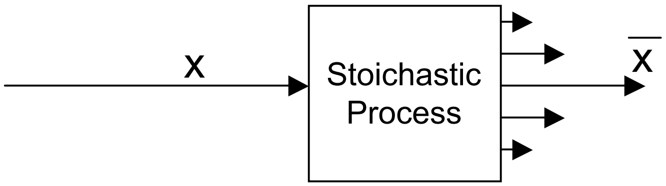
January 20, 06

Experimental Errors

Statistical Error-

Caused by random process which experimenter can not control. A symmetric type of error.



$x = 1/N \, \Sigma \, xi$	Average
$\Delta x = \{ 1/N \, \Sigma \, (xi - x)^2 \}^{1/2}$	RMS Error
$G(x) = 1/\sigma \, (1/2\pi)^{1/2} \exp\{-(x- x)^2/2\sigma^2\}$	Gaussian

As $N \rightarrow \infty$ $\quad \bar{x} \rightarrow x$ and $\Delta x \rightarrow \sigma$

Systematic Error-

Shifts in x up or down due to experimenter error, misalignment, apparatus, background noise. etc.

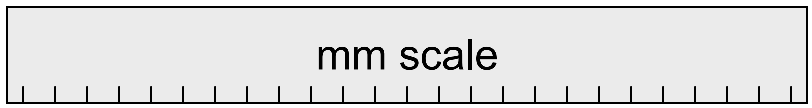
$+\Delta x1 \quad -\Delta x2 \quad \ldots\ldots$

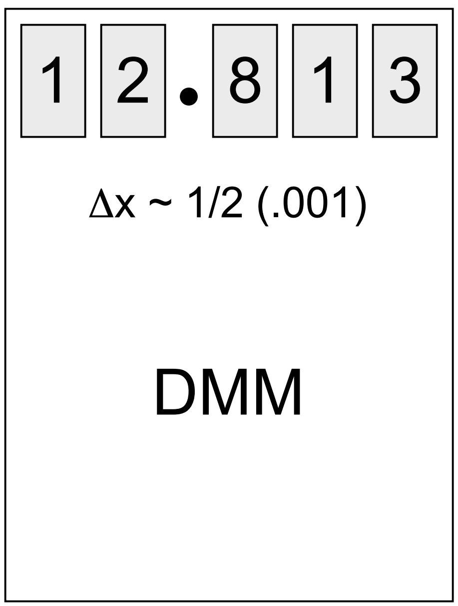
Measurement $= X \pm \Delta X(sys) \pm \Delta X(sys)$

Single Measurements

- Often we make a single measurement of a quantity. We must then make the best estimate of Δx from knowledge of the measuring device or other means.

$\Delta x \sim 1/2$ mm.





Counting Errors

$\Delta x \sim 1/ \text{Sqrt}(N)$

Propagation of Errors

•It is common that a number of measurements are made and functionally combined to obtain a final answer. We can use differential arithmetic to obtain the error on the combination. The bracket < > means average value.

Consider measuring factors a and b with associated errors $da \sim \Delta a$ and $db \sim \Delta b$:

Addition or Subtraction

$f = a + b$ or $f = a - b$

$df = da + db$ *-differential*

$(df)^2 = (da)^2 + (db)^2 + 2\,(da)(db)$ *-square*

$\langle (df)^2 \rangle = \langle (da)^2 \rangle + \langle (db)^2 \rangle + 2\,\langle (da)(db) \rangle$ *-take average (mean)*

variance ... *covariance* → 0

If measurements a and b are uncorrelated the cross term vanishes because alternating sign of da and db!

$\Delta f^2 = \Delta a^2 + \Delta b^2$ *-variance*

$\Delta f_{RMS} = (\Delta a^2 + \Delta b^2)^{1/2}$ -root-mean-square

Multiply or Ratio

$f = a\,b$ or $f = a/b$

$df = b\,da + a\,db$ — *-differential*

$(df/f)^2 = (da/a)^2 + (db/b)^2 + 2\,(da/a)(db/b)$ — -divide by f and *square*

$\langle (df/f)^2 \rangle = \langle (da/a)^2 \rangle + \langle (db/b)^2 \rangle$ — *-take average (mean)*

(the cross term $2\,(da/a)(db/b)$ averages to 0)

$(\Delta f/f)^2 = (\Delta a/a)^2 + (\Delta b/b)^2$ — -variance

$\Delta f/f = \{ (\Delta a/a)^2 + (\Delta b/b)^2 \}^{1/2}$ — -root-mean-square

Constants

$Z = n\,A$

$dZ = n\,dA$

$\Delta Z = n\,\Delta A$

Logarithm

$Z = \ln\{A\}$

$dZ = dA/A$

$\Delta Z = \Delta A/A$

Exponential

$Z = \exp\{A\}$

$dZ = dA\,\exp\{A\}$

$\Delta Z/Z = \Delta A$

Combinations

In a calculation where we have a combination of terms try to break the function in to fundamentals parts. For example:

$f = ab/c + d = A + B$ where $A = ab/c$ and $B = d$

$\Delta A/A = \{(\Delta a/a)^2 + (\Delta b/b)^2 + (\Delta c/c)^2\}^{1/2}$

$\Delta B = \Delta d$

$\Delta f = (\Delta A^2 + \Delta B^2)^{1/2} = (ab/c)\,\{(\Delta a/a)^2 + (\Delta b/b)^2 + (\Delta c/c)^2\}^{1/2} + \Delta d^2$

Example

Let f = ab + d where a=2.0+-1.0, b=6.0+-1.0, d=3.0+-2.0

Find f and Δf.

f = A + B A=ab B=d

$\Delta f = \{ (\Delta A)^2 + (\Delta B)^2 \}^{1/2}$

$\Delta A/A = \{ (\Delta a/a)^2 + (\Delta b/b)^2 \}^{1/2} = \{ (1/2)^2 + (1/6)^2 \}^{1/2} = 0.523$

$\Delta A = 0.523\ (12) = 6.32$

$\Delta B = 2.00$

f = 12 + 3 = 15.00

$\Delta f = (\Delta A)^2 + (\Delta B)^2 = \{ 6.32^2 + 2.0^2 \}^{1/2} = 6.63$

f =15.0 +-6.6

Wiggle Method

It is sometimes okay to just evaluate the function f(a,b,c,..) by varying a,b,c,…. By their errors Δa, Δb, Δc,….   Up and down one at a time.

$\Delta fa = | f(a+ \Delta a) - f(a- \Delta a)|/2$
$\Delta fb = | f(b+ \Delta b) - f(b- \Delta b)|/2$

Then define $\Delta f = \{ (\Delta fa)^2 + (\Delta fa)^2 + (\Delta fa)^2 + .. \}^{1/2}$

Use the Wiggle method to determine Δf in the before example.

Let   f = ab + d   where a=2.0+-1.0, b=6.0+-1.0, d=3.0+-2.0

Find f and Δf.