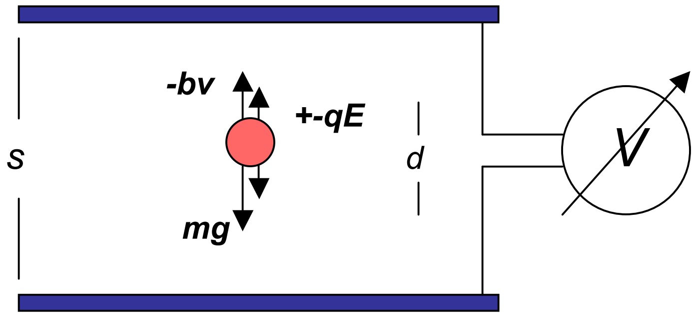
Millikan Oil Drop Experiment

- Millikan's apparatus first showed that charge $q = ne$ was quantized.
- Charged oil droplets were balanced in between electrostatic plates.
- The net force on a falling oil drop is due to the acceleration of gravity mg, a retarding drag force $-bv$ of air resistance, and elecrostatic force qE.

$$F = ma = mg - bv \pm qE$$



$$m = \frac{4}{3}\pi a^3(\rho_{oil} - \rho_{air})$$

$$b = 6\pi a\eta$$

$$E = V / s$$

$$q = ne\ ?$$

1. With V=0, after a brief fall time the drop reaches terminal velocity v_0 = d / t_0 .
2. V is adjusted +-V I and v measured again by determining the time t_{+-} to move distance d.
3. The data can be used to show that charge is quantized and to determine the fundamental charge $q_0 = e$.

Solving First order DE

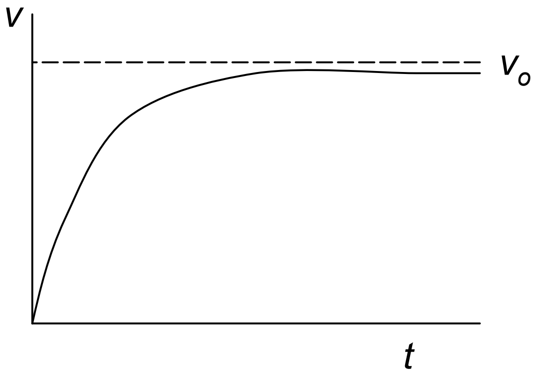
$$m\frac{dv}{dt} = F(v) \qquad \leftarrow F(v) = -(mg + bv)$$

$$\int_0^v \frac{dv}{F(v)} = \frac{1}{m}\int_0^t dt$$

$$\int_0^v \frac{dv}{(\frac{mg}{b} + v)} = \frac{-b}{m}\int_0^t dt$$

$$\ln(\frac{mg}{b} + v)\Big|_0^v = \frac{-b}{m}\ t \qquad \rightarrow \qquad v(t) = \left(\frac{mg}{b}\right)(1 - e^{\frac{-b}{m}t})$$

$$t >> 0 \quad v_0 = \frac{mg}{b} \quad \text{terminal velocity}$$



Millikan Oil Drop Experiment (2)

$$qE = mg + b\,\frac{d}{t} \qquad \textit{Equation of motion after terminal velocity reached.}\; v_0 = \frac{d}{t_0} \qquad (1)$$

$$\frac{1}{t_\pm} = \left(\frac{E}{b\,d}\right) q - \left(\frac{mg}{b\,d}\right) = \left(\frac{\pm V/s}{b\,d}\right) q + \frac{1}{t_0} \quad \rightarrow \quad \frac{1}{t_+} - \frac{1}{t_-} = 2\left(\frac{|V|/s}{b\,d}\right) q \qquad (2)$$

$$\frac{1}{t_0} = -\left(\frac{mg}{b\,d}\right) = \frac{2}{9}\,\frac{a^2(\rho_{oil} - \rho_{air})}{\eta\, d}\, g \qquad (3)$$

1) See data in Table 1.1 for your calculations.
2) Use Eq (3) t_0 to find the average drop radius *a* for each case. You will need this to find *b* in step 3).
3) Use Eq (2) to find *q* for each trial (1-12).
4) Assume fundamental charge $e=q_{min}$.
5) Determine $n = q/q_{min}$ and round off *n* to the nearest integer.
6) Graph *1/t* vs *n* (see Figure 1.4). How does this linear plot show that *q* is quantized?
7) Determine the best value of the electron charge $e = q_{min}$ from the 12 data points (drop-1 and drop-2).
8) Values of ρ–oil and ρ-air are in the text.

2. The Millikan Oil Drop Experiment

2.1 GENERAL

In 1909, R. Millikan reported a reliable method for measuring ionic charges. It consists of observing the motion of small oil droplets under the influence of an electric field. Usually the drops acquire a few electron charges and thus conventional fields impart to them velocities that permit isolation of a drop and continuous observation for a considerable length of time; further, the mass of the oil droplet remains almost constant (there is very slight evaporation) during these long observation times.

In principle, if we measure the force due to the electric field E,

$$F_e = qE = neE \tag{2.1}$$

we can obtain ne; repeating this measurement for several (or the same) drops but with different values of the integer n, we can extract the charge of the electron e.

The electric force can be measured either by a null method—that is, by balancing the drop against the gravitational force—or as will be described here, by observing the motion of the drop under the influence of both forces. Oil droplets in air, acted on by a constant force, soon reach a terminal velocity given by Stokes' law,

$$F = 6\pi a\eta v \tag{2.2}$$

where a is the radius of the (assumed spherical) droplet, η the viscosity of the oil, and v the terminal velocity. To obtain the radius of the drop (needed in Eq. 2.2) we observe the free fall of the drop; the gravitational force is

$$F_g = \tfrac{4}{3}\pi a^3(\rho - \sigma)g \tag{2.3}$$

with ρ and σ the density of air and oil and g the acceleration of gravity.

Schematically, as shown in Fig. 1.1, the apparatus consists of two parallel plates which can be alternatively charged to a constant potential $+V$, $-V$, or 0. The drop is then observed (with a telescope) and the time t it takes to travel through a distance d is measured. Let F_+ denote the force on a negatively charged drop with electric field up (time t_+; electric force aiding gravity) and F_- the force with electric field down (time t_-; electric force opposing gravity). Then

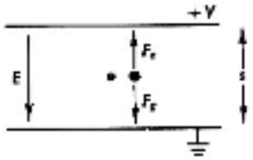


FIG. 1.1 Forces on a charged oil drop between the plates of the Millikan apparatus.

$$\begin{aligned} F_\pm &= \pm ne(V/s) - \tfrac{4}{3}\pi a^3(\rho - \sigma)g = 6\pi a\eta d(1/t_\pm^{(n)}) \\ F_0 &= -\tfrac{4}{3}\pi a^3(\rho - \sigma)g = 6\pi a\eta d(t_0)^{-1} \end{aligned} \tag{2.4}$$

where the sign conventions hold if t is considered >0 when the drop moves up, $t < 0$ when it is moving down. A convenient method of analysis is to write Eq. 2.4 as

$$\begin{aligned} \frac{1}{t_\pm^{(n)}} &= \pm An - B \qquad A = \frac{Ve}{6s\pi a\eta d} \\ \frac{1}{t_0} &= -B \qquad\qquad B = \frac{2}{9}\frac{a^2(\rho - \sigma)g}{\eta d} \end{aligned} \tag{2.5}$$

so that A and B can be easily determined.†

Indeed, a plot of $1/t_\pm^{(n)}$ against n reveals the linear relationship and the fact that only *integer* values of n appear, proving that the drop has acquired one, two, three, or more electric charges of value e, and never a fraction of that value. Thus we have clear evidence that the ionic charge picked up by the oil drops is *quantized*. Furthermore, the absolute value of this minimal electric charge is in good agreement with inferred measurements of the charge carried by the atomic electrons,‡ and therefore is accepted as the most accurate value of the charge of the electron.

2.2 THE EXPERIMENT

The apparatus used in this laboratory (Fig. 1.2) consists of two parallel brass plates $\frac{1}{4}$ in. thick and approximately 2 in. in diameter, placed in a lucite cylinder held apart by three ceramic spacers 4.7 mm long. This assembly is in turn enclosed in a cylindrical brass housing with provisions for electrical connections and containing two windows, one for illumination of the drops and one for observation. The top plate has a small hole in its

---

† These expressions are in cgs units so that V must be expressed in statvolts if e is to be obtained in esu.

‡ As in e/m experiments, shot noise measurements, etc.

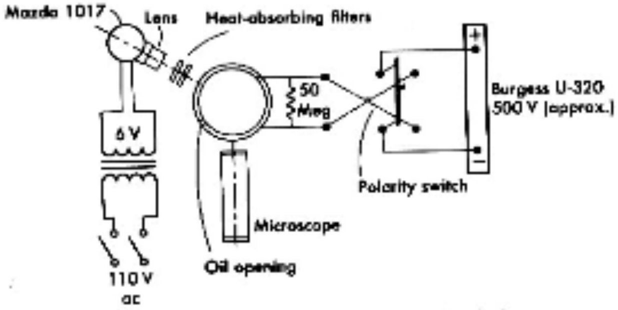


Fig. 1.2 Millikan oil drop experiment schematic of the apparatus.

center for the admission of the oil drops, which are produced by spraying oil with a regular atomizer.

To charge the plates, a 500-V battery and a reversing switch are used; the plates are shunted by a 50-megohm resistor to prevent them from remaining charged when the switch is open. For observations a 10-cm focal length microscope is used (Cenco 72925), while illumination is provided by a Mazda 1017-watt lamp and condensing lens. To avoid convection currents inside the apparatus, a heat-absorbing filter (Corning infrared-absorbing) is placed in the illuminating beam.

The plates should be made perpendicular to the gravitational field by means of the three leveling screws at the base of the apparatus and a level placed on the top plate. Being a cosine error, the deviation introduced by an angular displacement of the gravitational component from perpendicular by 8 degrees is 1 percent. A value for the plate spacing s may be obtained by using the stage micrometer. The micrometer should be focused on a wire inserted in the oil hole in the center of the top plate, and the crosshair of the micrometer should be moved along the length of the wire. Several measurements should be taken and their results averaged.

The velocities are determined by measuring with a stop watch the time required for the droplet to cover a specified number of divisions of the microscope scale. Care must be taken to avoid drafts and vibrations in the vicinity of the apparatus; for that reason and because of Brownian motion, the drop may wander or be displaced out of the field of the microscope. It may then be necessary to *reposition* the microscope between measurements on a single drop. Moreover, the drop should be kept in focus to avoid parallax errors.

Both the microscope and the light source may be adjusted by viewing a small wire inserted in the oil hole. The light should be adjusted so that the focal point is somewhat ahead or behind the wire and the wire is more or less evenly illuminated. To light the scale, a small light is placed next to the slit just ahead of the eyepiece of the microscope. The actual distance to which a scale division corresponds may be found by using a microscope slide on which a subdivided millimeter scale has been scratched.† The eyepiece focus of the microscope should not be changed during a run, since moving the eyepiece changes the effective distance of the scale. (To bring the drop back into focus the entire microscope should be moved.)

It is important to be *sparing* in the amount of oil sprayed into the chamber. In addition to gumming up the interior more quickly, large quantities create so many particles in the microscope field that without excessive eyestrain it is virtually impossible to single out and follow a single droplet.

Under the influence of gravity, droplets will fall at various limiting speeds. If the plates are charged, some of the drops will move down more rapidly, whereas others will reverse their direction of motion since in the process of spraying some drops become positively charged and others negatively charged. By concentrating on one drop which can be controlled by the field, and manipulating the sign of the electric field so that this particular drop is retained, it is possible to remove all other drops from the field. The limiting velocity is reached very quickly and the measurement should be started near the top or bottom of the plate. Measurement should be completed before the drop has reached a point in its travel where application of the reverse potential is insufficient to save the drop from being "gobbled up."

The density in air of the oil used was‡ 0.883 ± 0.003 gm/cm³. It is desirable to take measurements in the shortest possible time since, as previously mentioned, the mass of the drop changes through evaporation.

It is obviously desirable to make measurements on as many different charges on the same or different drops as possible. Thus after four or five measurements of $t_r^{(+)}$, $t_r^{(-)}$, and t_g have been taken, the charge on the drop must be changed; this is accomplished by bringing close to one of the windows a radioactive source (10 to 100 μCi of Co^{60} will do).§ The droplet should be brought close to the top plate and allowed to fall with the *field off*; on its way down it will sweep up a few ions created by the source. This can be checked by occasionally turning the field on to see if the charge has changed; rarely will a drop pick up any charge when the field is on.

The battery voltage should be checked with a 1-percent resistor divider and potentiometer; microscope calibration should be checked before and

---

† Note that the focal length of the microscope must not be changed, but instead the slide should be brought into the focal plane.

‡ This may be found by a simple measurement.

§ Ci = curie.

after the measurements; the same holds true for air temperature and pressure, which are needed for a correction to Stokes' law.

Indeed, when the diameter of the drop is comparable to the mean free path in air, the viscosity η in Eq. (2.2) should be replaced by

$$\eta(T) = \eta_0(T)\left[1 + \frac{b}{aP}\right]^{-1} \tag{2.6}$$

where $\eta_0(T)$ is the viscosity of air as a function of T (Fig. 1.3), $b = 6.17 \times 10^{-4}$, P is the air pressure in centimeters of mercury, and a is the radius of the drop in centimeters (of the order of 10^{-4} cm). In analyzing the data it is convenient to calculate a_0 by letting $\eta = \eta_0(T)$ in Eq. 2.5; a_0 is then inserted in Eq. 2.6 to obtain $\eta(T)$ and thus a more accurate value for a.

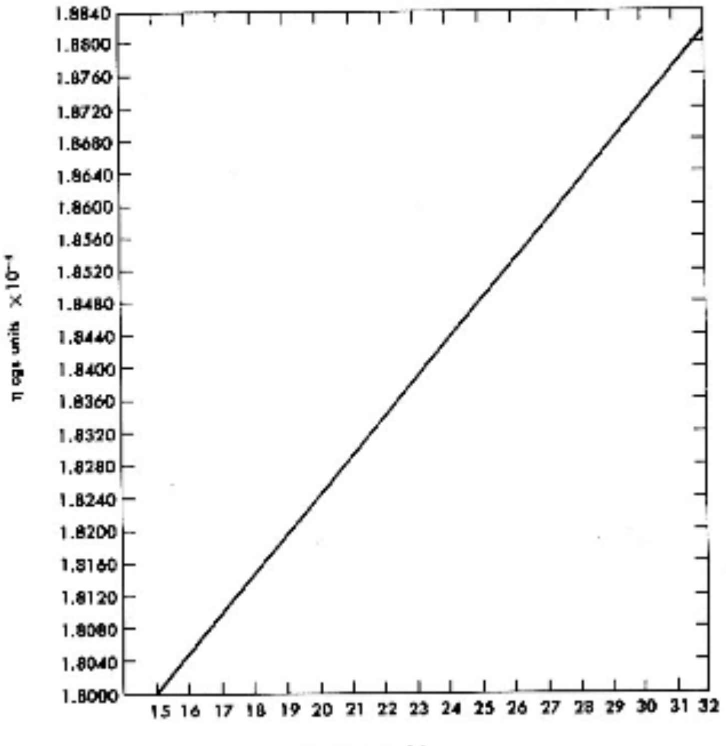


FIG. 1.3 Viscosity of dry air as a function of temperature.

TABLE 1.1

DATA ON MILLIKAN OIL DROP EXPERIMENT

	t_g	t_- (s)	t_+ (s)	$(1/t_+)-(1/t_0)$	$(1/t_-)-(1/t_0)$	n	$\frac{(1/t_+)-(1/t_-)}{2n}$
Drop 1	−27.9	+8.69	−5.65	−0.142	+0.150	1	−0.146
	−29.6	+1.30	−1.18	−0.813	+0.770	5	−0.158
	−28.2	+3.66	−3.00	−0.299	+0.306	2	−0.152
	−29.3	+0.75	−0.716	−1.362	+1.368	9	−0.152
	−29.4	+2.35	−1.97	−0.473	+0.460	3	−0.155
$1/t_0 = -0.0346$							0.1535
	$a = 4.91 \times 10^{-5}$ cm			$\eta = 1.60 \times 10^{-4}$			
Drop 2	−24.22	+3.98	−3.071	−0.285	+0.291	2	−0.144
	−25.75	+9.73	−6.65	−0.137	+0.143	1	−0.140
	−25.4	+2.5	−2.12	−0.432	+0.440	3	−0.145
	−25.22	+9.67	−5.42	−0.145	+0.143	1	−0.144
	−25.22	+4.1	−3.07	−0.286	+0.288	2	−0.143
	−24.4	+1.73	−1.73	−0.538	+0.618	4	−0.144
	−24.4	+9.95	−6.02	−0.126	+0.141	1	−0.133
$1/t_0 = -0.0400$							0.1433
	$a = 5.06 \times 10^{-5}$ cm			$\eta = 1.59 \times 10^{-4}$			

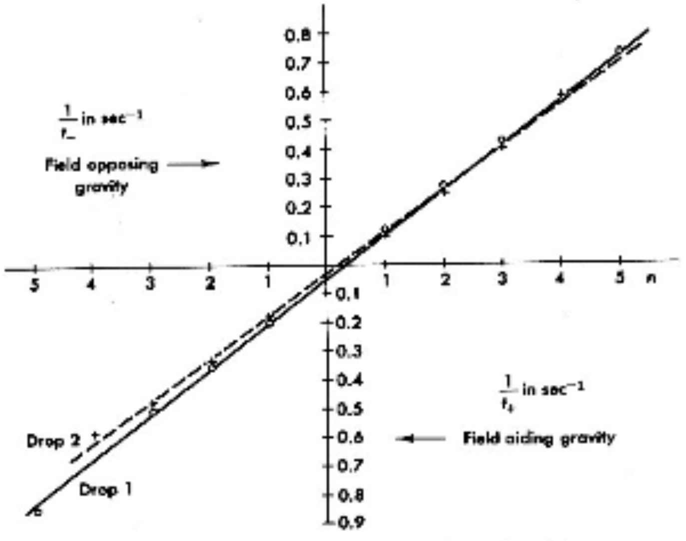


FIG. 1.4 Plot of $1/t_+$ and $1/t_-$ versus n where n is an integer.