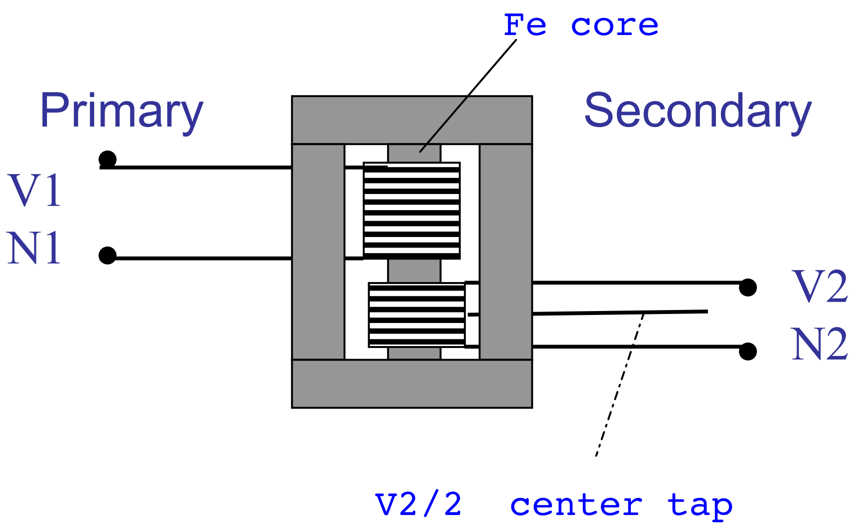
Chapter 4 - AC Circuits II

- Transformers and Impedance Matching
- Transmission Lines
- Fourier Waveforms

- Transformers- VAC



V1/V2 = N1/N2 ***voltage ratio = winding ratio***

I1 V1 = I2 V2 ***conservation of energy***

I1/I2 = V2/V1 = N2/N1

V=I Z ***Ohms AC Law***

Z1/Z2 = (V1/I1) / (V2/I2) =(I2/I1)(V1/V2)

Z1/Z2 =(N1/N2)^2 ***Impedance Matching***

Transformer core enhances performance

$L \sim \mu_{Fe}(\omega)\, \mu_o \times L_{air}$

Impedance matching - theory

$P = I V = I^2 R = [V/(r+R)]^2 R$

Maximum power is delivered at what value of R?

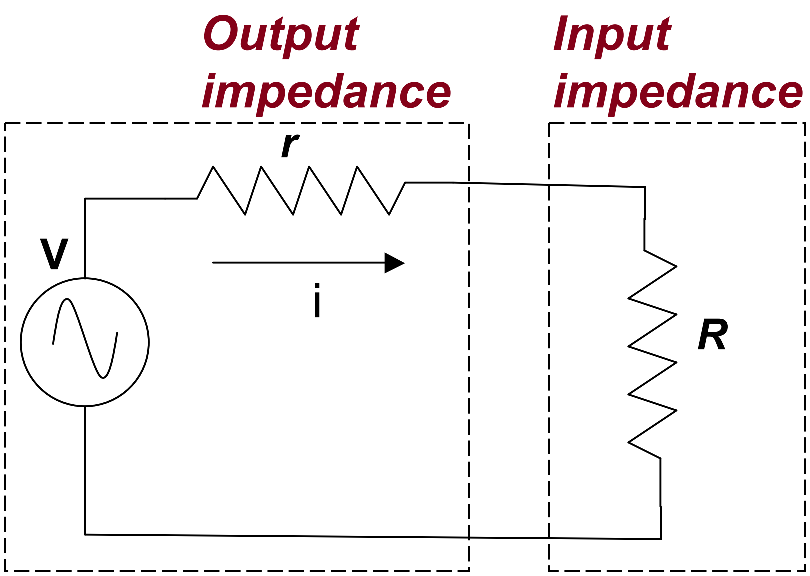
$$dP/dR = -2V^2R(r+R)^{-3} + V^2/(r+R)^{-2} = 0$$

$$-2V^2R(r+R)^{-3} + V^2/(r+R)^{-2} = 0$$

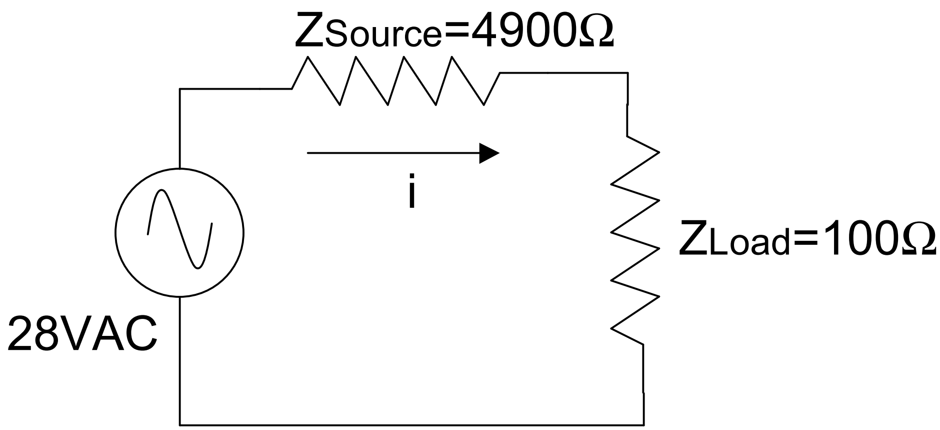
$$R/(r+R) = 1/2$$

$$R = 1/2r + 1/2R \;\text{-->}\; R = r\,!$$

Maximum power is transferred when Output impedance matches Input impedance!



Impedance matching - w transformer

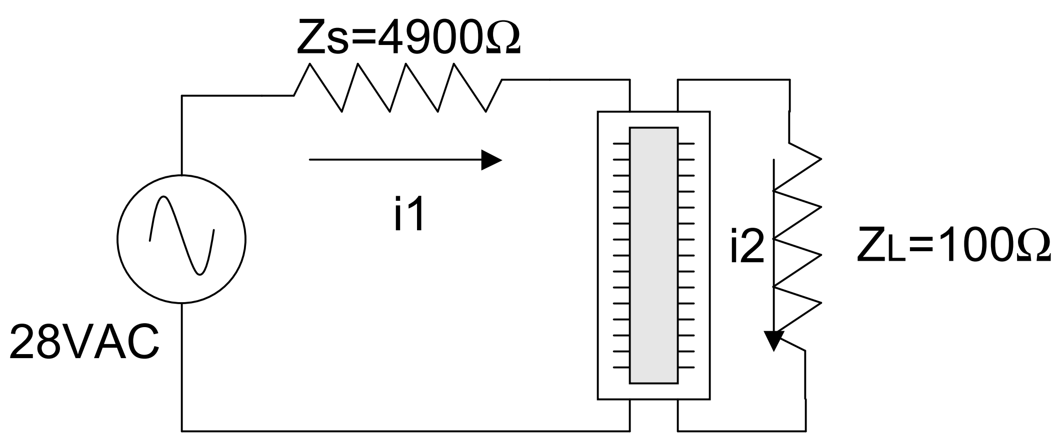


Without Transformer

i = 28/(4900+100) = 5.6mA

V_L = (100/5000)28V =0.56V

P_L = i V2 = 3.1 mW



With Transformer

$Z1/Z2 = (N1/N2)^2$

N1/N2 = sqrt(4900/100)=7

V2= (1/7)V1= (1/7) 28 = 4V

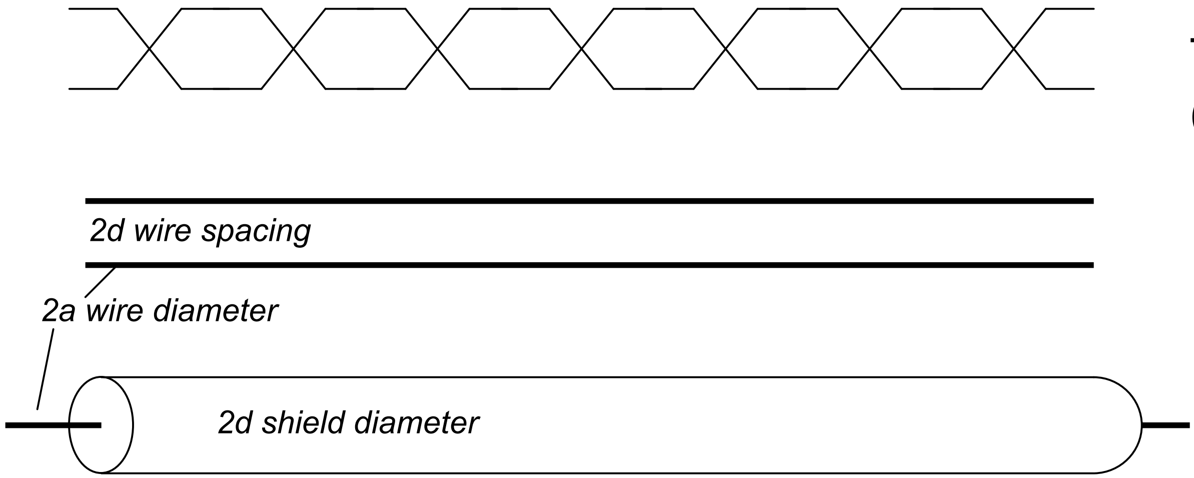
I2 = 4V/100Ω = .040A

P2 = (.040A)4V= 160mW !

I1 = i2(N2/N1)=.0057A

P1= (.0057A)28V = 160mW

Transmission Lines



Twisted pair (Z~100Ω)

Parallel conductor (Z~90Ω)

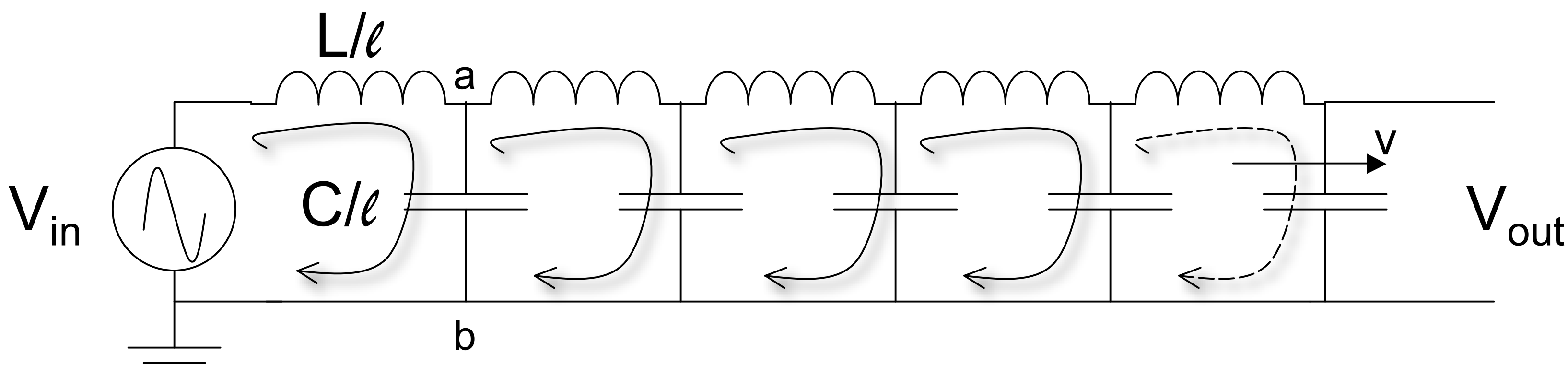
CoAxial (Z~50-100Ω)

$$Z_{parallel} = \left(\frac{\mu\mu_o}{\pi^2 \varepsilon\varepsilon_o} \right)^{1/2} \ln\left(\frac{2d}{a} \right)$$

$$Z_{coax} = \left(\frac{\mu\mu_o}{\pi^2 \varepsilon\varepsilon_o} \right)^{1/2} \ln\left(\frac{b}{a} \right)$$

http://www.eeweb.com/toolbox/wire-over-plane-inductance/

Transmission Lines



•We often use a cable or transmission line to transfer AC signals.

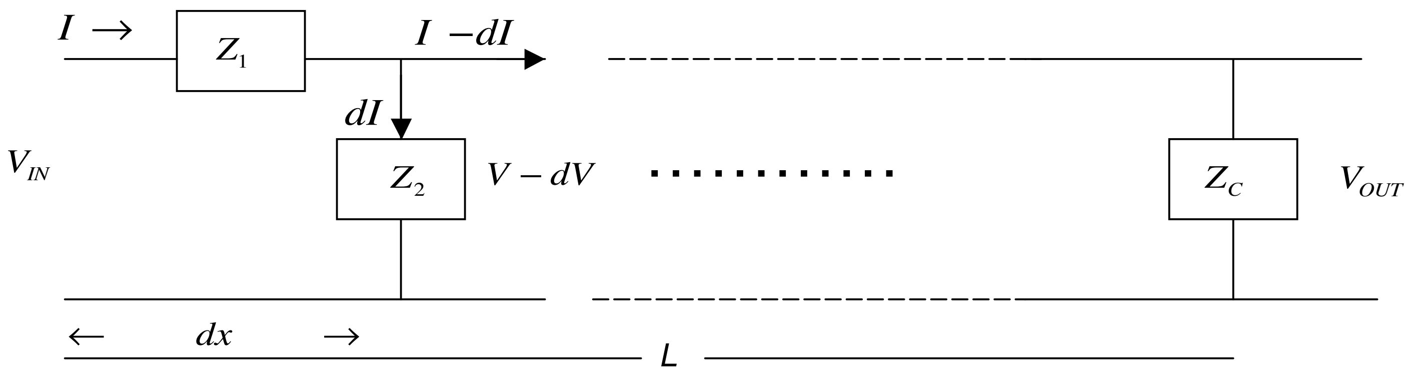
•The cable can be modeled as a series L=L/l and parallel C=C/l to ground. Sometimes a series R=R/l is also included.

$$V_{ab} = \frac{1/\omega C}{\sqrt{(1/\omega C)^2 + (\omega L)^2}} V_{in} = \frac{1}{\sqrt{1+(\omega^2 LC)^2}} V_{in} \qquad V_{out} = \left(\frac{1}{\sqrt{1+(\omega^2 LC)^2}} \right)^N V_{in}$$

•Since Vab ~ 1 for low frequencies (ω-> 0) generally signal propagation is only a concern at higher frequency.

•Resistance of the line will attenuate the signal at all frequencies!

Transmission Line Theory(1)



$$Z_1 = R_1/\ell + i\omega L/\ell$$
$$Z_2 = R_2/\ell + i\omega\left(C/\ell\right)$$

1) $dV/dx = -I\ Z_1 \;\rightarrow\; \dfrac{d^2V}{dx^2} = -Z_1\dfrac{dI}{dx} = \dfrac{Z_1}{Z_2}V \;\rightarrow\; \underline{V(x) = A\,e^{-\sqrt{Z_1/Z_2}\,x} + B\,e^{+\sqrt{Z_1/Z_2}\,x}}$

2) $dI/dx = -\dfrac{V}{Z_2} \;\rightarrow\; \dfrac{d^2I}{dx^2} = -Z_1\dfrac{dV}{dx} = \dfrac{Z_1}{Z_2}I \;\rightarrow\; I(x) = C\,e^{-\sqrt{Z_1/Z_2}\,x} - D\,e^{+\sqrt{Z_1/Z_2}\,x}$

By using 1) $I = \dfrac{1}{Z_1}\dfrac{dV}{dx} \;\rightarrow\; \underline{I(x) = \dfrac{1}{\sqrt{Z_1Z_2}}\left(A\,e^{-\sqrt{Z_1/Z_2}\,x} - B\,e^{+\sqrt{Z_1/Z_2}\,x}\right)}$ where $\underline{\mathbf{Z_C = \sqrt{Z_1Z_2}}}$ characteristic impedance

3) Assume we terminate the transmission at $x = L$ line with Z_C.

$V(L) = A\,e^{-\sqrt{Z_1/Z_2}\,L} + B\,e^{+\sqrt{Z_1/Z_2}\,L}$ and $I(L) = \dfrac{1}{Z_C}\left(A\,e^{-\sqrt{Z_1/Z_2}\,L} - B\,e^{+\sqrt{Z_1/Z_2}\,L}\right)$

$Z(L) = \dfrac{V(L)}{I(L)} = Z_C \;\rightarrow\; B = 0$

4) $V(0) = V_{IN} \;\rightarrow\; \underline{\underline{V(x) = V_{IN}\,e^{-\sqrt{Z_1/Z_2}\,x} \text{ and } I(x) = \dfrac{V_{IN}}{Z_C}\,e^{-\sqrt{Z_1/Z_2}\,x}}} \;\rightarrow\; \underline{\underline{\overbrace{Z_{IN} = \dfrac{V(0)}{I(0)} = Z_C}^{\text{Input Impedance} = Z_C \text{ independent of } L!}}}$

Transmission Line Theory(2)

- The characteristic impedance of a transmission line is

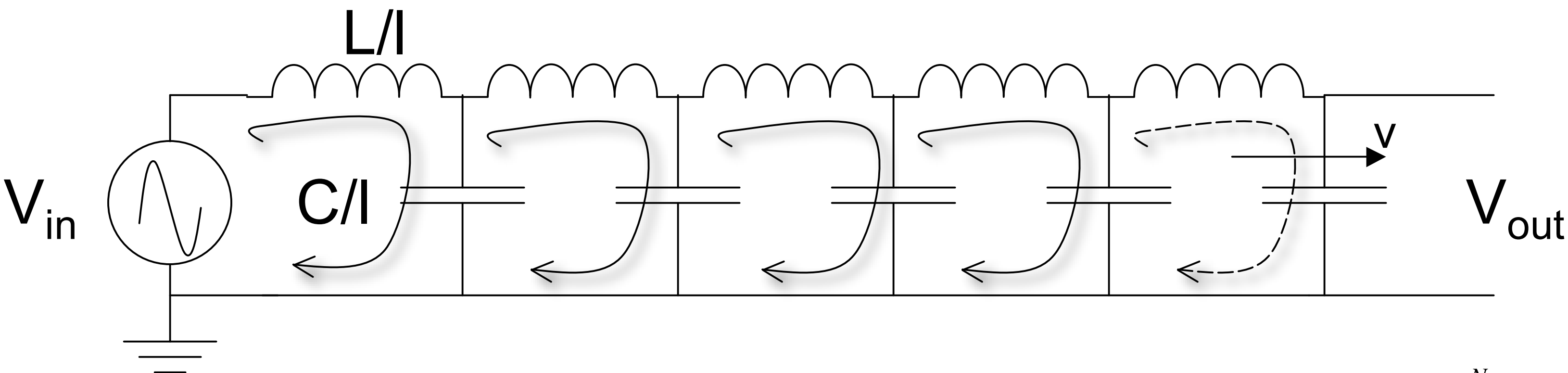
$$\mathbf{Z}_{\mathbf{C}harictersitic} = Z_C = \sqrt{\mathbf{Z}_1\mathbf{Z}_2}$$

- The input impedance of a transmission line is independent of *L* terminated if terminated in characteristic impedance is Z_C.

- The signal speed of propagation

$$\mathbf{v} = \mathbf{1} / \sqrt{\mathbf{L'C'}} = \mathbf{c} / \sqrt{\mu\varepsilon} = c / n$$

Speed of Signal Propagation



$$V_{ab} = \frac{1/\omega C}{\sqrt{(1/\omega C)^2 + (\omega L)^2}} V_{in} = \frac{1}{\sqrt{1+(\omega^2 LC)^2}} V_{in} \qquad V_{out} = \left(\frac{1}{\sqrt{1+(\omega^2 LC)^2}}\right)^N V_{in}$$

The LC *circuit rings when* $\omega^2 LC = 1$ *or* $\omega = 1/\sqrt{LC}$

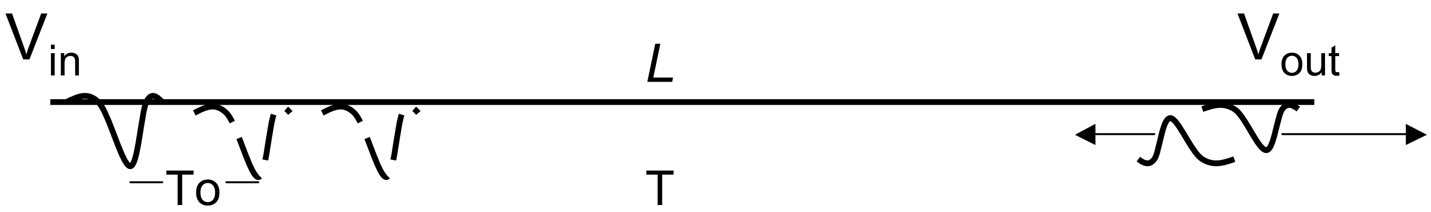
This is the optimum condition for signal transport.

Using the ringing period as $T = \sqrt{LC}$

$$\omega = 1/\sqrt{LC} \quad \Rightarrow \quad 1/\mathbf{v} = \overbrace{\sqrt{LC}}^{T}/l = \sqrt{(L/l)(C/l)}$$

$$\mathbf{v = 1/\sqrt{L'C'} = c/}\sqrt{\mu\varepsilon} = c/n$$

Reflection Coefficient



f =1/To =signal frequency

If a transmission line of characteristic impedance Zc is terminated with impedance $Z \neq Zc$ a reflected signal will develop!

$K = A/Ao$ = [(Z-Zc)/(Z+Zc)] reflection coef (If Z=Zo then no reflection!)

If T ~ 2(L/v) = transit time for 2L reflection then signal cancellation likely.
If T<<To then no termination necessary.
If T ~ To then cancellation or distortion and termination necessary.

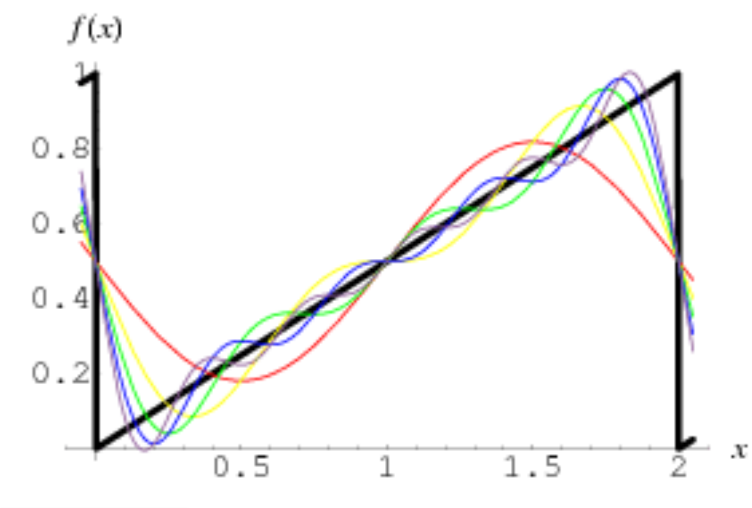
Will a 100 MHz signal traveling down a 2 m RG58/U 50Ω cable need termination?

To = 10 ns T= [2(2m)/2x10^{8} m/s] = 20 ns : Yes cable should be terminated in 50Ω!

Complex Signal and Fourier Decomposition

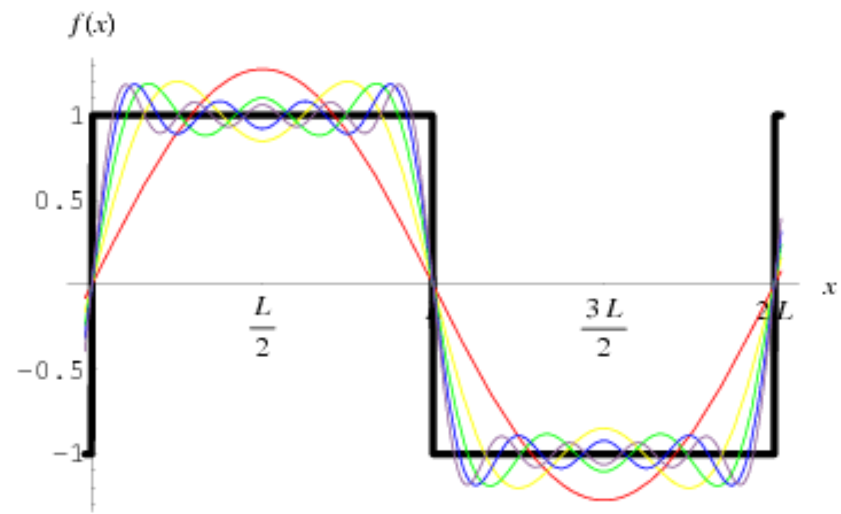
$$V(t) = \underbrace{\frac{A_o}{2}}_{DC} + \underbrace{\sum A_n \sin(2n\pi\ t/T) + \sum B_n \cos(2n\pi\ t/T)}_{AC} \qquad \omega_n = 2n\pi f$$

Fourier – Any waveform $V(t)$ *can be decomposed in to* sin*e and* cos*ine waves.*



Sawtooth Wave

$$A_n = \frac{1}{n\pi} \quad n = 1,3,5,\ldots.\ odd$$
$$B_n = 0$$

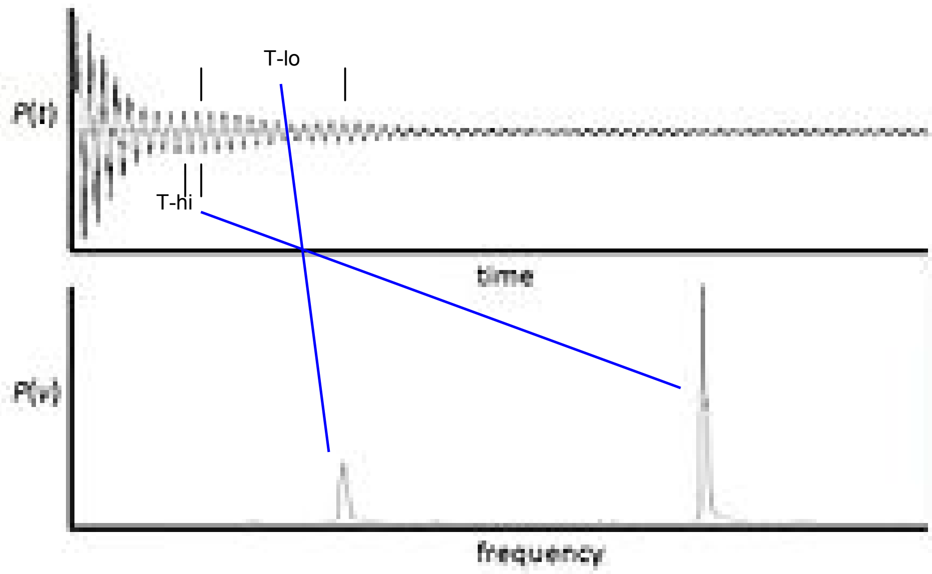


Square Wave

$$A_n = \frac{4}{n\pi} \quad n = 1,3,5,\ldots.\ odd$$
$$B_n = 0$$

Fourier Decomposition- FFT

• Fast Fourier Transform (FFT) is an algorithm to compute a discrete Fourier transform. This signal *P(t)* is decomposed in to a high frequency and low frequency *P(ν).*



• Noise and signal can be separated in the frequency domain