II. Comparing 2 Numbers (Confidence Limits)
Consider that we want to compare two numbers B and C (counts) to determine if the difference is statistically significant.

$$A = B - C$$

$$\text{Err(A)} = \ = \sqrt{B + C}$$

The ratio A/Err(A) gives the significance of the difference B-C in standard deviations. If N/Err(N) >2σ standard deviations we are >95% confident that B-C are really different at the 95%CL . N/Err(N) >3σ 99.7%CL (confidence limit).

Example:
Are B and C really different? B=100 C= 80

B-C = 20
Err(B-C)= $= \sqrt{180} = 13.4$
(B-C)/ Err(B-C)= 1.5 < 2 so we are not very sure that B and C are not statistically significant.

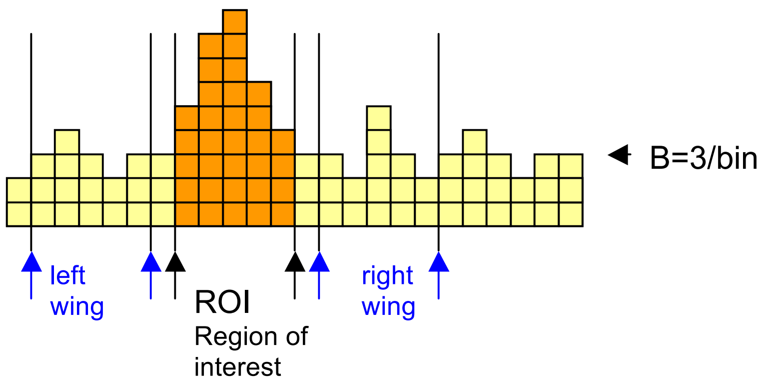
Fractional Error The fractional error on a number err(N)/N = $1/\sqrt{N}$ grows smaller as N is increased. When 1000 people are polled the fractional error is about 3% or there is a +/-3% error.

Example: Mary got 450 votes and Sam got 550 votes in a pole of 1000 people. Mary 45% and Sam 55%. A 10% margin! The poling error is 3% = $1/\sqrt{1000}$ so the 10% difference represent 3.3 σ which we are >90% sure Sam will be elected.

III. **Statistical significance of a signal**

Imagine one is searching for a rare signal above background counts. One defines a region of interest ROI. This may be constrined by some physics or measurement resolution. One needs a background B estimate in the region. Define a left wing and right wing. B = #of counts/ # bins = (15+15)/10 =3/bin. One assumes this is the same backgound level in the ROI. Now count the total events N in the ROI, giving N = 32. We estimate B=15 in the ROI. The signal is then S = 32-15 =17. The error on S err(S) = $\sqrt{47}$ =6.9

N=32, B=15, S = 17



We have sighted 17.0+/-6.9 events or 17/6.9 = 2.5σ standard errors of significance. What does it mean?

We interpret the result by assuming the null hypohtesis. What is the probablility that the background will fluctuate up to this level? Assuming random events and gaussian statistics, the conservative thing to do is assume all the events in the ROI are background so a +/-6.9 event fluctuation is 1σ and we have to determine the probability of a gaussian fluctuating up to 17 events. This is a 2.5σ fluctuation.

The probability of the count to be within +/-1σ is 68%, 2σ 90%, 3σ 99%, etc. So there is < 5% chance that a 2.5σ fluctuation is not real (95%CL). Odds are good that it is a signal of significance(95%CL). But scientist generally require a >5σ statistical significance before one considers a something a “new discovery”.