

## Digital Basics

- Analogue Communications over long distances often were hampered by noise, fluctuations, etc.
- Analogue communications can have high band width. AM and FM radio! (information transfer/sec)
- Digital communications are more robust but can suffer from band width limitations. Drums, Smoke Signals, Morse Code, Internet, etc.



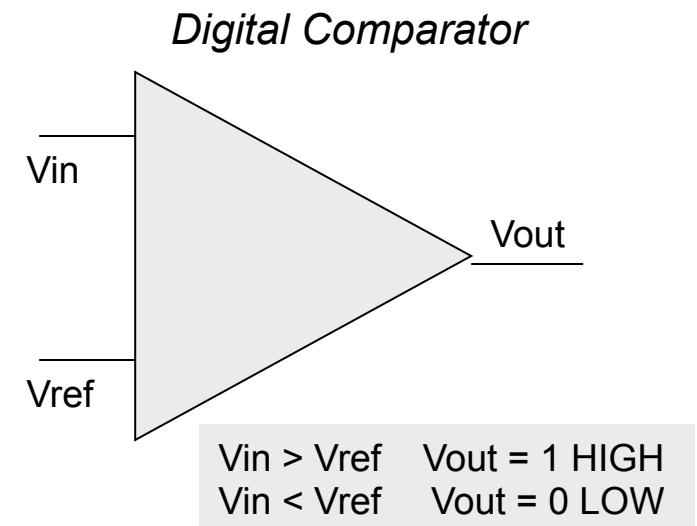
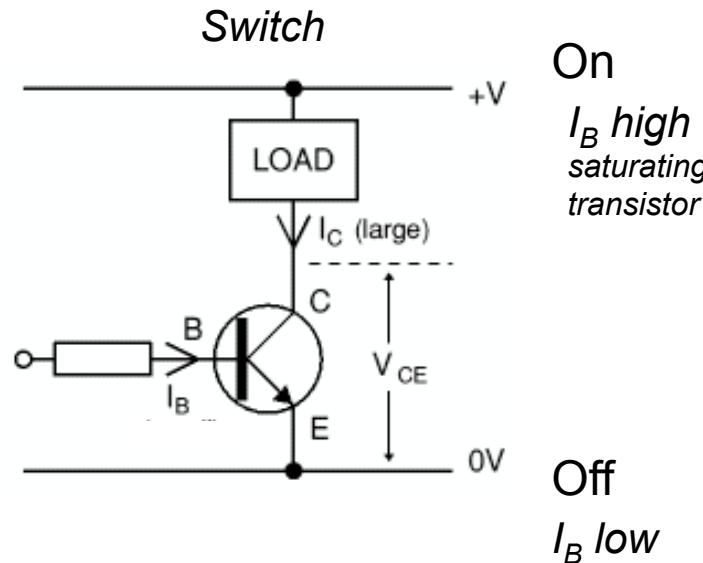
- Analogue-to-digital & Digital-to-Analogue translators important for research and communications.

# Transistor Switch and Comparator

In the transistor switch we can create a situation where a small input current can turn a transistor “On” or “Off”, the two states needed for digital electronics.

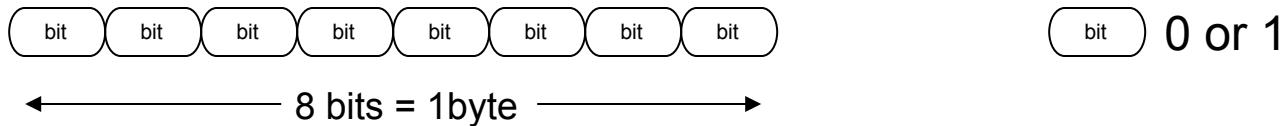
Early computers used the vacuum tubes, but in the 1950’s Silicon rectifiers and diodes were being developed for this purpose.

The “On” “Off” state is referred to as a “bit” short for “binary digit”.



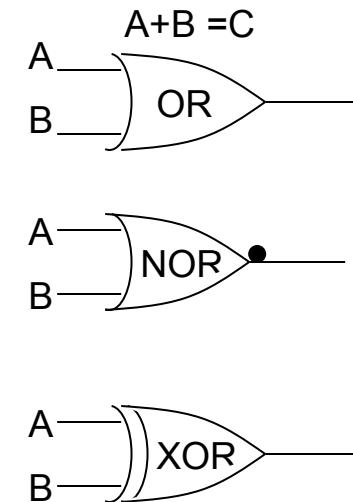
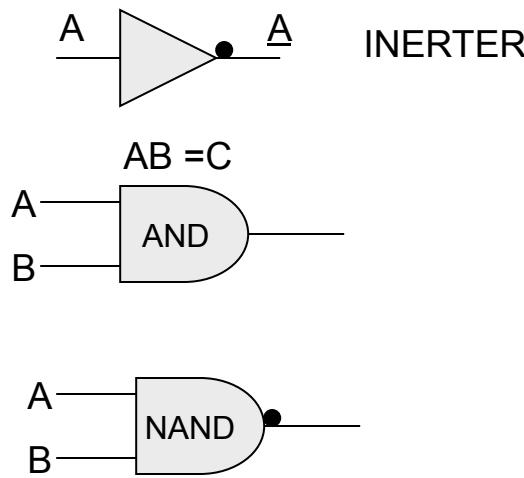
# BITS and BYTES

- Digital information is sent over data packets.



- Internet packet = 32 bits
- Computer word packet = 32 bits long word  
Computer word packet = 64 bits double precision
- Computer memory = 1 Gbyte =  $10^9 \times 8$  bits
- Hard disk = >50 GB 2 TB ( $10^{12}$ )
- CD = 700 MB ( 80 minutes of audio)
- DVD = 4.7 GB (2 hrs HR video )
- Blue Ray = 100-125 GB

# Digital Gates



TRUTH TABLE

A	B	AND	NAND
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

TRUTH TABLE

A	B	OR	NOR	XOR
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	0	0

# Boolean Algebra

$$A \cdot B = C \quad A \text{ .and. } B = C$$

$$A+B = C \quad A \text{ .or. } B = C$$

$$\underline{A} \cdot \underline{B} = \underline{C} \quad \underline{A} \text{ .and. } \underline{B} = \underline{C}$$

$$\underline{A} + \underline{B} = \underline{C} \quad \underline{A} \text{ .or. } \underline{B} = \underline{C}$$

## Boolean Theorems

$$(1) A+A = A$$

$$(2) A \cdot A = A$$

$$(3) A + 1 = 1$$

$$(4) A \cdot 1 = A$$

$$(5) A + 0 = A$$

$$(6) A \cdot 0 = A$$

$$(7) A + \underline{A} = A$$

$$(8) A \cdot \underline{A} = 0$$

$$(9) \underline{\underline{A}} = A$$

$$(10) A + A \cdot B = A$$

$$(11) A \cdot (A + B) = A$$

$$(12) A \cdot (\underline{A} + B) = A \cdot B$$

$$(13) A + \underline{A} \cdot B = A + B$$

$$(14) \underline{A} + A \cdot B = \underline{A} + B$$

$$(15) \underline{A} + A \cdot \underline{B} = \underline{A} + \underline{B}$$

$$(16) \underline{A} \cdot B = \underline{A} + \underline{B} \quad DeMorgan's$$

$$(17) \underline{A} + B = \underline{A} \cdot \underline{B} \quad Theorems$$

## Boolean Proofs by Truth Table

$$(1) A + A = A$$

A	A	$A + A$
0	0	0
1	1	1

$$(2) A \cdot A = A$$

A	A	$AA$
0	0	0
1	1	1

$$(8) A \cdot \underline{A} = 0$$

A	$\underline{A}$	$A\underline{A}$
0	1	0
1	0	0

$$(11) A(A+B) = A$$

A	B	$A+B$	$A(A+B)$
0	0	0	0
1	0	1	1
0	1	1	0
1	1	1	1

$$(16) \underline{A}\underline{B} = \underline{A} + \underline{B}$$

A	B	$AB$	$\underline{AB}$
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

$\underline{A}$	$\underline{B}$	$\underline{A} + \underline{B}$
1	1	1
0	1	1
1	0	1
0	0	0

## Boolean Proofs by DeMorgan's Theorems

$$\begin{aligned}\bullet F &= A(\underline{A+B}) \\ &= \underline{AA} + \underline{AB} \\ &= 0 + AB & -8 \\ &= AB & -5\end{aligned}$$

$$\begin{aligned}\bullet \#6(a) \\ \bullet F &= ABCD + \underline{ABCD} \\ &= A(B+\underline{B})CD \\ &= A(1)CD & -7 \\ &= ACD\end{aligned}$$

$$\begin{aligned}\bullet \#6(b) \\ \bullet F &= AB + \underline{AC} + \underline{ABC}(AB+C) \\ &= AB + \underline{AC} + \underline{ABC}AB + \underline{ABC}C \\ &= AB + \underline{AC} + (AA)(\underline{BB})C + \underline{ABC}C & 2,8 \\ &= AB + \underline{AC} + (A)(0)C + \underline{ABC} \\ &= A(B+\underline{BC}) + \underline{AC} \\ &= A(B+C) + \underline{AC} & 13 \\ &= AB + AC + \underline{AC} = \\ &= AB\end{aligned}$$

## **BOOLEAN LOGIC PROBLEMS**

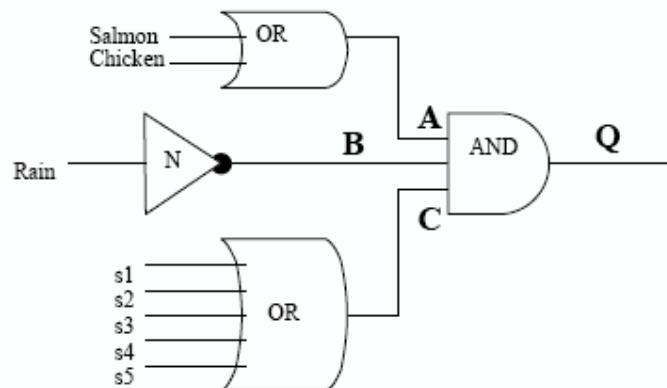
### **George Boole**

(1815-1864) was an English mathematician and logician who devised a system for representing logical symbolic relationships now known as **Boolean algebra**. The logical relationships, called **Boolean expressions**, use the logical operators AND, OR, and NOT between entities. These expressions have application in computer circuit design, information retrieval strategies, and logic problems such as this.

#1- I want to go to a picnic. There are conditions to be met before I can go. Construct a logic gate circuit to determine if I may go on to the picnic. Gates with more than 2 inputs may be used. 1 = yes 0 = no

- Krogers must have either baked chicken or salmon on sale.
- It must not be raining
- At least one of my five students has to go with me.

$$Q = \{\text{Chicken} \oplus \text{Salmon}\} \cdot \{\overline{\text{rain}}\} \cdot \{s_1 \oplus s_2 \oplus s_3 \oplus s_4 \oplus s_5\}$$



TRUTH TABLE

A	B	C	Q
1	1	1	1
0	1	1	0
1	0	1	0
1	1	0	0

## Binary - base2

$$N_2 = a_0 2^0 + a_1 2^1 + a_2 2^2 + a_3 2^3 + \dots = (a_3 a_2 a_1 a_0)_2$$

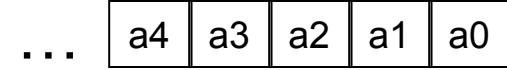
Decimel	$2^7$	$2^6$	$2^5$	$2^4$	$2^3$	$2^2$	$2^1$	$2^0$
0	0	0	0	0	0	0	0	<b>0</b>
1	0	0	0	0	0	0	0	<b>1</b>
2	0	0	0	0	0	0	<b>1</b>	<b>0</b>
3	0	0	0	0	0	0	<b>1</b>	<b>1</b>
4	0	0	0	0	0	<b>1</b>	<b>0</b>	<b>0</b>
5	0	0	0	0	0	<b>1</b>	<b>0</b>	<b>1</b>
6	0	0	0	0	0	<b>1</b>	<b>1</b>	<b>0</b>
7	0	0	0	0	0	<b>1</b>	<b>1</b>	<b>1</b>
8	0	0	0	0	<b>1</b>	<b>0</b>	<b>0</b>	<b>0</b>

MSB                                   LSB

## Other Systems

Binary 2-digi 0,1

$$N_2 = a_0 2^0 + a_1 2^1 + a_2 2^2 + a_3 2^3 +$$



Octal 8-digi 0,1,2,3,4,5,6,7

$$N_8 = a_0 8^0 + a_1 8^1 + a_2 8^2 + a_3 8^3 +$$

Decimal 10-digi 0,1,2,3,4,5,6,7,8,9

$$N_{10} = a_0 10^0 + a_1 10^1 + a_2 10^2 + a_3 10^3 +$$

Hexadecimal 16-digi 0,1,2,3,4,5,.....A,B,C,D,E,F

$$N_{16} = a_0 16^0 + a_1 16^1 + a_2 16^2 + a_3 16^3 +$$

BCD binary coded decimal

$$99_{10} = 1001\ 1001_{BCD}$$

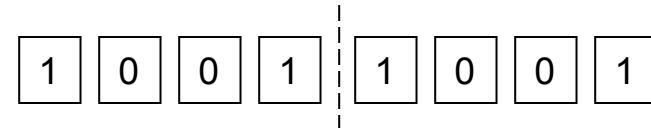


TABLE 11.13 Comparison of Numbering Systems

Decimal	Binary	BCD	Octal	Hexadecimal
0	0	0000	0	0
1	1	0001	1	1
2	10	0010	2	2
3	11	0011	3	3
4	100	0100	4	4
5	101	0101	5	5
6	110	0110	6	6
7	111	0111	7	7
8	1000	1000	10	8
9	1001	1001	11	9
10	1010	0001 0000	12	A
11	1011	0001 0001	13	B
12	1100	0001 0010	14	C
13	1101	0001 0011	15	D
14	1110	0001 0100	16	E
15	1111	0001 0101	17	F
16	10000	0001 0110	20	10
17	10001	0001 0111	21	11
18	10010	0001 1000	22	12
19	10011	0001 1001	23	13
20	10100	0010 0000	24	14
21	10101	0010 0001	25	15
22	10110	0010 0010	26	16
23	10111	0010 0011	27	17
24	11000	0010 0100	30	18
25	11001	0010 0101	31	19
99	110011	1001 1001	143	63

ASCII Hex Symbol	ASCII Hex Symbol	ASCII Hex Symbol	ASCII Hex Symbol
0 0 NUL	16 10 DLE	32 20 (space)	48 30 0
1 1 SOH	17 11 DC1	33 21 !	49 31 1
2 2 STX	18 12 DC2	34 22 "	50 32 2
3 3 ETX	19 13 DC3	35 23 #	51 33 3
4 4 EOT	20 14 DC4	36 24 \$	52 34 4
5 5 ENQ	21 15 NAK	37 25 %	53 35 5
6 6 ACK	22 16 SYN	38 26 &	54 36 6
7 7 BEL	23 17 ETB	39 27 '	55 37 7
8 8 BS	24 18 CAN	40 28 (	56 38 8
9 9 TAB	25 19 EM	41 29 )	57 39 9
10 A LF	26 1A SUB	42 2A *	58 3A :
11 B VT	27 1B ESC	43 2B +	59 3B ;
12 C FF	28 1C FS	44 2C ,	60 3C <
13 D CR	29 1D GS	45 2D -	61 3D =
14 E SO	30 1E RS	46 2E .	62 3E >
15 F SI	31 1F US	47 2F /	63 3F ?
ASCII Hex Symbol	ASCII Hex Symbol	ASCII Hex Symbol	ASCII Hex Symbol
64 40 @	80 50 P	96 60 `	112 70 p
65 41 A	81 51 Q	97 61 a	113 71 q
66 42 B	82 52 R	98 62 b	114 72 r
67 43 C	83 53 S	99 63 c	115 73 s
68 44 D	84 54 T	100 64 d	116 74 t
69 45 E	85 55 U	101 65 e	117 75 u
70 46 F	86 56 V	102 66 f	118 76 v
71 47 G	87 57 W	103 67 g	119 77 w
72 48 H	88 58 X	104 68 h	120 78 x
73 49 I	89 59 Y	105 69 i	121 79 y
74 4A J	90 5A Z	106 6A j	122 7A z
75 4B K	91 5B [	107 6B k	123 7B {
76 4C L	92 5C \	108 6C l	124 7C
77 4D M	93 5D ]	109 6D m	125 7D }
78 4E N	94 5E ^	110 6E n	126 7E ~
79 4F O	95 5F _	111 6F o	127 7F □

## LOGIC FAMILIES

TTL Logic- Transistor-Transistor Logic gates

Transistors are fully turned on.

Excellent noise margin

High power (1-20)mW/gate

Single Vcc supply voltage.

High>3-5V    Low~0.2V

ECL Logic- Emitter Coupled Logic

High speed by not saturating transistors.

$T_d < 1\text{ns}$  (gate speed)

Low Power (0.5-1.0) mW/gate

Multiple supply voltages

Higher fan-in capability

CMOS- Complementary Metal Oxide Logic

Very Low Power

Excellent Noise Margin

Inexpensive

Slower than TTL    $T_d > 10\text{ns}$

Fragile

## Case of Binary Addition A+B=C

Carry	0	0	0	0	0	1	1	0
A register	0	0	0	0	1	0	1	1
+								
B register	0	0	0	0	0	0	0	1
=	0	0	0	0	1	1	0	0

registers

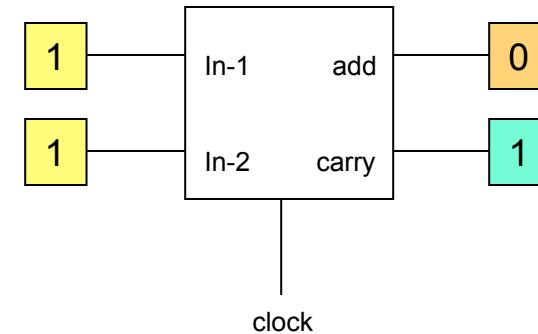
Base 2 notation

$$1011_2 + 0001_2 = 1100_2$$

Base 10 notation

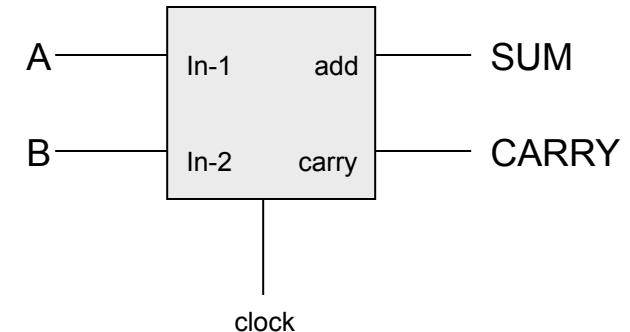
$$11_{10} + 01_{10} = 12_{10}$$

GATE/FLIP-FLOP



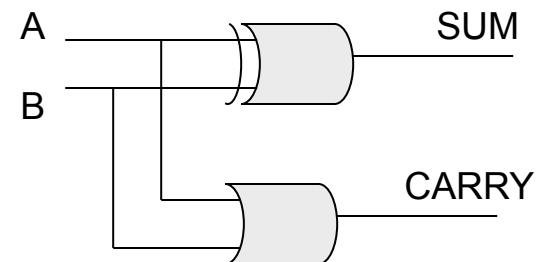
## TRUTH TABLE

A	B	AND	NAND	OR	NOR	XOR
0	0	0	1	0	1	0
0	1	0	1	1	0	1
1	0	0	1	1	0	1
1	1	1	0	1	0	0

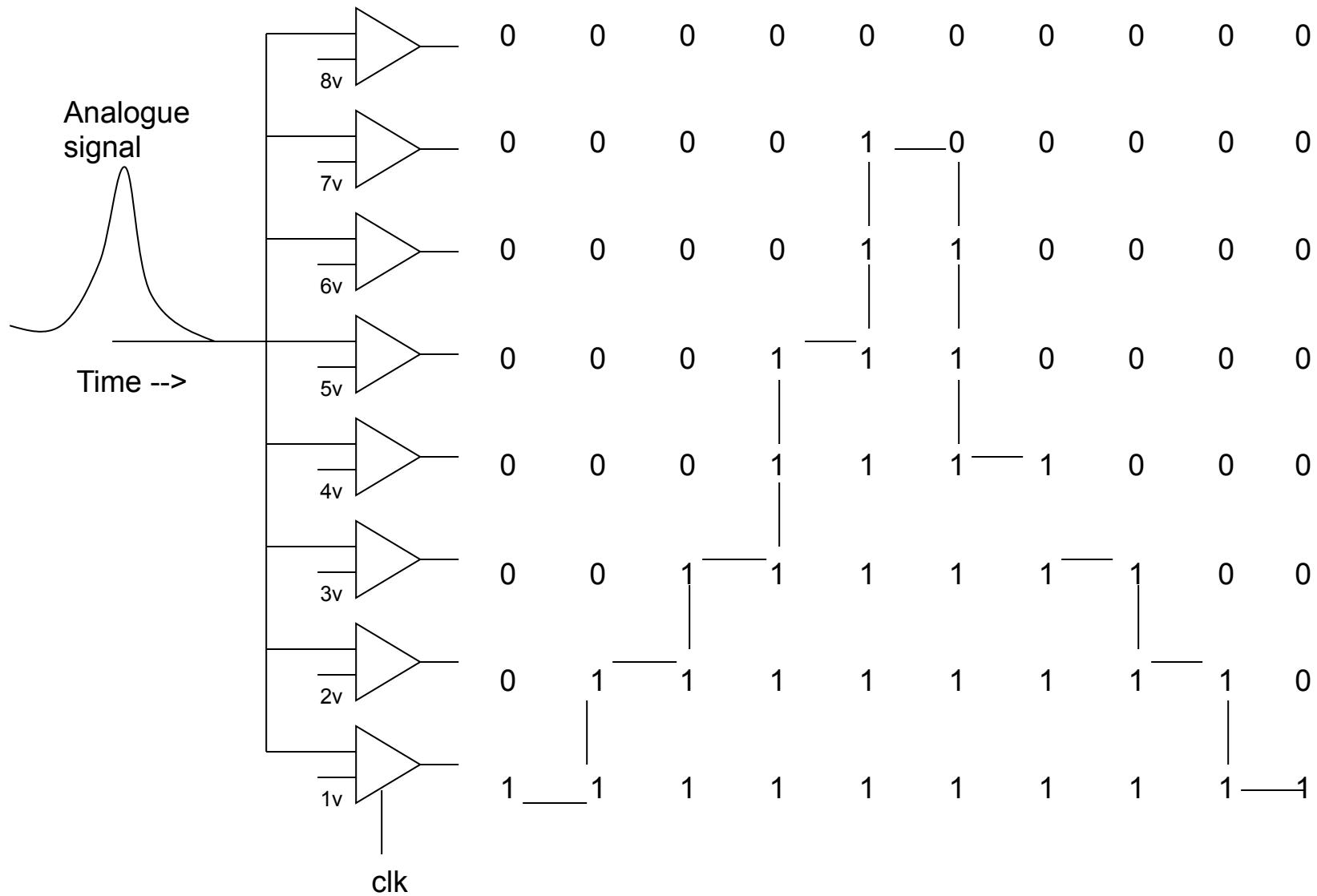


## ADDITION

A	B	SUM	CARRY
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1
		XOR	OR



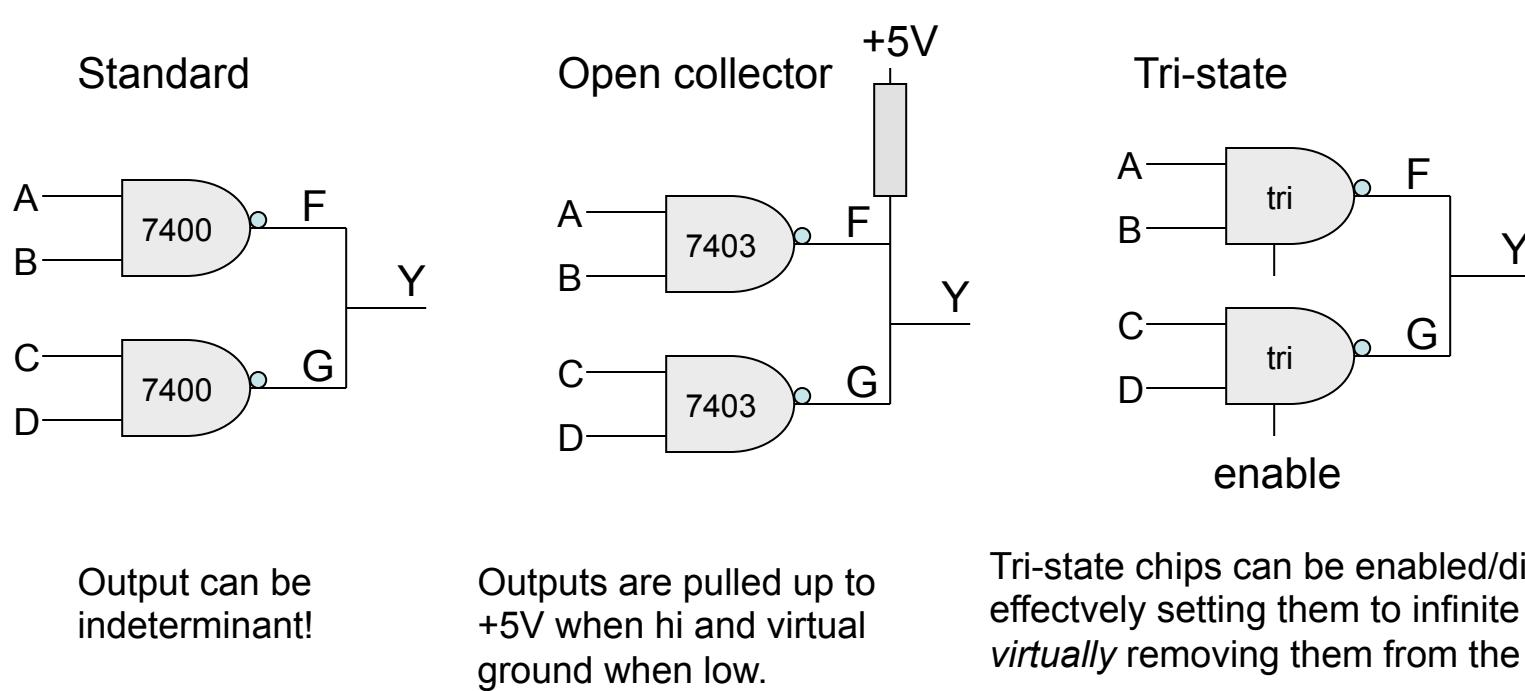
## Analogue-to-Digital



## Open Collector and Tri-state Logic

- In standard ttl or cmos logic there is a problem with concatenating gates for an output. Consider  $Y=ABCD$  formed below from 2 NAND Gates.

- When  $A=B=C=D=0$   $F=G=1$  and  $Y=1$   
When  $A=0$   $B=C=D=1$   $F=1$   $G=0$  and  $Y=??$  **indeterminant**



# Epitaxial Gates

