34-4 THE HALL EFFECT

In 1879, Edwin H. Hall* conducted an experiment that permitted direct measurement of the sign and the number density (number per unit volume) of charge carriers in a conductor. The *Hall effect* plays a critical role in our understanding of electrical conduction in metals and semiconductors.

Consider a flat strip of material of width w carrying a current i, as shown in Fig. 18. The direction of the current i is the conventional one, opposite to the direction of motion of the electrons. A uniform magnetic field **B** is established perpendicular to the plane of the strip, such as by placing the strip between the poles of an electromagnet. The charge carriers (electrons, for instance) experience a magnetic deflecting force $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$, as shown in the figure, and move to the right side of the strip. Note that positive charges moving in the direction of i experience a deflecting force in the *same* direction.

The buildup of charge along the right side of the strip (and a corresponding deficiency of charge of that sign on the opposite side of the strip), which is the Hall effect, produces an electric field **E** across the strip, as shown in Fig. 18*b*. Equivalently, a potential difference $V = E/w$, called the *Hall potential difference* (or Hall voltage), exists across the strip. We can measure V by connecting the leads of a voltmeter to points x and y of Fig. 18. As we show below, the sign of V gives the sign of the charge carriers, and the magnitude of V gives their density (number per unit volume). If the charge carriers are electrons, for example, an excess of negative charges builds up on the right side of the strip, and point y is at a lower potential than point x. This may seem like an obvious conclusion in the case of metals; however, you should keep in mind that Hall's work was done nearly 20 years before Thomson's discovery of the electron, and the nature of electrical conduction in metals was not at all obvious at that time.

Let us assume that conduction in the material is due to charge carriers of a particular sign (positive or negative) moving with drift velocity $\mathbf{v}_d$. As the charge carriers drift, they are deflected to the right in Fig. 18 by the magnetic force. As the charges collect on the right side, they set up an electric field that acts inside the conductor to oppose the sideways motion of additional charge carriers. Eventually, an equilibrium is reached, and the Hall voltage reaches its maximum; the sideways magnetic force ($q\mathbf{v}_d \times \mathbf{B}$) is then balanced by the sideways electric force ($q\mathbf{E}$). In vector terms, the Lorentz force on the charge carriers under these circumstances is zero:

$$q\mathbf{E} + q\mathbf{v}_d \times \mathbf{B} = 0, \qquad (20)$$

or

$$\mathbf{E} = -\mathbf{v}_d \times \mathbf{B}. \qquad (21)$$

Since $\mathbf{v}_d$ and **B** are at right angles, we can write Eq. 21 in terms of magnitudes as

$$E = v_d B. \qquad (22)$$

From Eq. 6 of Chapter 32 we can write the drift speed as $v_d = j/ne$, where j is the current density in the strip and n is the density of charge carriers. The current density j is the current i per unit cross-sectional area A of the strip. If t is the thickness of the strip, then its cross-sectional area A can be written as wt. Substituting V/w for the electric field E, we obtain

$$\frac{V}{w} = v_d B = \frac{j}{ne} B = \frac{i}{wtne} B$$

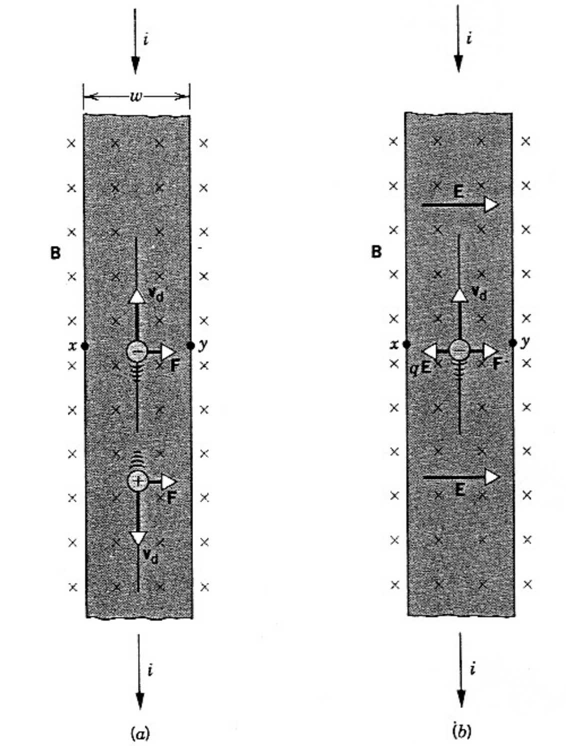

Figure 18 A strip of copper immersed in a magnetic field **B** carries a current i. (*a*) The situation just after the magnetic field has been turned on, and (*b*) the situation at equilibrium, which quickly follows. Note that negative charges pile up on the right side of the strip, leaving uncompensated positive charges on the left. Point x is at a higher potential than point y.

* At the time of his discovery, Hall was a 24-year-old graduate student at the Johns Hopkins University. His research supervisor was Professor Henry A. Rowland, who had a few years earlier shown that a moving electric charge produced the same magnetic effect as an electric current. See "Rowland's Physics," by John D. Miller, *Physics Today*, July 1976, p. 39.

TABLE 2 HALL EFFECT RESULTS FOR SELECTED MATERIALS

Material	n ($10^{28}/m^3$)	*Sign of V*	*Number per per atom*[a]
Na	2.5	−	0.99
K	1.5	−	1.1
Cu	11	−	1.3
Ag	7.4	−	1.3
Al	21	−	3.5
Sb	0.31	−	0.09
Be	2.6	+	2.2
Zn	19	+	2.9
Si (pure)	1.5×10^{-12}	−	3×10^{-13}
Si (typical n-type)	10^{-7}	−	2×10^{-8}

[a] The number of charge carriers per atom of the material as determined from the number per unit volume and the density and molar mass of the material.

or, solving for the density of charge carriers,

$$n = \frac{iB}{etV}. \tag{23}$$

From a measurement of the magnitude of the Hall potential difference V we can find the number density of the charge carriers. Table 2 shows a summary of Hall effect data for several metals and semiconductors. For some monovalent metals (Na, K, Cu, Ag) the Hall effect indicates that each atom contributes approximately one free electron to the conduction. For other metals, the number of electrons can be greater than one per atom (Al) or less than one per atom (Sb). For some metals (Be, Zn), the Hall potential difference shows that the charge carriers have a *positive* sign. In this case the conduction is dominated by *holes*, unoccupied energy levels in the valence band (see Section 32-7 and Chapter 53 of the extended text). The holes correspond to the absence of an electron and thus behave like positive charge carriers moving through the material. For some materials, semiconductors in particular, there may be substantial contributions from both electrons and holes, and the simple interpretation of the Hall effect in terms of free conduction by one type of charge carrier is not sufficient. In this case we must use more detailed calculations based on quantum theory.

Sample Problem 3 A strip of copper 150 μm thick is placed in a magnetic field $B = 0.65$ T perpendicular to the plane of the strip, and a current $i = 23$ A is set up in the strip. What Hall potential difference V would appear across the width of the strip if there were one charge carrier per atom?

Solution In Sample Problem 2 of Chapter 32, we calculated the number of charge carriers per unit volume for copper, assuming that each atom contributes one electron, and we found

$$n = 8.49 \times 10^{28} \text{ electrons/m}^3.$$

From Eq. 23 then,

$$V = \frac{iB}{net} = \frac{(23 \text{ A})(0.65 \text{ T})}{(8.49 \times 10^{28} \text{ m}^{-3})(1.60 \times 10^{-19} \text{ C})(150 \times 10^{-6} \text{ m})}$$

$$= 7.3 \times 10^{-6} \text{ V} = 7.3 \,\mu\text{V}.$$

This potential difference, though small, is readily measurable.

The Quantized Hall Effect* *(Optional)*

Let us rewrite Eq. 23 as

$$\frac{V}{i} = \frac{1}{etn} B. \tag{24}$$

The quantity on the left has the dimension of resistance (voltage divided by current), although it is not a resistance in the conventional sense. It is commonly called the *Hall resistance.* We can determine the Hall resistance by measuring the Hall voltage V in a material carrying a current i.

Equation 24 shows that the Hall resistance is expected to increase linearly with the magnetic field B for a particular sample of material (in which n and t are constants). A plot of the Hall resistance against B should be a straight line.

In experiments done in 1980, German physicist Klaus von Klitzing discovered that, at high magnetic fields and low temperatures (about 1 K), the Hall resistance did not increase linearly with the field; instead, the plot showed a series of "stair steps," as shown in Fig. 19. This effect has become known as the *quantized Hall effect,* and von Klitzing was awarded the 1985 Nobel Prize in physics for his discovery.

The explanation for this effect involves the circular paths in which electrons are forced to move by the field. Quantum mechanics prevents the electron orbits from overlapping. As the field increases, the orbital radius decreases, permitting more orbits to bunch together on one side of the material. Because the orbital motion of electrons is quantized (only certain orbits being allowed), the changes in orbital motion occur suddenly, corresponding to the steps in Fig. 19. A natural unit of resistance

* See "The Quantized Hall Effect," by Bertrand I. Halperin, *Scientific American,* April 1986, p. 52.

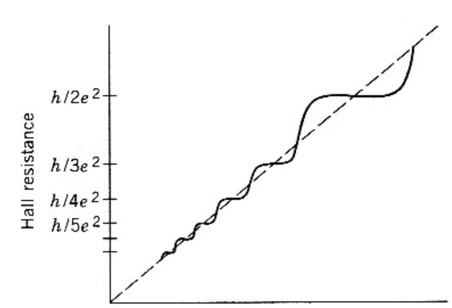

Figure 19 The quantized Hall effect. The dashed line shows the expected classical behavior. The steps show the quantum behavior.

corresponding to orbital motion is h/e^2, where h is Planck's constant, and the steps in Fig. 19 occur at Hall resistances of $h/2e^2$, $h/3e^2$, $h/4e^2$, and so on.

The quantized Hall resistance h/e^2 has the value 25812.806 Ω and is known to a precision of less than 1 part in 10^{10}, so the quantized Hall effect has provided a new standard for resistance. This standard, which can be duplicated exactly in laboratories around the world, became the new representation for the ohm in 1990. ■

34-5 THE MAGNETIC FORCE ON A CURRENT

A current is a collection of moving charges. Because a magnetic field exerts a sideways force on a moving charge, it should also exert a sideways force on a wire carrying a current. That is, a sideways force is exerted on the conduction electrons in the wire, but since the electrons cannot escape sideways, the force must be transmitted to the wire itself. Figure 20 shows a wire that passes through a region in which a magnetic field **B** exists. When the wire carries no current (Fig. 20*a*), it experiences no deflection. When a current is carried by the wire, it deflects (Fig. 20*b*); when the current is reversed (Fig. 20*c*), the deflection reverses. The deflection also reverses when the field **B** is reversed.

To understand this effect, we consider the individual charges flowing in a wire (Fig. 21). We use the free-electron model (Section 32-5) for current in a wire, assuming the electrons to move with a constant velocity, the drift velocity $\mathbf{v}_d$. The actual direction of motion of the electrons is of course opposite to the direction we take for the current i in the wire.

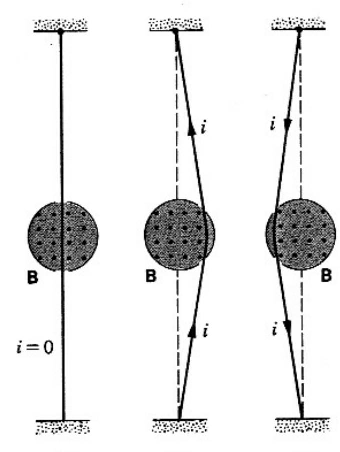

Figure 20 A flexible wire passes between the poles of a magnet. (*a*) There is no current in the wire. (*b*) A current is established in the wire. (*c*) The current is reversed.

The wire passes through a region in which a uniform field **B** exists. The sideways force on each electron (of charge $q = -e$) due to the magnetic field is $-e\mathbf{v}_d \times \mathbf{B}$. Let us consider the total sideways force on a segment of the wire of length L. The same force (magnitude and direction) acts on each electron in the segment, and the total force **F** on the segment is therefore equal to the number N of electrons times the force on each electron:

$$\mathbf{F} = -Ne\mathbf{v}_d \times \mathbf{B}. \quad (25)$$

How many electrons are contained in that segment of wire? If n is the number density (number per unit volume) of electrons, then the total number N of electrons in the segment is nAL, where A is the cross-sectional area of the wire. Substituting into Eq. 25, we obtain

$$\mathbf{F} = -nALe\mathbf{v}_d \times \mathbf{B}. \quad (26)$$

Equation 6 of Chapter 32 ($v_d = i/nAe$) permits us to write Eq. 26 in terms of the current i. To preserve the vector relationship of Eq. 26, we define the vector **L** to be equal in magnitude to the length of the segment and to point in the direction of the current (opposite to the direction of electron flow). The vectors $\mathbf{v}_d$ and **L** have opposite directions, and we can write the scalar relationship $nALev_d = iL$ using vectors as

$$-nALe\mathbf{v}_d = i\mathbf{L}. \quad (27)$$

Substituting Eq. 27 into Eq. 26, we obtain an expression for the force on the segment:

$$\mathbf{F} = i\mathbf{L} \times \mathbf{B}. \quad (28)$$

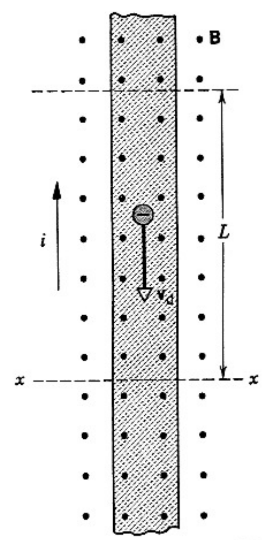

Figure 21 A close-up view of a length L of the wire of Fig. 20*b*. The current direction is upward, which means that electrons drift downward. A magnetic field emerges from the plane of the figure, so that the wire is deflected to the right.

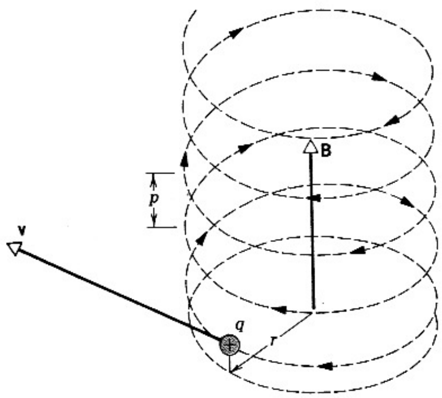

Figure 33 Problem 31.

wise? Assume that the orbit radius does not change. [*Hint*: The centripetal force is now partially electric ($\mathbf{F}_E$) and partially magnetic ($\mathbf{F}_B$) in origin.] (*c*) Show that the change in frequency of revolution caused by the magnetic field is given approximately by

$$\Delta\nu = \pm\frac{Be}{4\pi m}.$$

Such frequency shifts were observed by Zeeman in 1896. (*Hint*: Calculate the frequency of revolution without the magnetic field and also with it. Subtract, bearing in mind that because the effect of the magnetic field is very small, some—but not all—terms containing B can be set equal to zero with little error.)

33. Estimate the total path length traveled by a deuteron in a cyclotron during the acceleration process. Assume an accelerating potential between the dees of 80 kV, a dee radius of 53 cm, and an oscillator frequency of 12 MHz.

34. Consider a particle of mass m and charge q moving in the xy plane under the influence of a uniform magnetic field $\mathbf{B}$ pointing in the $+z$ direction. Write expressions for the coordinates $x(t)$ and $y(t)$ of the particle as functions of time t, assuming that the particle moves in a circle of radius R centered at the origin of coordinates.

35. Consider the particle of Problem 34, but this time *prove* (rather than *assuming*) that the particle moves in a circular path by solving Newton's law analytically. (*Hint*: Solve the expression for F_y to find v_x and substitute into the expression for F_x to obtain an equation that can be solved for v_y. Do the same for v_x by substituting into the F_y equation. Finally, obtain $x(t)$ and $y(t)$ from v_x and v_y.)

Section 34-4 The Hall Effect

36. In a Hall effect experiment, a current of 3.2 A lengthwise in a conductor 1.2 cm wide, 4.0 cm long, and 9.5 μm thick produces a transverse Hall voltage (across the width) of 40 μV when a magnetic field of 1.4 T is passed perpendicularly through the thin conductor. From these data, find (*a*) the drift velocity of the charge carriers and (*b*) the number density of charge carriers. From Table 2, identify the conductor. (*c*) Show on a diagram the polarity of the Hall voltage with a given current and magnetic field direction, assuming the charge carriers are (negative) electrons.

37. Show that, in terms of the Hall electric field E and the current density j, the number of charge carriers per unit volume is given by

$$n = \frac{jB}{eE}.$$

38. (*a*) Show that the ratio of the Hall electric field E to the electric field E_c responsible for the current is

$$\frac{E}{E_c} = \frac{B}{ne\rho},$$

where ρ is the resistivity of the material. (*b*) Compute the ratio numerically for Sample Problem 3. See Table 1 in Chapter 32.

39. A metal strip 6.5 cm long, 0.88 cm wide, and 0.76 mm thick moves with constant velocity $\mathbf{v}$ through a magnetic field $B = 1.2$ mT perpendicular to the strip, as shown in Fig. 34. A potential difference of 3.9 μV is measured between points x and y across the strip. Calculate the speed v.

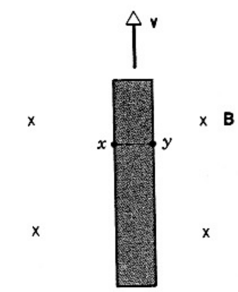

Figure 34 Problem 39.

Section 34-5 The Magnetic Force on a Current

40. A horizontal conductor in a power line carries a current of 5.12 kA from south to north. The Earth's magnetic field in the vicinity of the line is 58.0 μT and is directed toward the north and inclined downward at 70.0° to the horizontal. Find the magnitude and direction of the magnetic force on 100 m of the conductor due to the Earth's field.

41. A wire of length 62.0 cm and mass 13.0 g is suspended by a pair of flexible leads in a magnetic field of 440 mT. Find the magnitude and direction of the current in the wire required to remove the tension in the supporting leads. See Fig. 35.

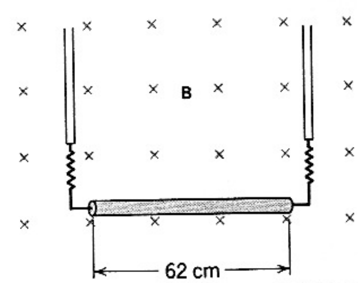

Figure 35 Problem 41.

42. A metal wire of mass m slides without friction on two horizontal rails spaced a distance d apart, as in Fig. 36. The track lies in a vertical uniform magnetic field $\mathbf{B}$. A constant

Calculate (*a*) the radius of the cyclotron and (*b*) the corresponding oscillator frequency. Relativity considerations are not significant.

19. In a nuclear experiment a proton with kinetic energy K_p moves in a uniform magnetic field in a circular path. What energy must (*a*) an alpha particle and (*b*) a deuteron have if they are to circulate in the same orbit? (For a deuteron, $q = +e$, $m = 2.0$ u; for an alpha particle, $q = +2e$, $m = 4.0$ u.)

20. A proton, a deuteron, and an alpha particle, accelerated through the same potential difference V, enter a region of uniform magnetic field, moving at right angles to **B**. (*a*) Find their kinetic energies. If the radius of the proton's circular path is r_p, what are the radii of (*b*) the deuteron and (*c*) the alpha particle paths, in terms of r_p?

21. A proton, a deuteron, and an alpha particle with the same kinetic energy enter a region of uniform magnetic field, moving at right angles to **B**. The proton moves in a circle of radius r_p. In terms of r_p, what are the radii of (*a*) the deuteron path and (*b*) the alpha particle path?

22. Figure 32 shows an arrangement used to measure the masses of ions. An ion of mass m and charge $+q$ is produced essentially at rest in source S, a chamber in which a gas discharge is taking place. The ion is accelerated by potential difference V and allowed to enter a magnetic field **B**. In the field it moves in a semicircle, striking a photographic plate at distance x from the entry slit. Show that the ion mass m is given by

$$m = \frac{B^2 q}{8V} x^2.$$

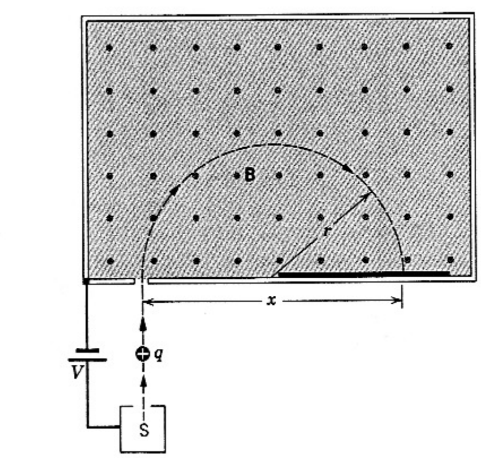

Figure 32 Problem 22.

23. Two types of singly ionized atoms having the same charge q and mass differing by a small amount Δm are introduced into the mass spectrometer described in Problem 22. (*a*) Calculate the difference in mass in terms of V, q, m (of either), B, and the distance Δx between the spots on the photographic plate. (*b*) Calculate Δx for a beam of singly ionized chlorine atoms of masses 35.0 and 37.0 u if $V = 7.33$ kV and $B = 520$ mT.

24. In a mass spectrometer (see Problem 22) used for commercial purposes, uranium ions of mass 238 u and charge $+2e$ are separated from related species. The ions are first accelerated through a potential difference of 105 kV and then pass into a magnetic field, where they travel a 180° arc of radius 97.3 cm. They are then collected in a cup after passing through a slit of width 1.20 mm and a height of 1.14 cm. (*a*) What is the magnitude of the (perpendicular) magnetic field in the separator? If the machine is designed to separate out 90.0 mg of material per hour, calculate (*b*) the current of the desired ions in the machine and (*c*) the internal energy dissipated in the cup in 1.00 h.

25. A neutral particle is at rest in a uniform magnetic field of magnitude B. At time $t = 0$ it decays into two charged particles each of mass m. (*a*) If the charge of one of the particles is $+q$, what is the charge of the other? (*b*) The two particles move off in separate paths both of which lie in the plane perpendicular to **B**. At a later time the particles collide. Express the time from decay until collision in terms of m, B, and q.

26. A deuteron in a cyclotron is moving in a magnetic field with an orbit radius of 50 cm. Because of a grazing collision with a target, the deuteron breaks up, with a negligible loss of kinetic energy, into a proton and a neutron. Discuss the subsequent motions of each. Assume that the deuteron energy is shared equally by the proton and neutron at breakup.

27. (*a*) What speed would a proton need to circle the Earth at the equator, if the Earth's magnetic field is everywhere horizontal there and directed along longitudinal lines? Relativistic effects must be taken into account. Take the magnitude of the Earth's magnetic field to be 41 μT at the equator. (*b*) Draw the velocity and magnetic field vectors corresponding to this situation.

28. Compute the radius of the path of a 10.0-MeV electron moving perpendicular to a uniform 2.20-T magnetic field. Use both the (*a*) classical and (*b*) relativistic formulas. (*c*) Calculate the true period of the circular motion. Is the result independent of the speed of the electron?

29. Ionization measurements show that a particular nuclear particle carries a double charge ($= 2e$) and is moving with a speed of $0.710c$. It follows a circular path of radius 4.72 m in a magnetic field of 1.33 T. Find the mass of the particle and identify it.

30. The proton synchrotron at Fermilab accelerates protons to a kinetic energy of 500 GeV. At this energy, calculate (*a*) the speed parameter and (*b*) the magnetic field at the proton orbit that has a radius of curvature of 750 m. (The proton has a rest energy of 938 MeV.)

31. A 22.5-eV positron (positively charged electron) is projected into a uniform magnetic field $B = 455$ μT with its velocity vector making an angle of 65.5° with **B**. Find (*a*) the period, (*b*) the pitch p, and (*c*) the radius r of the helical path. See Fig. 33.

32. In Bohr's theory of the hydrogen atom the electron can be thought of as moving in a circular orbit of radius r about the proton. Suppose that such an atom is placed in a magnetic field, with the plane of the orbit at right angles to **B**. (*a*) If the electron is circulating clockwise, as viewed by an observer sighting along **B**, will the angular frequency increase or decrease? (*b*) What if the electron is circulating counterclock-