THE TRANSFORMER EXERCISE

Through Lenz's Law we can introduce some of the basic formalism for the transformer. The magnetic flux $\Phi=\Phi_M$ and emf ε through a circuit or coil (N turns) is given by

$$\varepsilon = -\frac{d\Phi}{dt} \quad \text{and} \quad \Phi_M = N\,B\,A$$

The inductance L of the circuit is defined by $L = \dfrac{d\Phi}{dI}$. The emf can now be written as

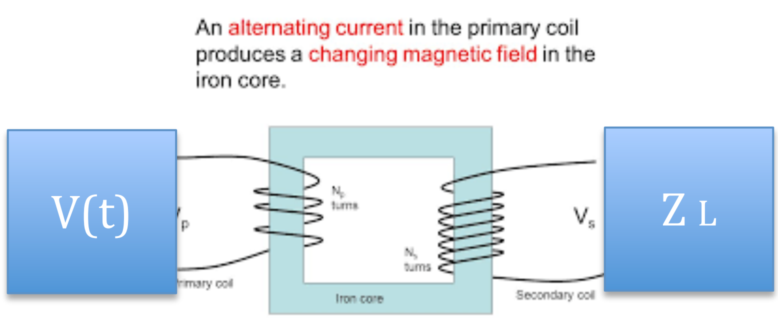
$$\varepsilon = -\frac{d\Phi}{dI}\frac{dI}{dt} = -L\frac{dI}{dt}$$

Consider a simple coil of *N1* turns on the primary and *N2* turns on the secondary. Assume the coils each have length ℓ and cross sectional area A. The steele laminations of the transformer have permeability κ_M *(κ~5000 μ_0 for Iron)*. We can write

$$B = \kappa_M \frac{N\,I}{\ell} \quad \text{and} \quad \Phi = \kappa_M \frac{N^2\,I}{\ell}\cdot A$$

$$L_{1,2} = \frac{d\Phi}{dI} = \kappa_M \frac{N_{1,2}^2}{\ell}\cdot A$$

Since the coils are strongly coupled together there is a strong mutual inductance between the primary and secondary coils $M_{12} = M_{21} = M \simeq k\sqrt{L_1 \cdot L_2}$ where coupling strength k varies from 0-1; k~0.95 for a well designed transformer.



Assume $V_p(t) = V_1 e^{i\omega t}$ and a load Z_L is attached to the secondary. Let I1 and I2 be clockwise currents in the primary and secondar coils. We can write Kirchoff's:

$$\varepsilon_1 = \overbrace{(i\omega L_1)}^{Z_{L1}}\,I_1 + \overbrace{(i\omega M)}^{Z_M}\,I_2$$

$$0 = \overbrace{(i\omega L_2)}^{Z_{L2}}\,I_2 + \overbrace{(i\omega M)}^{Z_M}\,I_1 + Z_L\,I_2$$

$$\varepsilon_1 = (i\omega L_1)\, I_1 + (i\omega M)\, I_2 \;\rightarrow \qquad \varepsilon_1 = (i\omega L_1)\, I_1 + (i\omega M)\, I_2$$
$$0 = (i\omega L_2) I_2 + (i\omega M) I_1 + Z_L\, I_2 \rightarrow \qquad 0 = \;(i\omega L_2 + Z_L)\, I_2 + i\omega M\, I_1$$

$$I_1 = \frac{-(i\omega L_2 + Z_L)}{i\omega M} I_2$$
$$\varepsilon_1 = \left((i\omega L_1) \frac{-(i\omega L_2 + Z_L)}{i\omega M} + (i\omega M) \right) I_2$$
$$\varepsilon_1 = \left(\frac{(i\omega L_1)(i\omega L_2 + Z_L) - \omega^2 M^2}{-i\omega M} \right) I_2$$
$$I_2 = \frac{-i\omega M}{(i\omega L_1)(i\omega L_2 + Z_L) - \omega^2 M^2} \varepsilon_1 \quad \text{and} \quad |I_2| = \sqrt{I_2 I_2^{\,*}}$$
$$I_1 = \frac{(i\omega L_2 + Z_L)}{((i\omega L_1)(i\omega L_2 + Z_L) - \omega^2 M^2} \varepsilon_1 \quad \text{and} \quad |I_1| = \sqrt{I_1 I_1^{\,*}}$$

Exercises:

1) Find expressions for $|I_1|$ and $|I_2|$.

2) If $L_1 = L_2 = 1\text{mH}$ and $Z_L = 50\Omega$, what angular frequency ω would $|I_2|$ be maximum. Let k=1?

3) Show that $|I_2|/|I_1| = N_1 / N_2$ with $Z_L=0$ and $L_1 = L_2$.