## CHAPTER 14- NUCLEAR PHYSICS APPLICATIONS

**NUCLEAR REACTIONS** $a + A \rightarrow B + C$

Energy and momentum must be conserved in all reactions, leading to

$$KE_a + M_a c^2 + KE_A + M_A c^2 = KE_B + M_B c^2 + KE_C + M_C c^2$$

$$Q = ( M_a c^2 + M_A c^2 - M_B c^2 - M_C c^2 ) = KE_B + KE_C - KE_a - \cancelto{0}{KE_A}$$

If Q > 0 we have an *exothermic* reaction which liberates energy.
If Q < 0 we have an *endothermic* reaction which needs energy to go.

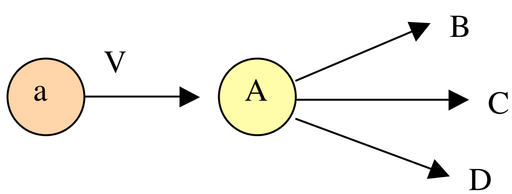
*Example 14-1 Find Q of the reaction* $^1H_1 + {}^7Li_3 \rightarrow {}^4He_2 + {}^4He_2$ *and the kinetic energy of the products.*

$Q = M(Li) + M(H) - 2\,M(He)$
$= 7.016003u + 1.007825u - 2(4.002603)u \times 931.5 \text{ MeV} = 17.3 \text{ MeV}$

$KE_{PRODUCTS} = Q + KEa = 17.3\text{MeV} + 0.6 \text{ MeV} = 17.9 \text{ MeV}$

**THESHOLD for the ENDOTHERMIC Q< 0 REACTION** **$a+A \rightarrow B+C+D+\ldots$**

Consider particle "a" moving with velocitv V toward particle "A" which is at rest. Particles "B", "C", "D", etc. emerge.




A reaction of the type $a+A \rightarrow B+C+D$ will only proceed if the total energy $E_{CM}$ of a+A is just large enough to create B+C+D at rest in the center of momentum (CM) frame

$$KE_a > | Q | [ 1 + M_a / M_A ]$$

*Example of Endothermic Reactions*
*Consider the reaction* $^7Li(p,n)^7Be$ *with Q = -1.645 MeV* *endothermic*
What is the threshold energy of the proton that will make this go?

$KE_p > -Q ( 1 + M(H)/M(Li) ) = 1.645 ( 1+ 1.008u/7.016u) \text{ MeV} = 1.645 (1.144)\text{MeV}$

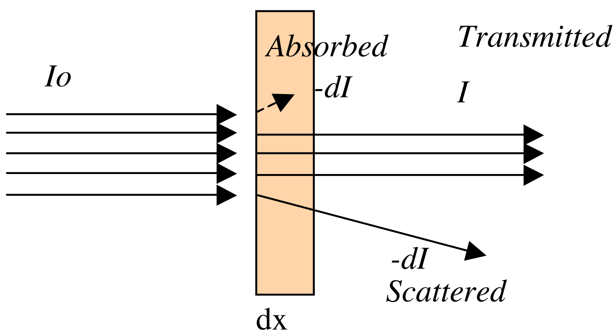
$KE_p > 1.88 \text{ MeV}$

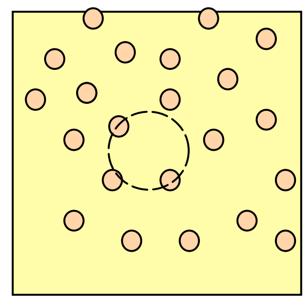
## REACTION CROSS SECTION

Consider a beam of particles of initial intensity *Io* (number/s) hitting a thin slab of material of thickness $\Delta z$. Some particles will be scattered or absorbed from the beam. Let *dI* be those scattered or absorbed out of the beam.

*dI/Io* must be proportional to the number of atoms encountered along the beam spot.

$$dI / Io = - (\rho N_A / A)\ \sigma_{scat}\ \Delta z$$



$n = (\rho N_A / A)$ = number of scatterers per cm$^3$ = number density of scatterers
$\sigma$ = scattering cross section $1\text{b} = 10^{-24}\ \text{cm}^2$ = 1 barn
= the scattering area that a projectile sees on approaching the target.
$\Delta z$ = target thickness

$$\int dI/Io = -n\ \sigma \int dz$$

$$\ln (I) = -n\ \sigma\ \Delta z + C$$

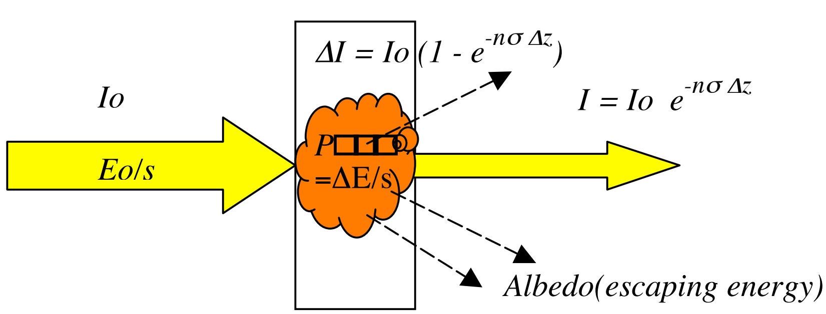
$$I = Io\ \exp[-n\ \sigma\ \Delta z]$$

The rate of particles scattered or absorbed out of the beam is simply given by

$$I_{ABS} = Io - I = Io\ (1 - \exp[-n\ \sigma\ z]\ \ \#/s$$

The *absorbed power* is approximately $Pabs = \Delta E/s = Eo\ \Delta N_{ABS}/s \approx Eo\ I_{ABS}$

$$Pabs = Eo\ I_{ABS}$$



Example and Quiz Topic:

A beam of energy Eo = 200MeV and intensity Io = 5M/s is incident on a thin slab of $^{27}Al_{13}$ ($\rho$=2.7 g/cm$^3$) of thickness z = 5mm. The interaction cross section $\sigma$ = 21b. (1 b =$10^{-24}$ cm$^2$)

a. What is the beam intensity emerging from the slab?

$n = \rho N_A/A = (2.7)(6.02\times10^{23})/27 = 6.02\times10^{22}$ atoms/cm$^3$

$I = Io \exp[-n\sigma z] = Io \exp[-(6.02\times10^{22} \text{ atoms/cm}^3)(21 \times 10^{-24} \text{ cm}^2)(.5\text{cm})]$
$= Io \exp[-.632] = 0.54\, Io$
$I = Io\,(0.54) = 2.7 \times 10^6$/s emerging from the slab.

b. How much energy per second is released in to the slab by the interacting beam?

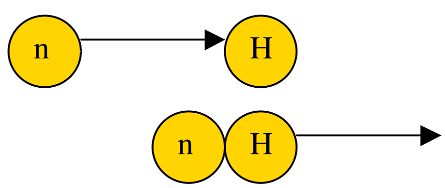
$Iabs = Io\,(1 - 0.54) = 0.46\, Io = 2.3 \times 10^6$/s
$Pabs = Iabs\, Eo = (200\text{MeV})(2.3 \times 10^6\text{/s}) = 4.60 \times 10^{14}$ eV/s $= 1.2 \times 10^{-5}$ J/s
$Pabs = 12\ \mu W$

*We have ignored energy from particles which escape the slab (albedo).*

**NEUTRONS**

Neutrons from reactions are classified as
- -Fast neutrons KE > 1MeV
- -Slow neutrons KE < 1MeV
- -Thermal neutrons KE ~ 3/2 kT ( kT = 1/40 eV @ room temperature )

Fast neutrons can be *thermalized* by passing through matter where they progressively slow down in collisions with atoms. Collisions with atoms of comparable size yield the maximum energy transfer, thus maximum neutron energy loss $\Delta E_n$!




$\Delta E_n \sim |(m1-m2)/(m1+m2)|$
*Neutron energy loss*

Materials which slow down neutrons effectively are called *moderators*.
Materials will hydrogen are very effective moderators. $H_2O$, Hydrocarbons, etc.

Once the neuron is thermalized it undergoes a neutron-capture reaction. The intermediate state ${}^{A+1}X_Z*$ quickly decays by gamma emission.

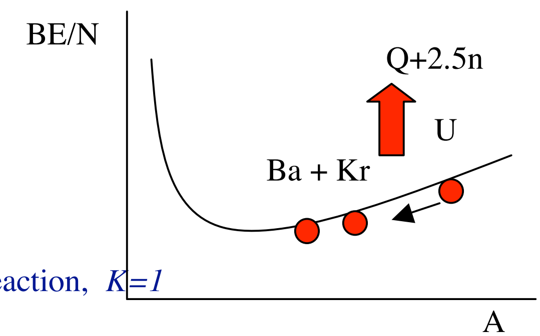
$${}^{1}n_0 + {}^{A}X_Z \rightarrow {}^{A+1}X_Z* \rightarrow {}^{A+1}X_Z + \gamma$$

**NUCLEAR FISSION**

Between 1900-1930 scientist understood that elements like Uranium and Radium Emitted $\alpha$, $\beta$ and $\gamma$ particles effusely. In each of these decays the $\alpha$, $\beta$ and $\gamma$ energy depend on the reaction *Q ~ few MeV* as we discussed.

In the mid-1930's U.S., Italian, and German scientists began detecting reactions in which the parent nuclei broke in to two large fragments. Since the parts may not easily emerge it took time to understand these *high-Q* ~ 100's MeV reactions. The *high-Q* was released in to the material as thermal energy, photons, atomic electron ionizations etc.

$n + {}^{235}U_{92} \rightarrow {}^{235}U_{92}* \rightarrow {}^{141}Ba_{56} + {}^{92}Kr_{36} + 3\,{}^{1}n_0$
$n + {}^{235}U_{92} \rightarrow {}^{235}U_{92}* \rightarrow {}^{140}Xe_{54} + {}^{94}Sr_{38} + 2\,{}^{1}n_0$
$n + {}^{235}U_{92} \rightarrow {}^{235}U_{92}* \rightarrow {}^{132}Sn_{50} + {}^{101}Mo_{42} + 3\,{}^{1}n_0$



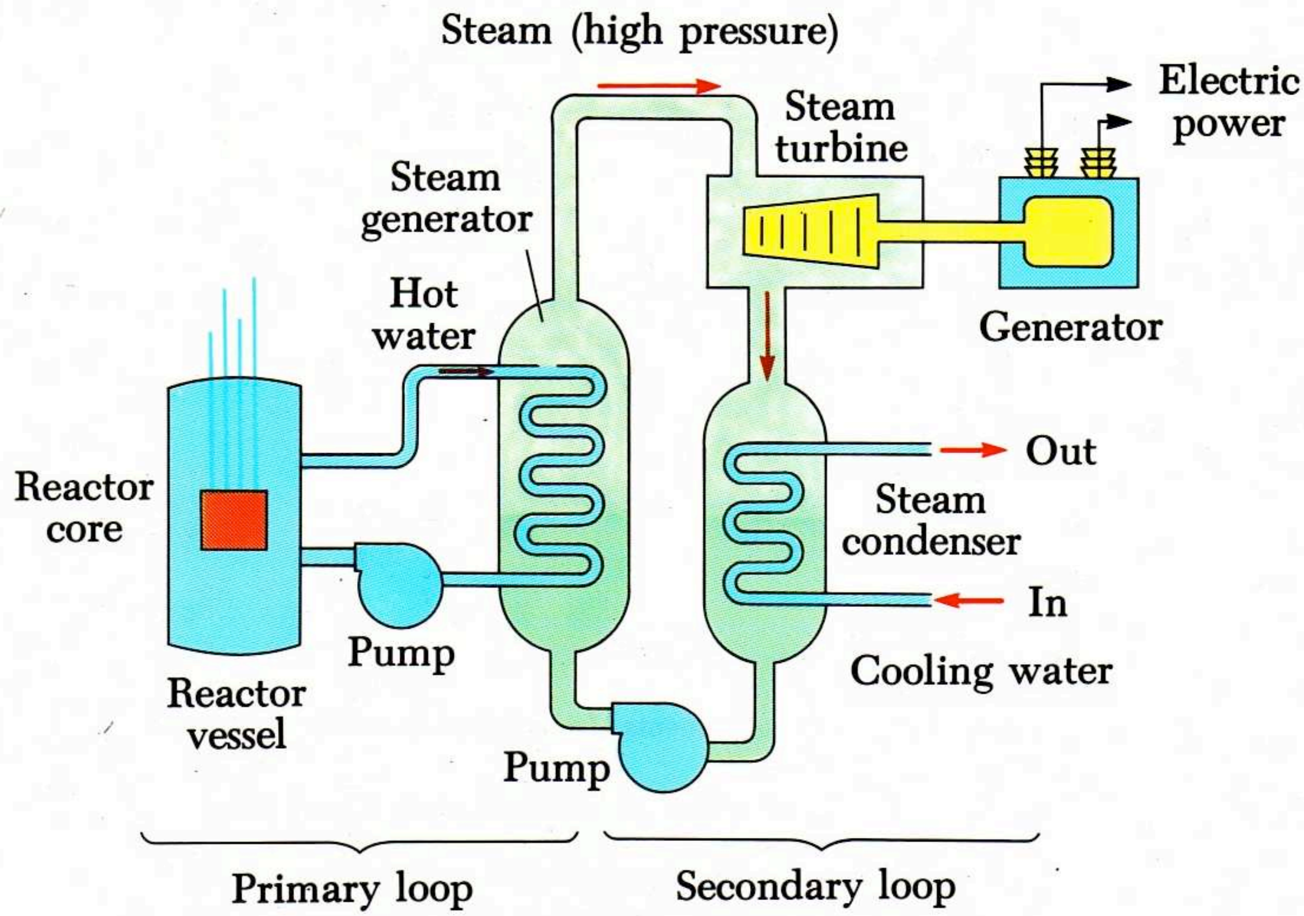

~ 2.5 neutrons released to initiate another reaction!

**FISSION REACTORS** - Self-Sustained Chain Reaction, *K=1*

*K= reproduction constant* = Average number of neutrons which cause another fission.

*K<1 Reactor sub-critical – not producing heat*
*K=1 Reactor critical – producing heat*
*K>1Reactor Super-critical- Runaway, Melting, etc*

- Neutron Leakage – Neutron Reflectors
- Neutron Moderation -
- Neutron Capture by $U^{238}/U^{235}$ - Enriched $U^{235}$ Fuel Rods
- Control of Power Level - Control Rods.
- Waste disposal- Fuel Rods



Steam (high pressure)
Steam turbine
Electric power
Steam generator
Generator
Hot water
Out
Reactor core
Steam condenser
In
Pump
Cooling water
Reactor vessel
Pump
Primary loop
Secondary loop

**FUSION REACTORS**

Proton-Proton cycle in the Sun

$^{1}H_{1} + {}^{1}H_{1} \rightarrow {}^{2}H_{1} + e^{+} + \nu_{e}$

$^{1}H_{1} + {}^{2}H_{1} \rightarrow {}^{3}He_{2} + \gamma$

$^{1}H_{1} + {}^{3}He_{2} \rightarrow {}^{4}He_{2} + e^{+} + \nu_{e}$

$^{3}He_{2} + {}^{3}He_{2} \rightarrow {}^{4}He_{2} + {}^{1}H_{1} + {}^{1}H_{1}$

*Hydrogen Burning*

Fusion Reactor Research

| | | |
|---|---|---|
| $^{2}H_{1} + {}^{2}H_{1} \rightarrow {}^{3}He_{2} + {}^{1}H_{1}$ | $Q = 3.27$ MeV | *p-D* |
| $^{2}H_{1} + {}^{2}H_{1} \rightarrow {}^{3}H_{1} + {}^{1}H_{1}$ | $Q = 4.03$ MeV | *D-D* |
| $^{2}H_{1} + {}^{3}H_{1} \rightarrow {}^{4}He_{2} + {}^{1}n_{0}$ | $Q = 17.59$ MeV | *D-T* |

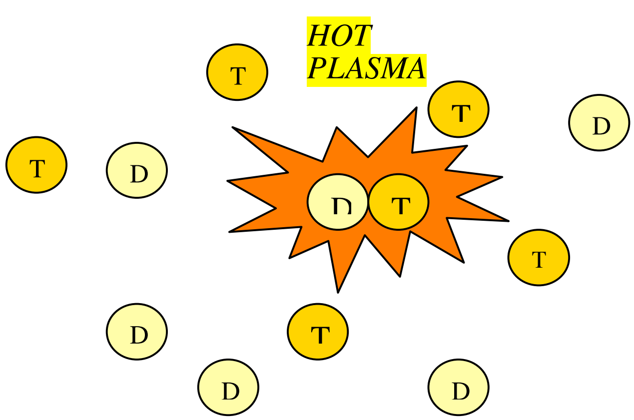
***FUSION RATE ~ Critical Temperature x Ion Density x Confinement Time***

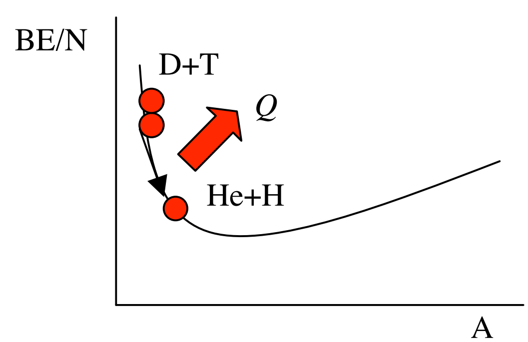
***Critical Ignition Temperature-*** *Tc is the temperature necessary to overcome the electrostatic repulsion between p-p, p-D, D-D, or D-T*

| | | | |
|---|---|---|---|
| *D-D* | $Tc = 4 \times 10^{8}$ K | $E = k_B T = 35$ keV | $n\tau > 10^{14}$ s/cm$^3$ |
| *D-T* | $Tc = 4.5 \times 10^{7}$ K | $E = k_B T = 4$ keV | $n\tau > 10^{16}$ s/cm$^3$ |

***Ion Density n = Number Density of Deuterium***

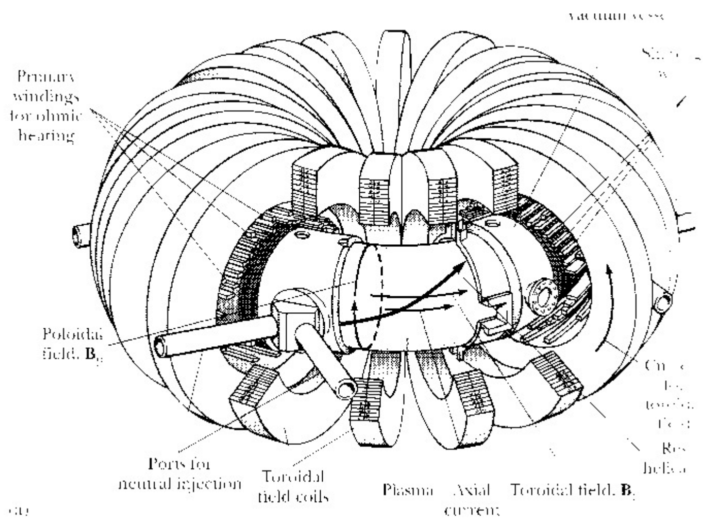
***Confinement Time*** - *Time $\tau$ at which the interacting ions maintain Tc.*



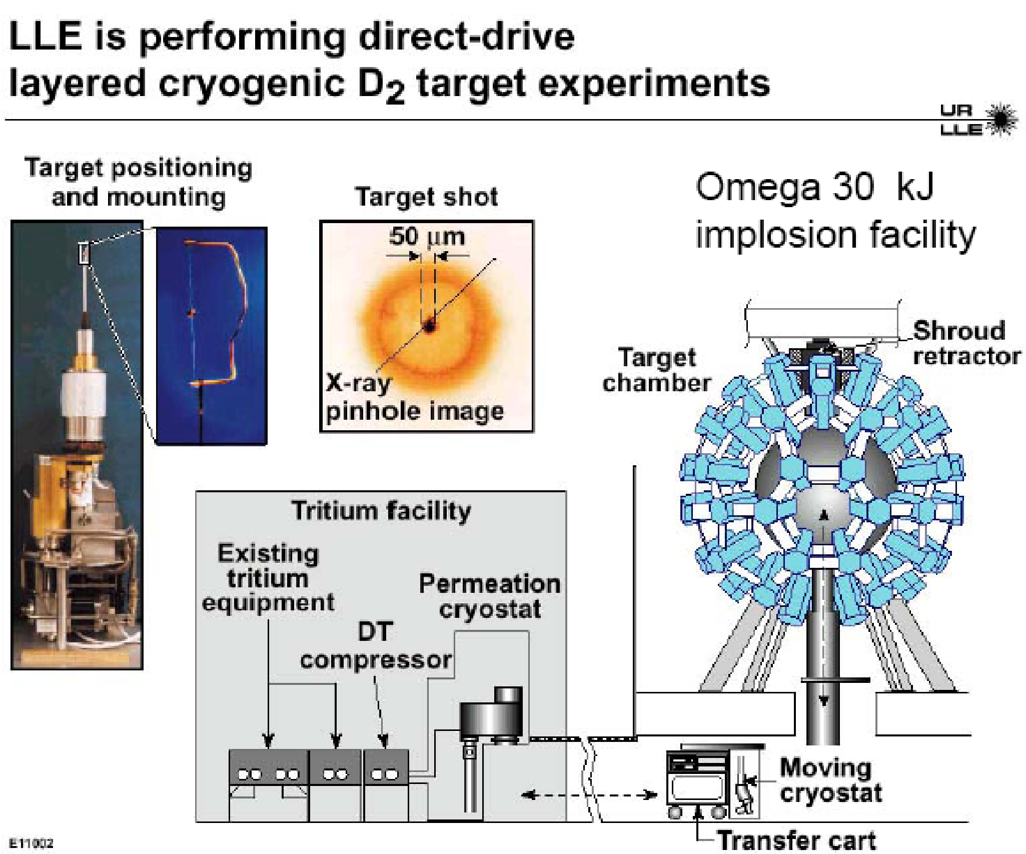



**TOKOMAC**s – Magnetic Confinement of Ions. – NOT SUSTAINED YET

*D-T fusions reported at* $10^{18}$*/s* $n\,\tau = 5 \times 10^{13}$ *s/cm*$^3$ *Tc = 30 keV*




**LASER IMPLOSION –** Lasers used to implode D-T pellets. - NOT SUSTAINED YET



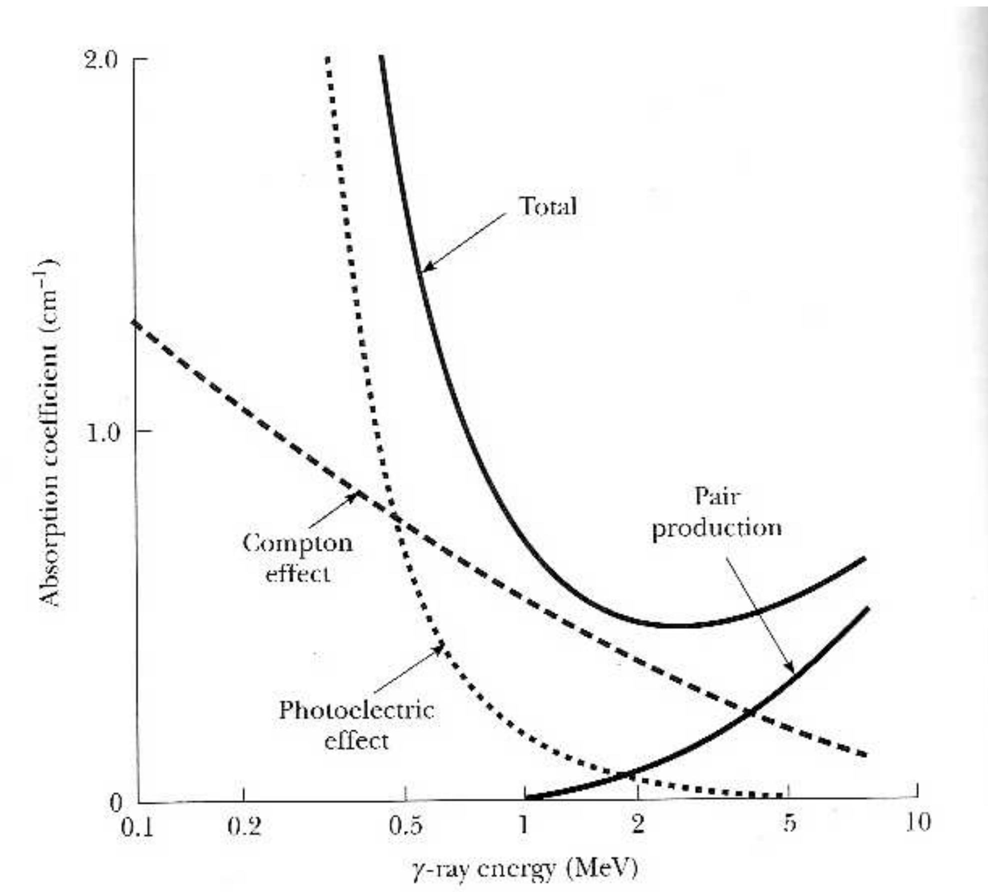
## ENERGY LOSS for PHOTONS in MATER

Gamma rays and Xrays are attenuated (absorbed) when passing through a material. The number surviving after traveling a distance x is:

$$I = Io\ exp[-\mu\ z]$$
$$I = Io\ exp[\ -n\ \sigma_{ABS}\ \ z\ ]$$

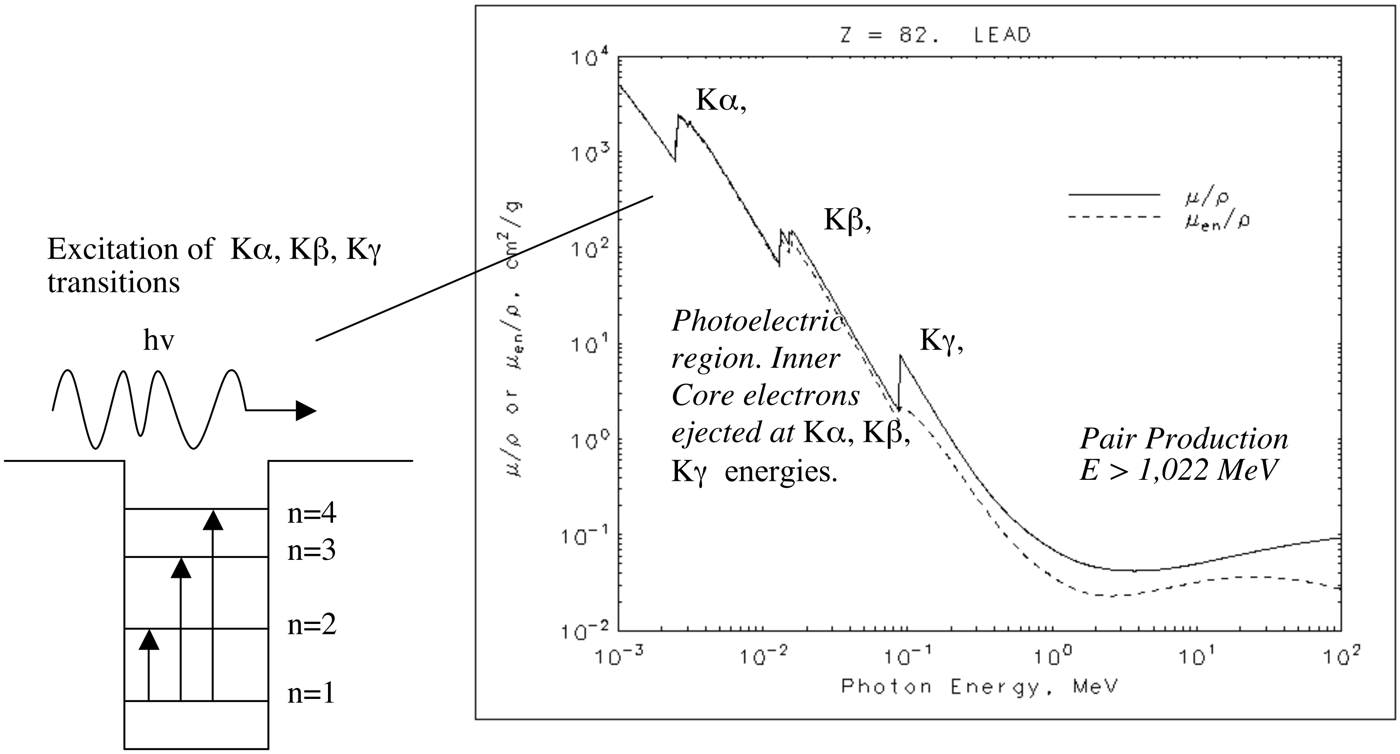
$\mu = \mu(E) = n\ \sigma$ is called the *absorption coeficient* is a strong function of energy *E*. Thus gamma radiation, xrays, etc can have vastly different absorption cross sections depending on *E*. Note this is exactly the same expression as derived for particle interactions in matter. The cross section $\sigma_{abs}$ is called the photon absorption cross section.

Three regions of energy loss can be identified depending on the photon energy *E*.




**(1)** Compton Scattering region (photons scatter from atomic electrons like billiard balls.
**(2)** PhotoElectric Effect (photons ionize atomic electroms)
**(3)** Pair Production - Photons have enough energy > 2 (511 KeV) to create an e+e- pair. ( *E* > 1.022 MeV )

In the following plots coefficient $\mu/\rho$ is plotted versus photon energy for Pb. The spikes in the graphs represent incident photon which excite X-ray transitions in Pb. Thus below 1 MeV lead is effective at stopping photons. Above 1 MeV the attenuation coefficient has dropped manner orders of magnitude. Thus Pb shields you well from Xrays but not gamma rays.




## HALF VALUE THICKNESS for Photons - $x_{1/2}$

$x_{1/2}$ = the thickness that will reduce the intensity of a photon beam by a factor of 2

$I = Io \exp\{-\mu x_{1/2})$ or $I/Io = 1/2 = \exp\{-\mu x_{1/2})$

$$x_{1/2} = \ln(2)/\mu = 0.693 / \mu$$

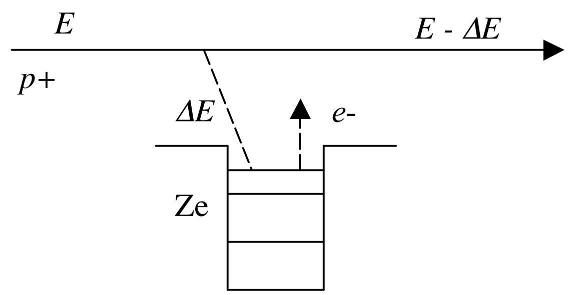
**See Example 14-7**

**$\mu = 55\ cm^{-1}$ for lead at a wavelength $\lambda$=20 pico meter (Table 14-2 )**

**$x_{1/2}$ = 0.693/ 55 = 0.0126 cm = 0.126 mm**

**ENERGY LOSS for Heavy of Charged Particles in Matter – Stopping Power**

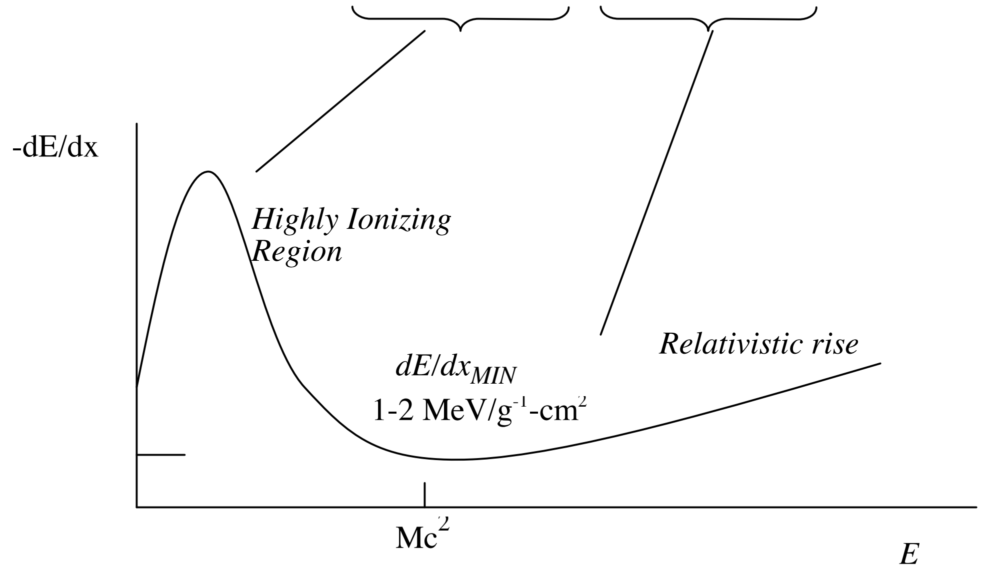
Heavy charged particles ($p^{\pm}$, $\alpha^{++}$, $\pi^{\pm}$, $\mu^{\pm}$) lose energy while traversing matter mainly by ionization of atomic electrons. Scientists Bethe and Bloch calculated this energy loss by ionization- called the **Bethe-Bloch formula**.




ze = charge of incident particle
Ze = charge of atomic matter
*I* ionzation potential of atom
n = number density of atomic matter
v = velocity of the incident particle.

**Bethe-Bloch Formula**

$$- dE/dx \propto 4\pi\, (ze)^2\, (Ze)^2\, n\, \frac{1}{m_e c^2 \beta^2} \left[\, \ln\left(\frac{m_e c^2 \beta^2}{I(1-\beta^2)}\right) - \beta^2 \,\right]$$




(1) For low energy protons $E < Mc^2$ the energy loss is due to highly ionizing interactions with the material.

(2) For protons $E \geq Mc^2$ *the proton reaction cross section with the atoms drops significantly to produce a* *$(1/\rho)dE/dx \sim 2$ MeV $cm^2/g$.*

***Example -***

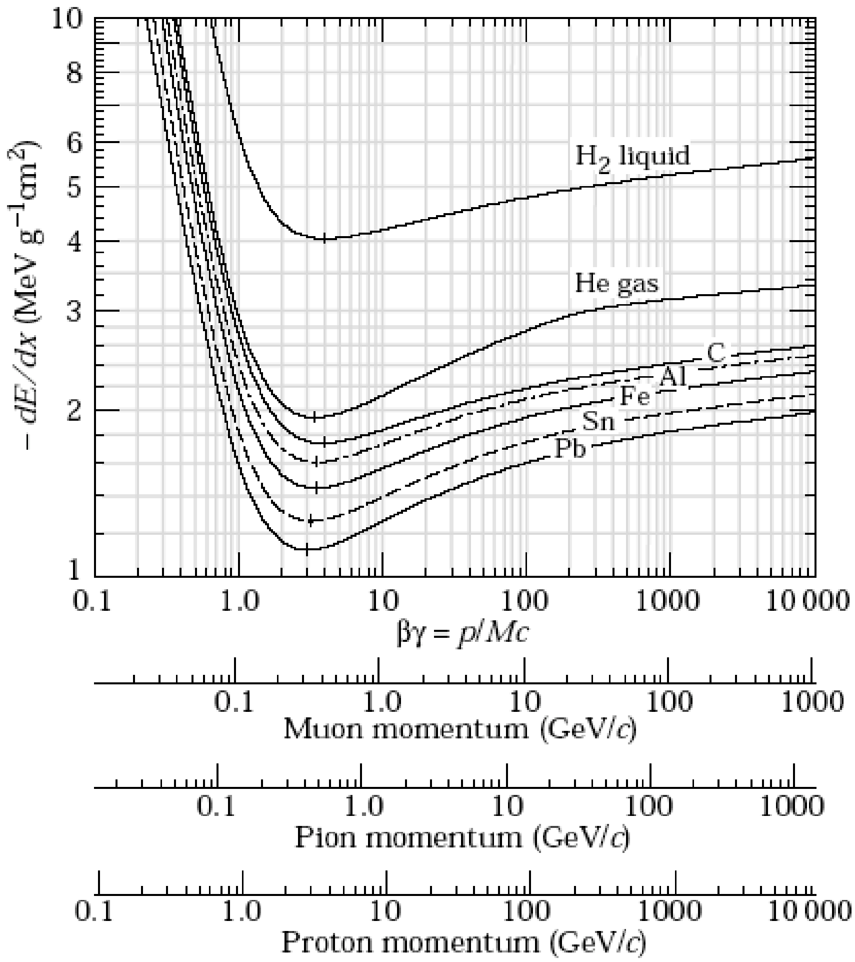
**How much energy will a 1000 MeV proton loose in traveling 50cm into water?**

**- dE/dx = 2 MeV/g$^{-1}$-cm$^2$ x 1 g/cm$^3$ in $H_2O$** $\quad$ **$\rho$ = 1 g/cm$^3$**

**- dE/dx = 2 MeV/cm**

**ΔE = dE/dx 50cm = 100 MeV** $\quad$ **of lost energy**

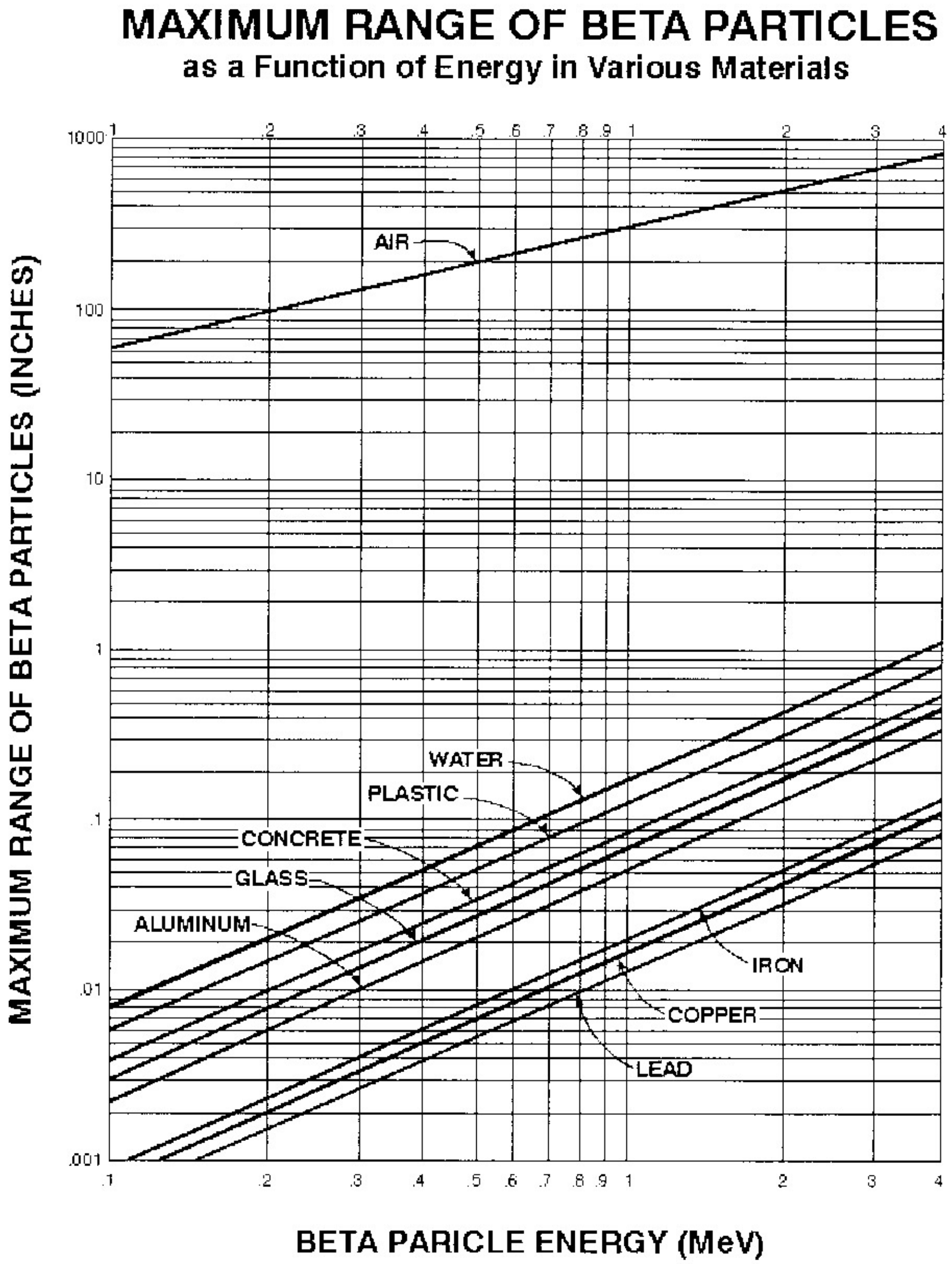
***ENERGY LOSS FOR HEAVY CHARGED PARTICLES vs MOMENTUM***



**RANGE for Protons/Electrons** $R = \int_0^E (dE/dx)^{-1}\, dE = \{1/(\mathrm{dE/dx})\}\ \Delta\mathrm{E}$

*Example-* What is the range of a 200 MeV proton in water?

$R \sim (\mathrm{dx/dE})\ \Delta\mathrm{E} = \{2\ \mathrm{MeV/cm}\}^{-1}\ 200\ \mathrm{MeV} = 100\ \mathrm{cm}$



## RADIATION DAMAGE IN MATTER

The ΔE energy loss mainly due to ionization of atoms.

- Weakening of bonds in materials- Material Fatigue.
- Color Centers in crystals – Discoloration, Light attenuation in Optical Fibers,
- Displacement centers in Si crystals - NASA worries about their computers in space.
- Somatic Damage to Cells and organs can cause organ failure.
- Genetic Damage to reproductive cells cause cancers.

## RADIATION ABSORBED DOSE - rad

**1 rad** = $10^{-2}$ J of energy released into 1 kg of absorbing material.

1 roentgen = $3.33 \times 10^{-10}$ C of electric charge produced in 1 $cm^3$ of air at STP.

RBE = Relative biological Effectiveness takes in to account biological damage to cells.

$$\text{DOSE(rem)} = N\,\text{(rads)} \times \text{RBE}$$

| | |
|---|---|
| Xrays and Gammas | RBE = 1 |
| Betas | RBE = 1-1.7 |
| Alphas | RBE =10-20 |
| Slow Neutrons | RBE = 4-5 |
| Protons, Fast Neutrons | RBE = 10 |
| Heavy Ions | RBE = 20 |

*Damage Factor*

| Natural Background Radiation | 130 mrem per yr |
|---|---|
| Radon | |
| Internal ($^{40}K$ ) | 40 |
| Terrestial (geology) | 28 (15-150) |
| Cosmic ($C^{14}$ , *T* ) | 27 |

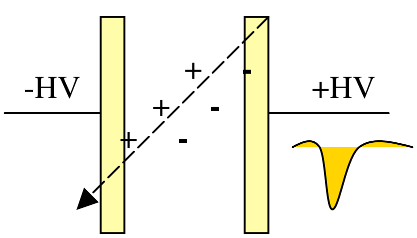
| Man-made radiation | 60 mrem per yr |
|---|---|
| X-rays | 39 |
| Nuclear Medicine | 14 |
| Consumer Products | 10 |

Government Regulation on Radiological Workers - < 500 mrem per yr

| An assortment of typical radiation doses (in mrem) | |
|---|---|
| Used to destroy the bone marrow in preparation for a marrow transplant (given over several days) | 1,000,000 |
| Approximate lethal dose ("$LD_{50}$") if no treatment and given to the entire body in a short period | 450,000 |
| Causes radiation sickness (when absorbed in a short period) | >100,000 |
| Increase in lifetime dose to most heavily exposed people living near Chernobyl | 43,000 |
| Average annual dose (excluding natural background) for medical x-ray technicians | 320 |
| Maximum permissible annual dose (excluding natural background and medical exposure) to general public | 170 |
| Natural background, Boston, MA, USA (per year)(excluding radon) | 102 |
| Natural background, Denver, CO, USA, (per year)(excluding radon) | 180 |
| Additional annual dose if you live in a brick rather than a wood house | 7 |
| Average dose to person living within 10 miles of Three-Mile Island (TMI) caused by the accident of 28 March 1979 | 8 |
| Most heavily exposed person (a fisherman) near TMI | <100 |
| Approximate dose received by a person spending 1 year at the fence surrounding a nuclear power station | 0.1–0.6 |
| Average dose to each person in the U. S. population from nuclear power plants (per year) | 0.002 |
| Received by the bone marrow during a set of dental x rays* | 9.4 |
| Annual dose to the gonads from TV sets | 0.2–1.5 |
| Received by the bone marrow during a barium enema | 875 |
| Received by the bone marrow during a chest x ray | 10 |
| Received by breast during mammogram | 50–700 |
| Average airline passenger (10 flights/year) | 3 |
| Flight crew and cabin attendants (per year) | 160 |
| Hourly dose to skin holding piece of the original "Fiesta Ware" (a brand of pottery) | 200–300 |
| Annual dose to each person in the U. S. population from fallout (former weapons testing plus Chernobyl) | 0.06 |

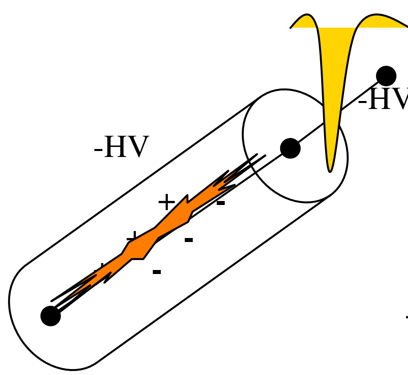
* Dose much higher (several thousand mrem) to the skin in the path of the beam, but bone marrow is more susceptible to damage (e.g., leukemia).

## RADIATION DETECTION



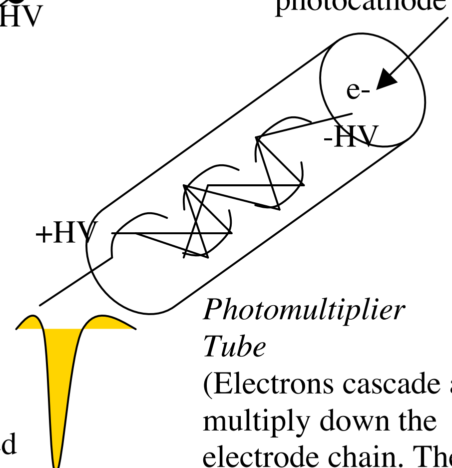

*Ionization Chamber*
(Ionized Charge collected at electrodes )

$$q = \int i \ dt$$




*Geiger Tube*
(Electrons Avalanche as electric field increases near the wire! A large induced signal is detected on the wire.)

Light excites electrons at the photocathode




*Photomultiplier Tube*
(Electrons cascade and multiply down the electrode chain. The final signal is $2^N$ times the initial electron charge seen at the faceplate. )

**PEACEFUL USES of RADIATION**

- Tracers in Medicine
- Neutron Activation for Element Analysis
  $^{1}n_{0} + {}^{65}Cu_{29} \rightarrow {}^{66}Cu^{*}_{29} \rightarrow {}^{66}Zn_{30} + {}^{0}e_{-1} + {}^{0}\nu_{0}$
- Radiation Therapy
- Food Preservation