Chapter - 1 Direct Current Circuits

- RESISTANCE
- KIRKOFFS LAW and LOOP-MESH METHOD
- VOLTAGE DIVIDER
- INTERNAL RESISTANCE and
OUTPUT IMPEDANCE
- HOW TO MEASURE OUTPUT IMPEDANCE OF A
DEVICE
- THEVENIN's THEOREM

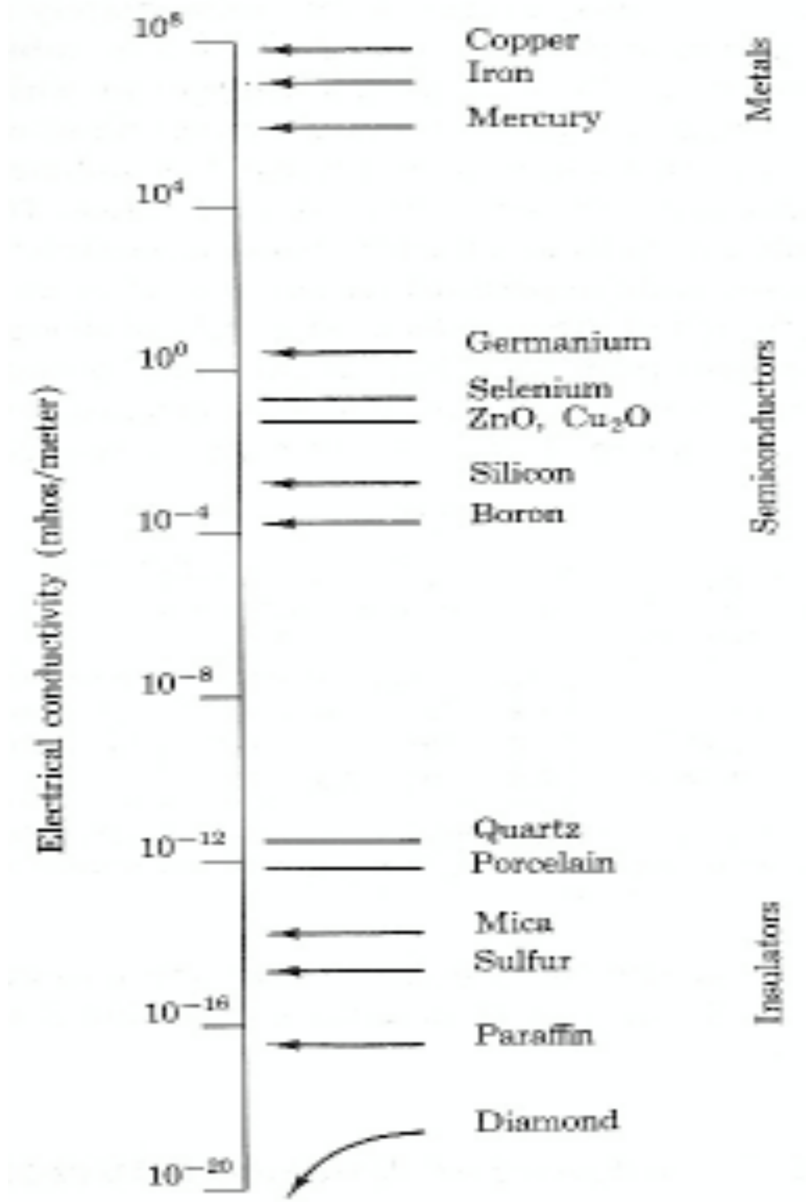
Resistivity

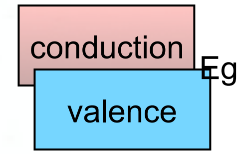
- ρ property of a material characterizing electron flow
- Σ=1/ρ conductivity

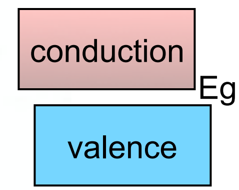
•Metal – Energy gap, Eg, between Valence and Conduction bands overlap.

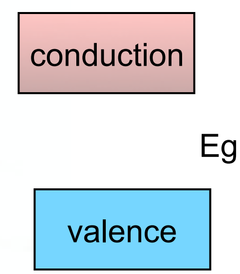
•Semiconductor - small Energy gap between valence and conduction bands, of order 1 eV.

•Insulator – large Energy gap between valence and conduction bands, of order 10 eV.





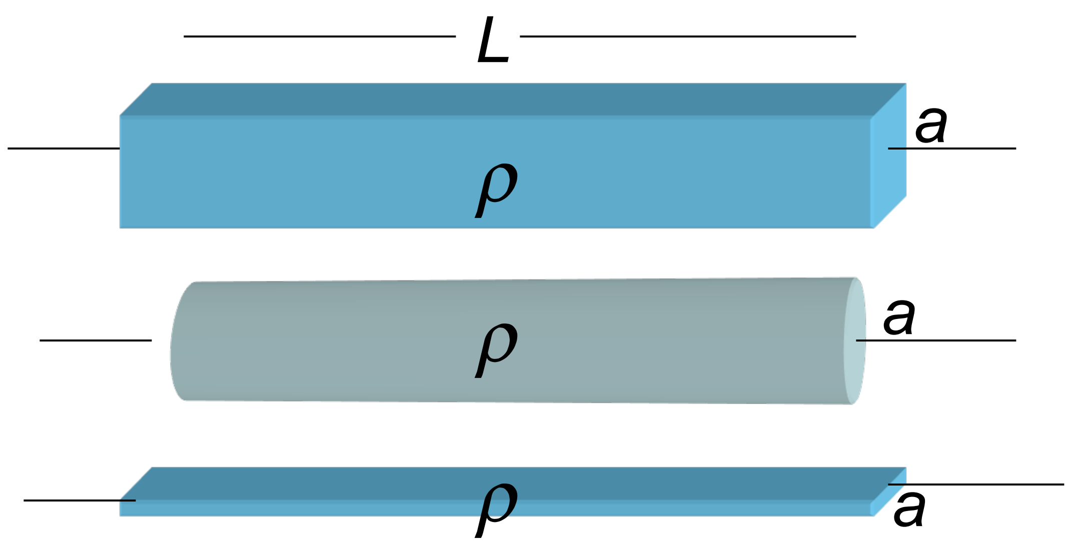




Resistance

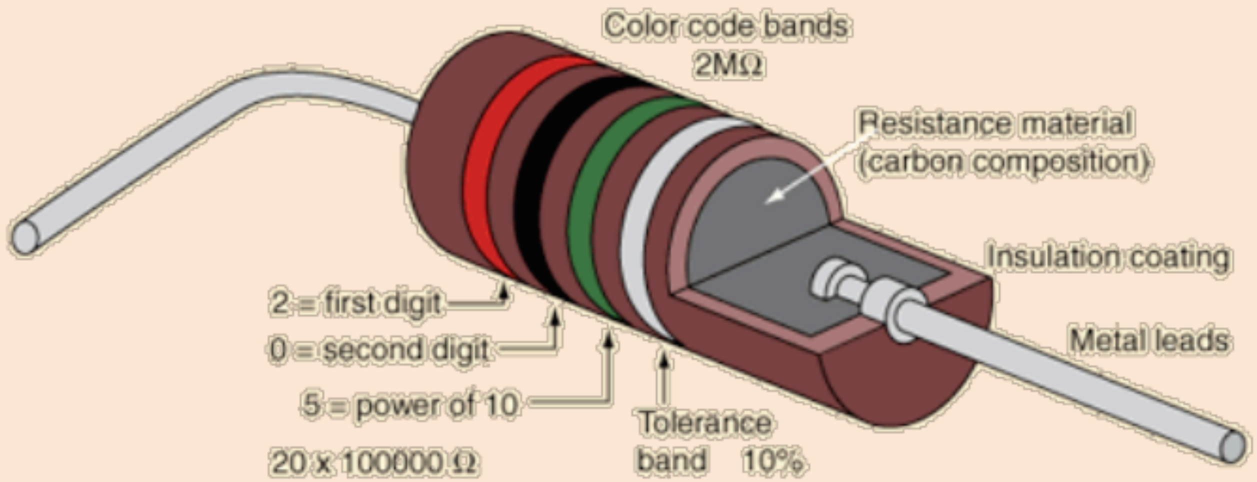
- Materials have different resistivities ρ
- $R = \rho(L/a)$ = resistance (Ohms)

  L=length a = cross sectional area



Carbon Resistors

There are many types of resistors, both fixed and variable. The most common type for electronics use is the carbon resistor. They are made in different physical sizes with power dissipation limits commonly from 1 watt down to 1/8 watt. The resistance value and tolerance can be determined from the standard resistor color code.

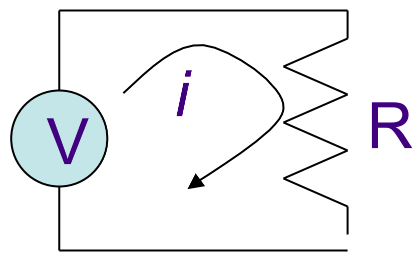


Resistor Color Code		
Resistance value, first three bands. 1st band - 1st digit 2nd band - 2nd digit 3rd band - number of zeros.	0	Black
	1	Brown
	2	Red
	3	Orange
	4	Yellow
	5	Green
	6	Blue
	7	Violet
	8	Gray
	9	White
4th band, tolerance	5%	Gold
	10%	Silver
	20%	No band

Ohm's Law

- The Voltage drop across a circuit element R is proportional to the current flowing through it.

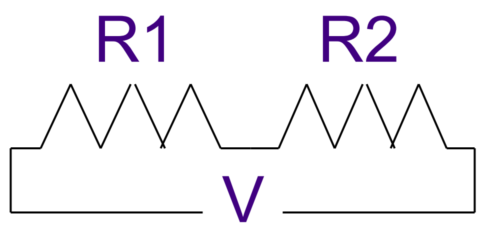
$$i = V/R \text{ or } V = i\,R$$



- The constant of proportionality is the resistance R or impedance.

Series and Parallel Connections

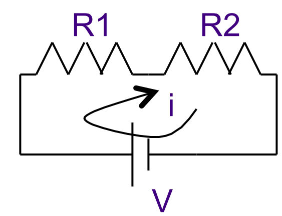
- Resistances in series directly add.



When a voltage V is placed across the pair in series we write

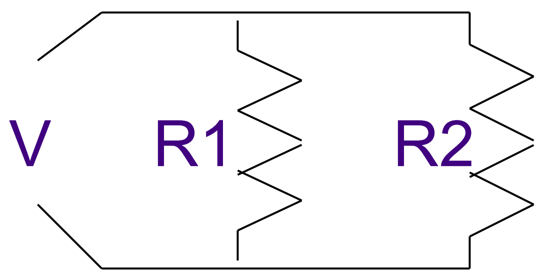
V = i(R1+R2) = iR

R = R1 + R2



R = R1+R2

- Resistances in parallel follow the inverse rule of addition.

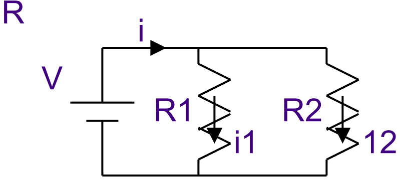


When a voltage V is placed across the pair in parallel we write

V = i1 R1 = i2 R2 = iR

i = i1 + i2

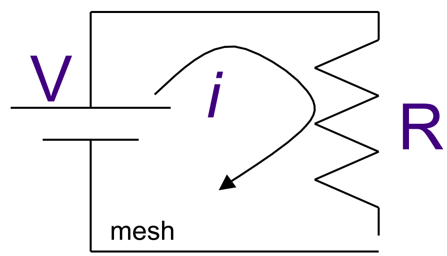
1/R =1/R1 + 1/R2



1/R = 1/R1+1/R2

Kirchoffs Circuit Laws (by Mesh Currents)

(1) Choose mesh currents to run CW in each circuit loop (convention).
(2) Add the battery emfs when in direction of mesh current, subtract when opposing mesh current.
(3) Write Kirkoff’s law for each mesh
(4) Subtract the voltage drops (-iR) across resistor in the mesh, add (+iR)the adjacent mesh.
(5) Physical currents through components can be determined from i1 and i2.

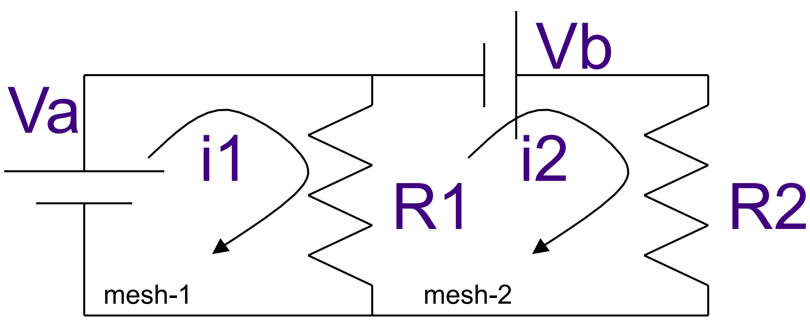


$+V - i\,R = 0$ (consevation of energy)

Battery emf In directio of Mesh current

Subtract Voltage drop

Add voltage from adj loop

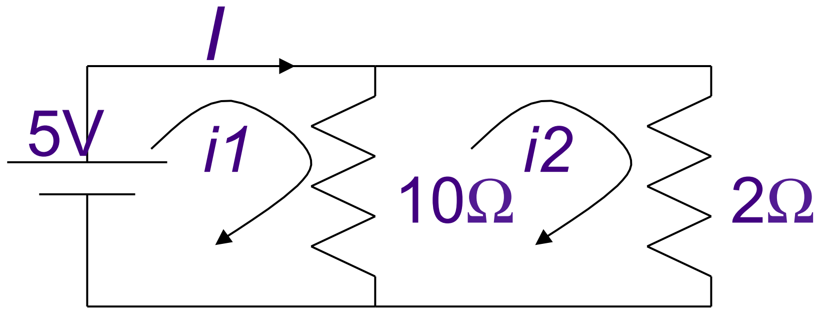


$$+Va - i1\,R1 + i2\,R1 = 0$$
$$+Vb - i2\,R2 - i2\,R1 + i1\,R1 = 0$$

$$Va = i1\,R1 - i2\,R1$$
$$Vb = -i1\,R1 + i2\,(R1+R2)$$

solve

Example



Find *I1*, *I2*, *I* ?

$5V - 10\,i1 + 10\,i2 = 0$

$0 - 2\,i2 - 10\,i2 + 10\,i1 = 0$

1) $5V = 10\,i1 - 10\,i2$

2) $\underline{0 = -10\,i1 + 12\,i2}$

$5V = 0 + 2\,i2$ adding 1) + 2) --> $i2 = 5/2$ A

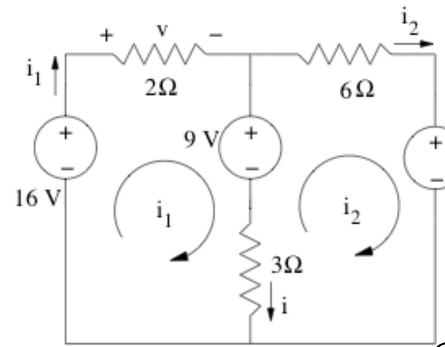
$i1 = (12/10)i2 = 3A$

$i2 = 5/2$ A

$I = i1 = 3A$

Example from web!

Example Find i and v.
Using mesh-current method:



Mesh 1: $\quad 2i_1 + 9 + 3(i_1 - i_2) - 16 = 0$

Mesh 2: $\quad 6i_2 + 6 + 3(i_2 - i_1) - 9 = 0$

$$\begin{cases} 5i_1 - 3i_2 = 7 \\ -3i_1 + 9i_2 = 3 \end{cases} \rightarrow \begin{cases} i_1 = 2 \text{ A} \\ i_2 = 1 \text{ A} \end{cases}$$

The problem unknowns, i and v can now be found from the mesh currents:

$$i = i_1 - i_2 = 1 \text{ A}$$

$$v = 2i_1 = 4 \text{ V}$$

i1 opposes battery polarity

16 - 2 i1 -9 -3 i1 +3 i2 = 0
-6 -3 i2 +3 i1 +9 -6 i2 = 0

i2 with battery polarity

7 = 5 i1 -3 i2
3 = -3 i1 +9 i2

21 = 15 i1 -9 i2
3 = -3i1 +9 i2

24 = 12 i1 **i1 = 2A**
7 = 10 -3 i2
-3 = -3 i2 **i2 = 1A**

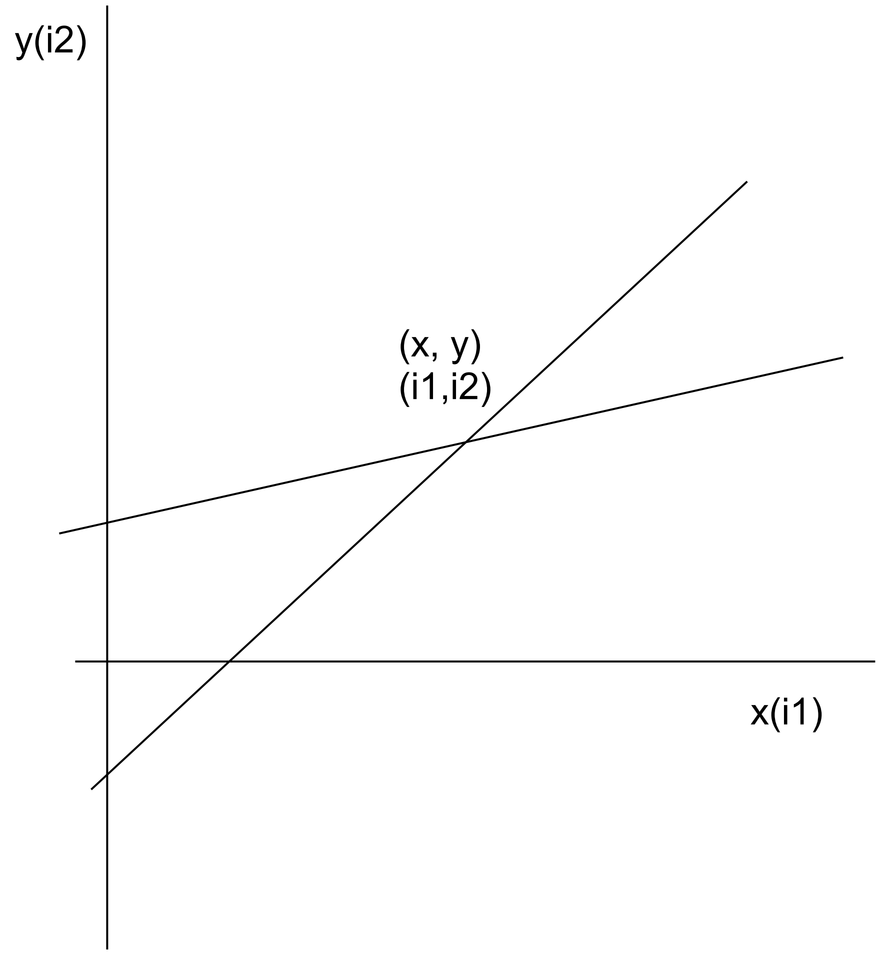
V = 2 1i = 4V

2LOOP Graphical Solution

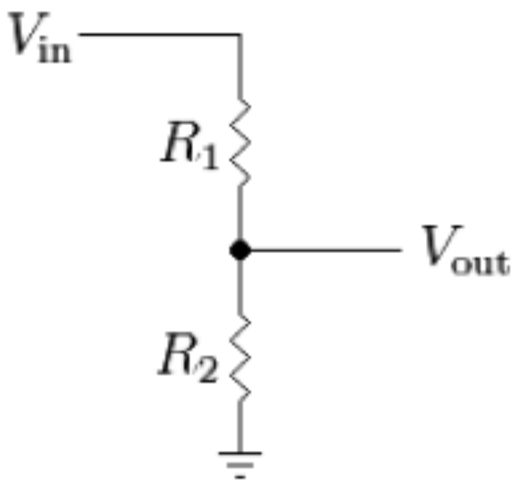
Rewrite equaions in terms of straight lines y= mx + b

$$y = x - \frac{Va}{R1}$$

$$y = \frac{R1}{(R1 + R2)}x + \frac{Vb}{(R1 + R2)}$$



Voltage Divider

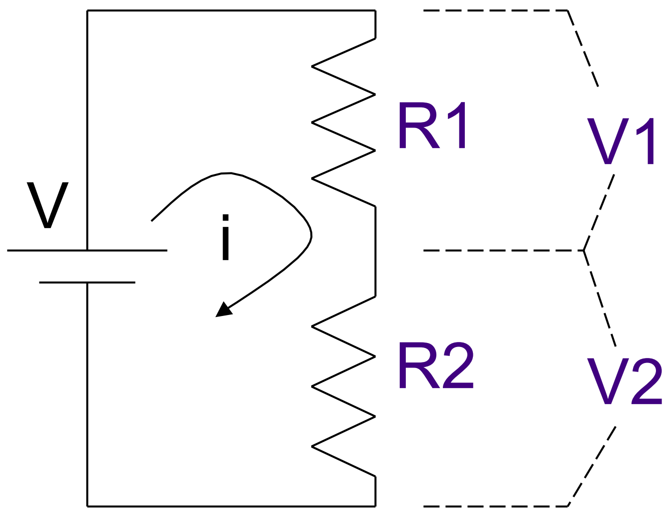


A simple resistive voltage divider allows us to adjust the input voltage to a lower level.

$$Vout = \left(\frac{R2}{R1 + R2} \right) Vin$$

This high voltage probe uses a voltage divider to allow us to measure a large voltage by dropping it to a lower range.

Voltage Divider



$V - i\,R1 - i\,R2 = 0$
$V = i(R1+R2)$
$i = V/(R1+R2)$

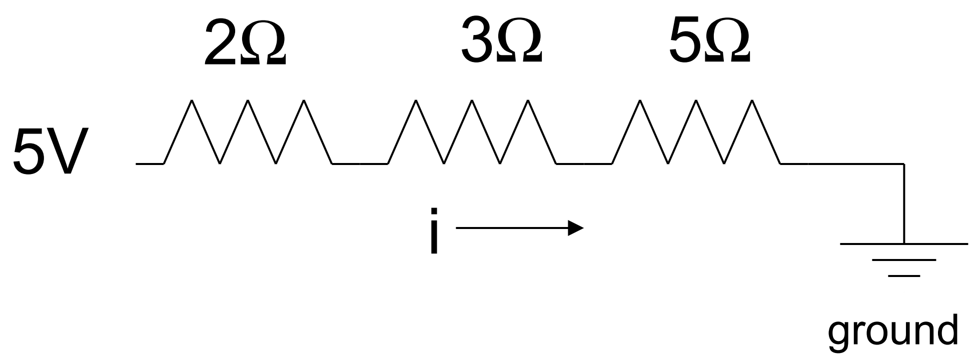
$V2 = i\,R2$
$= R2/(R1+R2)\ V$
$= (R2/Rtot)\ V$

Voltage drop on i[th] resistor is proportional to ratio of Ri to Rtot! $\quad Vi = \{ Ri/Rtot \}\ V$

$V1 = R1/Rtot \times V$

$V2 = R2/Rtot \times V$

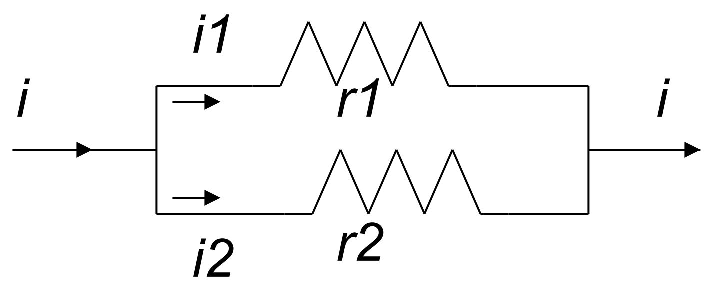
Example



•What is the voltage drop across the 5Ω resistor?
Ans: $V_{5\Omega}$ = (5/10) 5V = 2.5V

Current Division

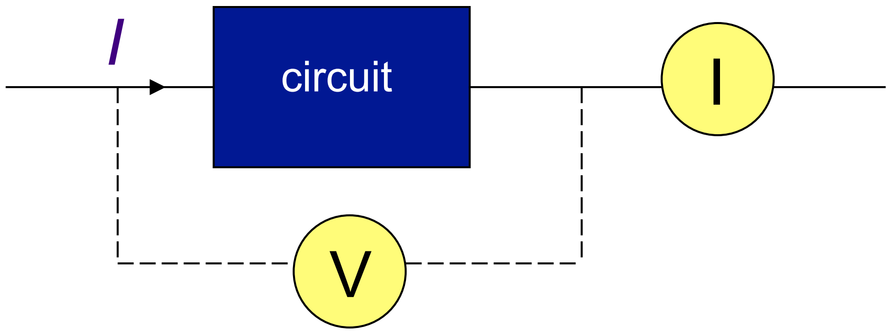
Current will take the path of least resistance, dividing Itself by inverse proportion with *i = i1 + i2*



$$i1 = \frac{V}{r1} = \frac{i\ r}{r1} = \frac{r_{tot}}{r1}i$$

$$i2 = \frac{V}{r2} = \frac{i\ r}{r2} = \frac{r_{tot}}{r2}i$$

Power Dissipated by a Circuit Element



P = I V

P = d/dt U   U=qV

P = d/dt qV

P = dq/dt V = I V

•The power dissipated by a circuit element is given by P=IV,
 I = the current passing through the element.
 V = voltage drop across the element.

$$How\ long\ does\ a\ 9volt\ last\ under\ 1ma\ load?$$

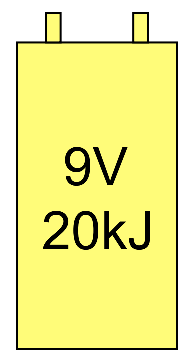
$$Let\ U = 20kJ \quad V = 9V \quad I = 1mA$$

$$P = dU/dt$$

$$dt = \frac{1}{IV}dU \quad \rightarrow t = \frac{1}{9mW}20kJ = 2.2e6s = 620\text{h}$$

•For Ohmic circuit elements, V=I R, we can also write:
 P=I^2R
 P=V^2/R

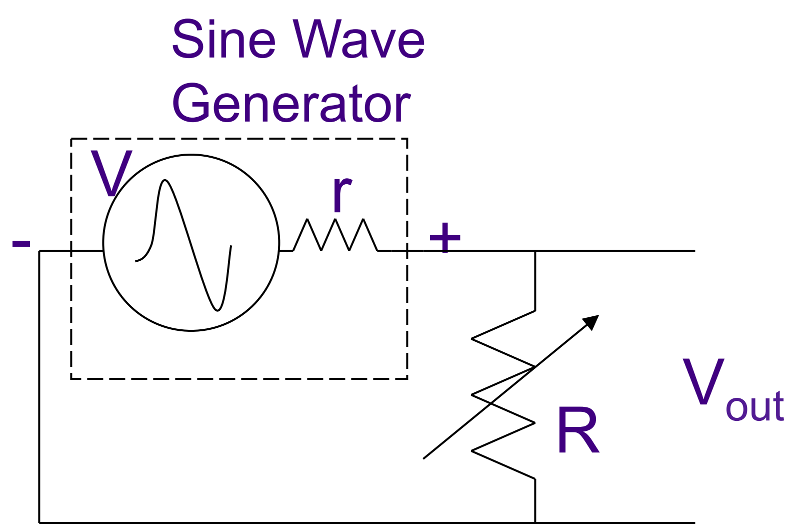
•Light bulbs (nonOhmic) Resistors(Ohmic)

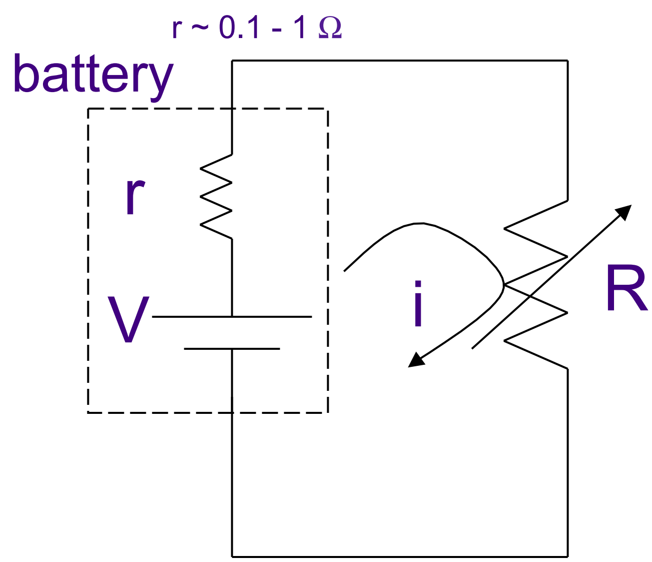


Internal Resistance and Output Impedance

- Every Source of Emf has some small internal resistance.
- A signal generator has an internal resistance related to its output Impedance r or z. (z~50 Ohm).
- A voltage divider circuit can be used to measure r and z.

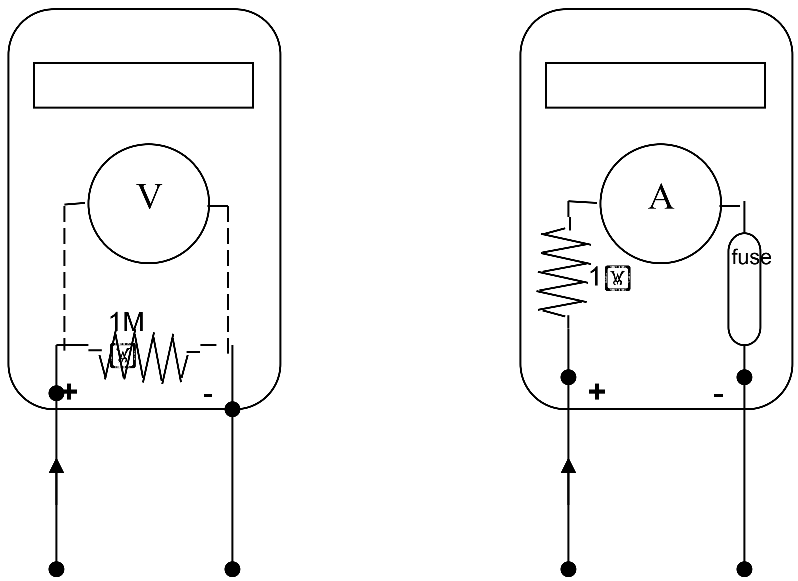
Adjust R until V_{out}=1/2 V ! Then R = internal resistance !





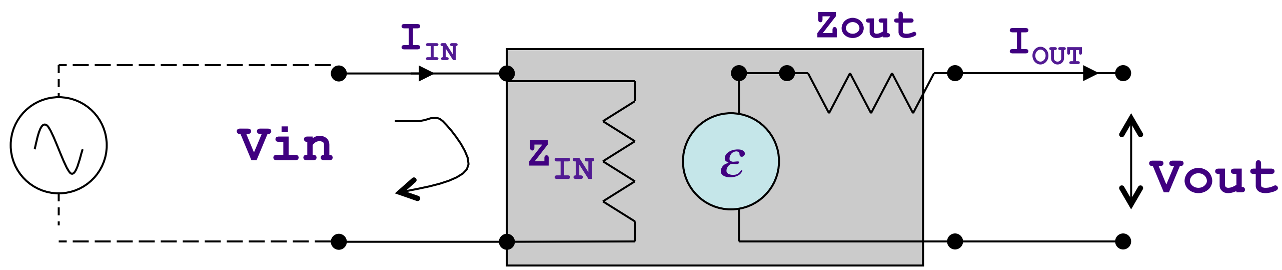
Input Impedance of Voltmeter and Ammeter

- All input devices has some small internal resistance to the current flowing into it. (Impedance to ground or negative terminal)
- A voltmeter has a **high input impedance** to limit the current flowing in to the measuring device.
- An ammeter wants to divert all the current into it and therefore has a very **low input impedance**.



Input and Output Impedance

- Consider an I/O circuit to be an element which transforms an input voltage waveform to and output voltage waveform.



- The input voltage source V_{IN} sees an effective input impedance Z_{IN} wrt to the input current $\mathbf{I}_{IN}=V_{IN}/Zi_{IN}$.

- The output current $\mathbf{I}_{OUT}$ is driven through an effective series resistance Zout with $V_{OUT}=\mathbf{I}_{OUT}Z_{OUT}$

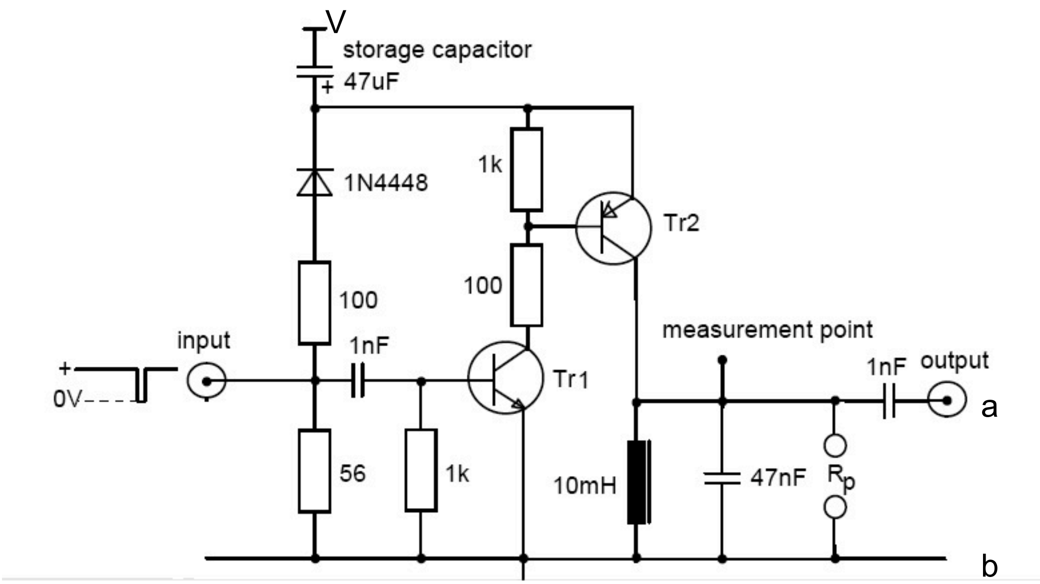
- All I/O devices can be characterized by an input and output impedance.

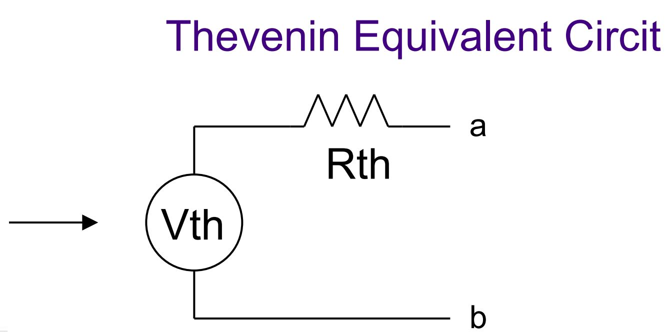
Thevenin's Theorem(1)

Consider a complex circuit of which we are dealing with a small part.

1) to calculate the current through (or voltage across) a component in any circuit.

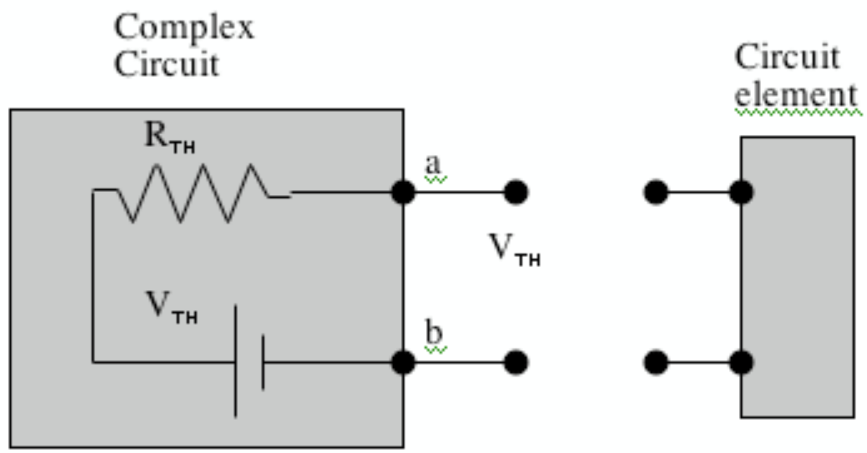
2) or develop a constant voltage equivalent circuit which may be used to simplify the analysis of a complex circuit





Thevenin's Theorem(2)

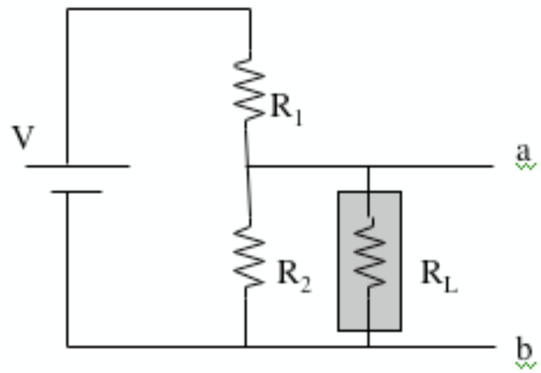
Any network as observed from two terminals is equivalent to a source of emf and impedance in series , connected to the two terminals.



(1) The element to be studied is removed from the circuit.
(2) $\mathbf{V_{TH}}$ is computed or measured from knowledge of the circuit
(3) All voltage sources are shorted and all current sources are opened=circuited.
(4) The equivalent resistance $\mathbf{R_{TH}}$ of the circuit is calculated or measured.
(5) The series connection of R_{TH} and V_{TH} are made and the element to be studied is reinserted into the circuit

Thevenin's Theorem(2)

Voltage Divider with Load



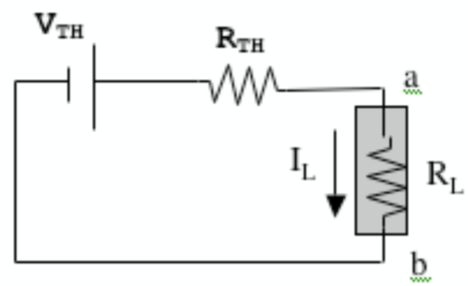
What is the current through the load?
V = 50V , R1= 100Ohm, R2=100Ohm
RL=50Ohm.

IL = **0.25A**

Vth = 50V (100Ohm/200Ohm) = 25V
Rth = 50 Ohm
Rtot = Rth + RL = 50Ohm + 50Ohm =100Ohm

Thevenin Circuit

(1) Remove load R_L (grey component)
(2) V_{TH} = Vab = V R2/(R1+R2).
(3) With battery shorted find Rab.
RTH = Rab = R1 R2/(R1+R2) parallel
(4) Replace Load in to series connection of V_{TH} and R_{TH}.

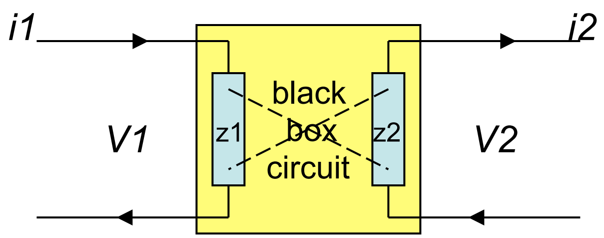


Thevenin Equivalent Circuit

http://esdstudent.gcal.ac.uk/Thevenin3.htm

2 Port Network (extra)

A two port network is a linear mathematical model that can be used to analyse a circuit and other systems, if the input and output voltages and currents can be isolated. This linear model can be generalized to other flows (I) and potentials (V).



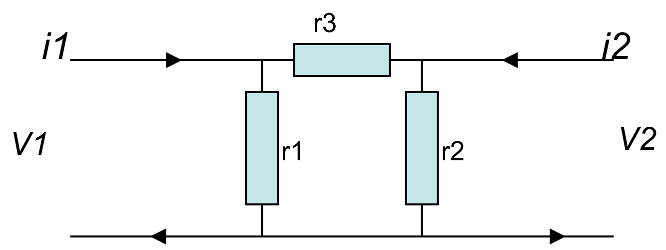
$$V1 = z_{11}\ I1 + z_{12}\ I2$$
$$V2 = z_{21}\ I1 + z_{22}\ I2$$

z's are the open circuit impedance parameters and can be evaluated zero current or no load conditions.

$$z_{11} = \frac{V1}{I1}\Big|_{I2=0} \qquad z_{12} = \frac{V1}{I2}\Big|_{I1=0} \qquad z_{21} = \frac{V2}{I1}\Big|_{I2=0} \qquad z_{22} = \frac{V2}{I2}\Big|_{I1=0}$$

2 Port Network (extra)

Consider the simple circuit. Given V1,V2, r1,r2,r3, find the z paramaters for the network?



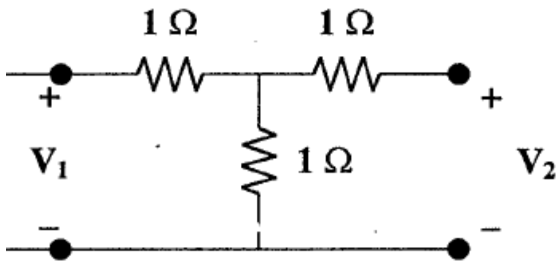
$$z_{11} = \left.\frac{V1}{I1}\right|_{I2=0} = R1 \parallel R2 + R3 = \left(\frac{1}{R1} + \frac{1}{R2+R3}\right)^{-1} = \frac{R1(R2+R3)}{R1+R2+R3}$$

$$z_{22} = \left.\frac{V2}{I2}\right|_{I1=0} = R2 \parallel R1 + R3 = \left(\frac{1}{R2} + \frac{1}{R1+R3}\right)^{-1} = \frac{R2(R1+R3)}{R1+R2+R3}$$

$$z_{12} = \left.\frac{V1}{I2}\right|_{I1=0} = \frac{I_{R1} \cdot R1}{I2} = \frac{\left(\frac{R1+R3}{R1+R2+R3}\right)\cancel{I2} \cdot R1}{\cancel{I2}} \quad \text{by current division}$$

$$z_{21} = \left.\frac{V2}{I1}\right|_{I2=0} = \frac{I_{R2} \cdot R2}{I1} = \frac{\left(\frac{R2+R3}{R1+R2+R3}\right)\cancel{I1} \cdot R2}{\cancel{I1}} \quad \text{by current division}$$

2 Port Network (example)



1) With side-2 open $z_{11} = V1 / I1 = 1 + 1 = 2\,\Omega$

$z_{11} = 2\Omega$

2) $z_{21} = V2 / I1 \qquad \text{where}\, V2 = I_{1\Omega} 1\Omega = I1$

$z_{21} = 1\Omega$

3) With side-1 open $z_{22} = 1 + 1 = 2\,\Omega$

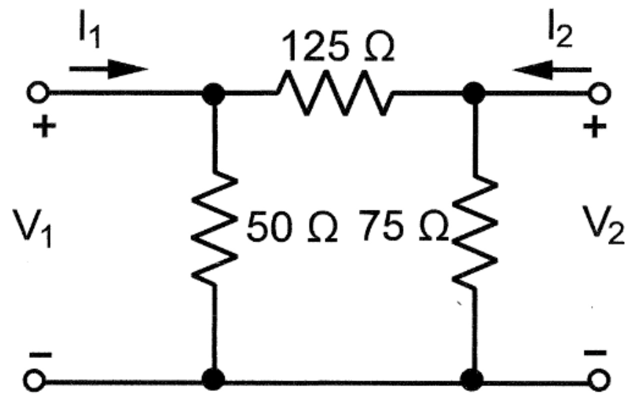
$z_{22} = 2\Omega$

4) $z_{12} = V1 / I2 \qquad where\, V1 = I_{1\Omega} 1\Omega = I2$

$z_{12} = 1\Omega$

For simple RLC neworks $z_{12} = z_{21}$. These are call **reciprocal networks**. Excitation of the input and output produce the veverable response in a reciprocal network.

2 Port Network (example)



1) With side-2 open circuited $z_{11} = V_1/I_1 = 50 \parallel (125+75) = 40\,\Omega$

   $z_{11} = 40\,\Omega$

2) z21= V_2/I_1 where $V_2 = 75\ I_{75}$

   $I_{75} = (\frac{50}{50+125+75}) I_1 = 0.2\ I_1$ by current division

   $z_{21} = 75 \times\ 0.2 I_1 / I_1 = 15\,\Omega$

3) With side-1 open circuited $z_{22} = V_1/I_1 = 75 \parallel (125+50) = 52.2\ \Omega$

4) z12= V_1/I_2 where $V_1 = 50\ I_{50}$

   $I_{50} = (\frac{75}{50+125+75}) I_2 = 0.3\ I_2$ by current division

   $z_{21} = 50 \times\ 0.3 I_1 / I_1 = 15\,\Omega$