Review of Complex Numbers

We define the symbol $\mathrm{i} = \sqrt{-1}$ **or** $\mathrm{i} \bullet \mathrm{i} = -1$.

Cartesian representation

$z = x + i\,y$

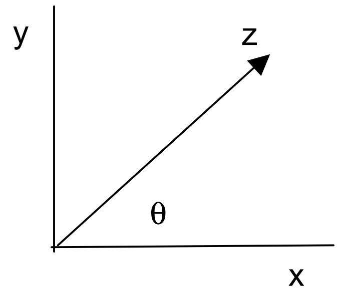
$x = \mathrm{Re}\{z\}$ **and** $y = \mathrm{Im}\{z\}$

Polar representation

$z = |z|\, e^{i\theta}$

$|z| = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}(y/x)$



Problems:

1) Find the magnitude and phase of

$z = \dfrac{10}{4+3i}$

$z = \dfrac{1+i}{1-i}$

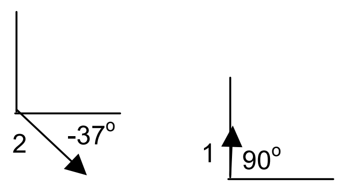
$z = 40/25 - 30/25i = 8/5 - 6/5i$, $|z| = (64/25+36/25)^{1/2} = 2$, $\phi = \tan^{-1}(-3/4) = -37^{o}$

$z = i$, $|z| = 1$ $\phi = \tan^{-1}(\infty) = 90^{o}$

2) Sketch in polar form

$z = \dfrac{10}{4+3i}$

$z = \dfrac{1+i}{1-i}$



3) Let $|Z|e^{i\theta}$. **Find the magnitude and phase of Z.**

$Z = \dfrac{-i/\omega C}{\omega L - i/\omega C}$

$$z = \frac{-i/\omega C}{\omega L - i/\omega C} = \frac{-i/\omega C(\omega L + i/\omega C)}{(\omega L - i/\omega C)(\omega L + i/\omega C)} = \frac{1/\omega^2 C^2 - iL/C}{\omega^2 L^2 + 1/\omega^2 C^2} = \frac{1 - i\omega^2 LC}{1 + \omega^4 L^2 C^2}$$

$$|z| = \sqrt{z^* z} = \sqrt{\frac{1 - i\omega^2 LC}{1 + \omega^4 L^2 C^2} \cdot \frac{1 + i\omega^2 LC}{1 + \omega^4 L^2 C^2}} = \frac{1 + \omega^4 L^2 C^2}{1 + \omega^4 L^2 C^2} = 1$$

$$\phi = \tan^{-1}(-\omega^2 LC/1)$$