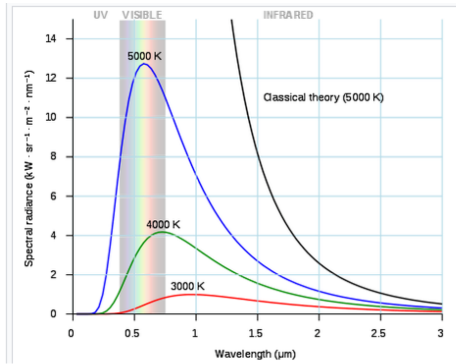
Black Body Radiation



https://en.wikipedia.org/wiki/Planck%27s_law

A black body is a hypothetical radiator that absorbs and emits all wavelengths of light uniformly. The emission spectrum B(λ,T)[W/sr-m2-Hz] for a radiator at temperature T is given by the Planck Distribution.

$$B(\lambda,T)=\frac{2hc^2}{\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}}-1}\left[W/sr-m^2-Hz\right] \qquad (1)$$

Rayleigh and Jeans used *classical oscillator theory* to predict the long wavelengths behavior. Atoms confined to oscillations in a region of dimension L would produce waves of harmonics of $\lambda \le L$, $L/2$, $L/3$ ... with $n\lambda \sim L$, similar to an organ pipe.

In 3D the number of modes per unit volume, $n/V \sim 1/\lambda^3$ ($V=L^3$). $\mathbf{dE/d\lambda \sim kT/\lambda^4}$ arriving at the Rayleigh and Jeans Law. The spectrum falls to zero at long wavelengths matching observation. But a short wavelengths the expression diverges. This was called the **UV Catastrophe**.

To solve the small wavelength problem Planck remembered the Maxwell-Boltzmann distribution for molecular velocities and proposed that the light emitted from a black body must follow a similar pattern. He proposed the oscillators would emit light in energy packets $\Delta E = hf = hc/\lambda$ implying 1) fewer vibrator modes with large energy and 2) many modes with small energies.

1)At short wavelengths the packet ΔE grew larger and larger, and only a few modes could exist by conservation of energy where $\Delta E < E_{BLACK\ BODY}$.
2)For large wavelengths the packet ΔE was very small and consistent with the Rayleigh-Jean approach.

Thus Planck's energy packet hypothesis $\Delta E=hf$ solved the Black Body Radiation problem. Einstein later ascribed each energy packet (mode) to a particle of light called a *photon*.

Wein Displacement Law Wein showed that for a black body with peak wavelength λ_{max} that $\lambda_{max}T = b$ with $b=2.898\text{x}10^6$nm-K. This follows from the fact that each vibrational mode in a radiator contrbutes on average about $5kT \approx hc/\lambda$ or $\lambda T \approx 1/5\ hc/k$

$$\lambda_{max}T = 2.898\text{x}10^6 \text{ nm-k}$$

By finding λ_{max} , the maximum wavelength of the black body, the temperature T of the black body radiator can be calculated.

Stephan-Boltzmann Law

Based on the Planck distribution one can integrate over the black body emission distribution and find the power radiated per meter^2 - Kelvin^4 is given by the Stephan-Bolzmann Law

$$J\,[\frac{W}{m^2-k^4}]=\varepsilon\sigma T^4$$

For perfect black bodies ε=1. For grey bodies ε<1. σ=5.67x10^{-8} Wm^{-2}K^{-4}

The power radiated at temperature T is $\boxed{P=J\cdot A=\varepsilon\sigma T^4\cdot A}$

Example-1 a 100 W light bulb filament of radius d=0.46mm and L=58-mm would have an outside area A=2πrL=1.676e-4 m^2. Assuming a black body emission with ε=1.

$$T=\sqrt[4]{\frac{P}{\sigma A}}=\sqrt[4]{\frac{100W}{5.67x108Wm^{-2}\,1.676x10^{-4}m^2}}\;K\;=\boxed{1800^o K}$$

Here the difficulty is determination of the filament size!

Example-2 Derive the Rayleigh-Jeans Law for a black body radiator from the Planck distribution (1) by Taylor serires expansion hc/λ << kT or hν << kT. (Low energy photons as compared to thermal temperatures!)

$$e^x=1+x+\frac{x^2}{2}+....\frac{x^n}{n!}\approx 1+x\quad x<<1\;\; where\;\; x=\frac{h\nu}{kT}$$

$$B^{RJ}(\lambda,T)=\frac{2h}{c^2\lambda^5}\frac{1}{e^{\frac{hc/\lambda}{kT}}-1}\approx\frac{2h}{c^2\lambda^5}\frac{1}{\cancel{1}+\underbrace{\frac{hc/\lambda}{kT}}_{x}+...-\cancel{1}}=\boxed{\frac{2kT}{c^3\lambda^4}}$$