12- Stephan's Law for Black Body Radiation

Object:
Measure how the current through an electric light bulb varies as the applied voltage is changed. This will allow you to establish Stephan's Law for Black Body Radiation.

Introduction:
When an electric current flows through the filament in a light bulb the filament heats up. The filament loses heat in two ways: electromagnetic radiation (mainly visible light and invisible heat radiation) and conduction (through the base of the bulb). The heat conducted away from the filament increases linearly with filament temperature. The air in the bulb is pumped out during manufacture so little heat is lost by convection.

Since it is difficult to measure the temperature of the filament directly, we use the fact that the filament resistance is approximately proportional to the filament temperature at T>>To. (*Ro*=resistance at room temperature *T=To*)

$$R(T)=Ro[\ 1+\alpha(T-To)\ ]$$
$$R(T\ ) = Ro + \alpha\ R\ T - \alpha\ Ro\ To\ \sim \alpha\ R\ T \quad (\ T>>.To)$$
$$R \sim T\ \ (\ T>>.To)$$

Part 1: Stefan's Law:
Stefan's Law states that the radiated power density (W/m2) of a black body is proportional to its absolute temperature *T* raised to the fourth power.

$$E = e\ \sigma\ T^4$$

The emissivity *e* is a correction for an approximate black body radiator, where *e* = *1 – R,* is the fraction of the light reflected (*R)* by the black body. For a true black body *R* = 0 and *e* = 1 or total absorbtion!

(σ = 5.66e-8 W/m2-K4 = Stephan-Boltzmann constant).

Using the power~temperature relationship:

$$E = e\ \sigma\ T^4 = e\ \sigma\ R^4$$

and

$$ln(E)\ = ln(e) + ln\ (\sigma) + 4\ ln(R)$$

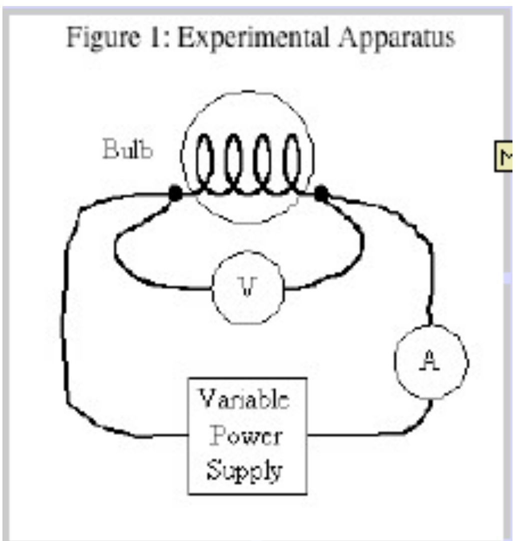
By plotting ln(P) vs ln(R) we should find a linear relationship y=a + bx where the slope b=4.

Directions:

1) Connect a variable power supply in series to the terminals of a the small utility light bulb as in Figure 1.

2) Attach a digital multimeter meter (DVM) in parallel across the light bulb + and – terminals to measure the voltage drop *V* across the filament.

3) A second DVM is place in series with the power supply to measure the current *I* passing through the filament.



4) Record the voltage *V* and current *I* for a number of voltages while keeping the current *I* below 200mA.

5) Determine the resistance *R* by Ohm;s Law.

6) Graph ln(*P*) vs ln(*R*)

7) Measure the slope fhe graph and confirm Stephan;s Law.

slope = 4 ± err

8) Determine the error in the slope either by analytical or graphical means.

V(volts)	*I(amps)*	*R=V / I*
0.5		
1.0		
1.5		
2.0		
2.5		
3.0		
3.5		
4.0		
5.0		
6.0		
7.0		
8.0		
9.0		
10.0		
11.0		
12.0		

Part II. Measurement of Power with a Tungsten Lamp and Radiometer.
We can directly measure the power output of a tungsten filament at each voltage using the Daedalon Stephan Boltzmann Source and Radiiometer.

Use the procedure supplies in the Daedalon Experiment Manual. In this experiment a tungsten lamp is used of known α = 0.0045 Ohms/degK

$$T = (R/Ro-1)/\alpha + To$$

V	*I*	*R=V / I*	*P(W/m²)* radiiometer	*T*
0.5				
1.0				
1.5				
2.0				
2.5				
3.0				
3.5				
4.0				
5.0				
6.0				
7.0				
8.0				
	< 1.5 A			

Instruction Manual

for

EH-15 Stefan-Boltzmann Source

Introduction

In the early studies of radiation one of the significant steps forward was the connection between the radiated energy from a hot body and its temperature. Such a radiating hot body is said to emit "black body" radiation, a term you will encounter many times in the study of radiation. Measuring the energy as a function of wavelength and temperature occupied many investigators. This data is presented as the "black body radiation" curve. Light was considered a wave motion when these measurements were made. In 1879, Stefan, using purely thermodynamic arguments, was able to deduce that the total emitted radiation is proportional to the fourth power of the temperature of the radiator; the hotter the body, the more it radiates.

$$E = \sigma T^4 \qquad (1)$$

The scaling constant in the equation is known as Stefan's constant and has a modern value of

$$\sigma = 0.56686 \times 10^{-7} \ W \ m^{-2} \ {}^{\circ}K^{-4}$$

In this experiment we will measure the radiation from a hot tungsten filament as a function of its temperature. The radiation will be measured using a Daedalon EG-45 Radiometer. The temperature of the filament will be determined by the change in resistance of the tungsten filament wire. Over a fairly wide range, the resistance of a metallic filament can be described by

$$R = R_{293} \ (1 + \alpha (T - 293)) \qquad (2)$$

where R_{293} is the resistance at 20° C, α is the temperature coefficient of resistance, and T is the temperature in degrees Kelvin. The resistance of the filament can easily be determined by measuring the current through it and the voltage across it. Since lamp sockets often have some resistance, the leads to the lamp are soldered in place. The voltage connections are made at the lamp base so that they do not carry the filament current. With these precautions, the true resistance of the lamp can be found.

The radiation from the source is measured with a Daedalon EG-45 Radiometer. This wide range instrument will measure the radiation from objects at room temperature to the radiation from direct sunlight. Plotting the data from the Radiometer as a function of the deduced temperature of the filament yields a plot which confirms Stefan's conclusion.

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Experimental Procedure

In addition to the EH-15 Source and the EG-45 Radiometer, two multimeters and a power supply with an adjustable output are required. Meters with a digital display are preferred but analog meters can be used with little loss in accuracy.

1. Set the Source and Radiometer facing each other on a rigid surface as illustrated in Figure One. The Source should be taped to the surface since the leads tend to move the lamp assembly. Measure the distance from the filament to the middle of the shutter control rod on the Radiometer. Adjust this distance to 10 cm.

This distance does not actually matter in the experiment as long as it remains constant. Remember that the radiation received by the detector varies as the inverse square law, so small changes in position of the source or detector make large differences in measured radiation. Check the separation several times during the experiment to make sure that it hasn't changed.

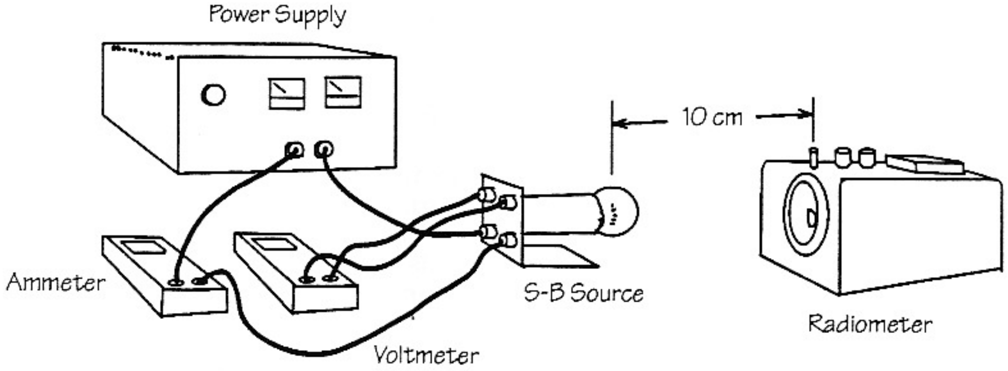

Figure One

2. Connect the power supply to the ammeter and the current jacks on the source support bracket.

3. Connect the voltmeter to the voltmeter jacks on the support bracket for the source.

4. Connect the power supply to an outlet and turn the output to zero. This step is important. If the power supply is turned on with the output turned up, the lamp could burn out immediately, prematurely ending the experiment, and possibly your lab deposit.

5. Turn on the power supply. Be careful that Step 4 has been carried out.

The power supply used to collect the sample data in this manual was a Daedalon EV-12. This supply is easy to use because it has a current limit adjustment that can be set to protect the lamp and a smooth voltage adjustment control. Simpler power supplies can be used as long as their output at a given setting is constant while the measurement is being made.

6. Measure the resistance of the filament at a very low current value, between 50 and 70 mA, for instance. The voltage will be around 0.03 Vdc. The resistance of the filament measured from these values will be considered the room temperature resistance.

7. Measure the room temperature.

8. Set the Radiometer to the 1 scale and with the shutter closed, set the scale zero.

9. Set the source voltage to about 1.3-1.5 Vdc. The filament will not glow but it will be radiating. Record the voltage and current readings. Open the Radiometer shutter and record the meter reading.

10. Repeat Step 10, increasing the voltage in small steps, while recording the voltage, current and radiation. Check the scale zero often, particularly if the sensitivity range is changed.

When adjusting the current, do not exceed 1.7 A to avoid damage to the filament and reduction of its life.

Data Reduction

The calculations are simple and repetitive. The best way to carry them out depends on the calculator or computer that is available. A computer spread sheet program is ideal. The sample calculation was done in MathCAD. Since both the current and voltage were measured, the value of the filament resistance can be calculated. By rearranging Equation 2

$$T_i = (R_i - R_{293}) / (\alpha \; R_{293}) + 293 \qquad (3)$$

The subscripts in Equation 3 assume the room temperature was 20° C or 293 K. These values should be adjusted to the actual room temperature measured during the experiment. The temperature coefficient, α, for tungsten is 0.0045. For a typical experiment, the filament temperature varies from room temperature to 2,500 K.

Since only the exponent in Equation One is being tested in this experiment, it is necessary to rewrite Equation 1 as

$$\log E = \log \sigma + 4 \log T \quad (4)$$

Plotting log E versus Log T and measuring the slope of the line determines the value of the exponent in the equation. Typical results are shown in Figure Two. The measured radiation data fits a straight line very satisfactorily. The slope of this line is 4.5 which is higher than the theory predicts but a reasonable value for such a short experiment.

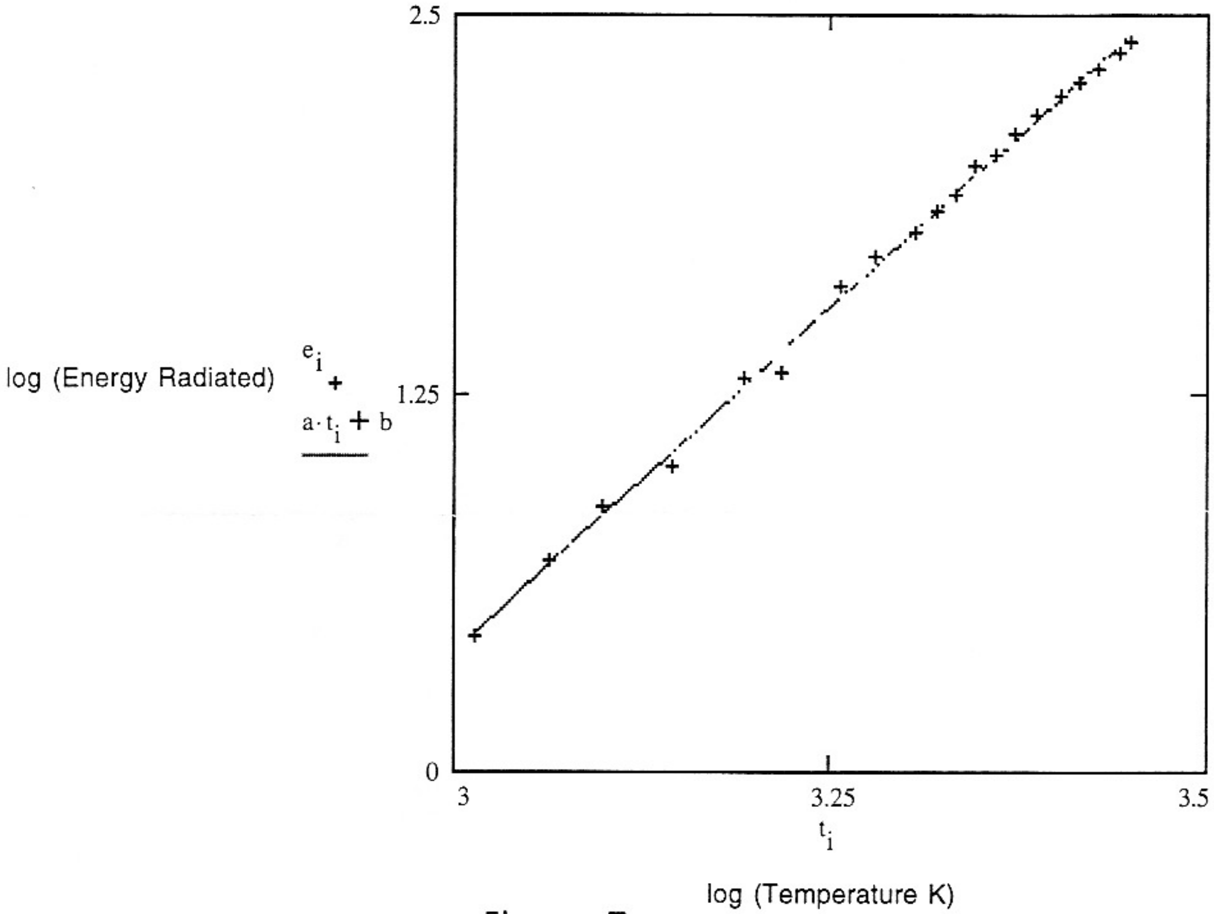

Figure Two

Discussion

The theoretical considerations used by Stefan in reaching his conclusion were based upon a special form of radiation source. It is described as "black body radiation from a hollow heated enclosure". To produce this radiation, visualize a closed furnace with a small hole drilled in the wall. Radiation inside of the furnace is radiated by the walls and absorbed by the walls so there is an equilbrium of energy being absorbed and radiated from the walls. The small hole lets a little bit of this radiation out of the furnace, but not enough to upset the equilibrium inside. This radiation has the spectral properties of an ideal "black body radiator" and it was radiation meeting these conditions that Stefan worked with.

Our lamp filament is far from the theoretical ideal so if the exponent is a little too high it is not surprising. It is close enough to verify the radiation law considering the approximate physical conditions.

WRJB-July 1998