CHAPTER 13 – NUCLEAR STRUCTURE

NUCLEAR FORCE

The nucleus is help firmly together by the nuclear or strong force. We can estimate the nuclear force by observing that protons residing about $1 \text{fm} = 10^{-15}$ m apart are held in place by a binding potential of BE ~ V = k e²/ R (Force Balance by Newton's Laws)

 $V = k e^{2} / R = (8.9 x 10^{9} \text{ N-m}^{2}/\text{C}^{2}) (1.6 \text{ c} 10^{-19} \text{ C})^{2} / (10^{-15} \text{ m}) = 2.3 x 10^{-13} \text{ J}$ = 2.3 x 10⁻¹³ J / 1.6 x 10⁻¹⁹ J/eV = **1.4 MeV per pair**



NUCLEAR SIZE

From Alpha Rutherford determined that the nucleus was very small in comparison to the atom. The alpha particles scattering from a Au foils indicated that they deflected from a point-like positive charge at the atom's center.

$$\frac{KE_{\alpha}}{1/2M} = \frac{PE_{\alpha}}{v^2} = k q 1 q 2 / R$$

More detailed studies showed resulted in a model in which the nucleus can be described by a radius $R = 1.07 A^{1/3}$ fm and surface thickness a~0.5 fm.



LINE OF NUCLEAR STABILITY Z = N

The nuclear force requires a balance between the number of protons and neutrons to a large extent. Some of this is due to the large binding energy of the alpha particle block p-p-n-n. As the atomic number increases more neutrons are needed to keep protons apart, thus reducing the Coulomb repulsion.



MAGNETIC MOMENT and NUCLEAR SPIN

The magnetic moment of a sample is defined as the magnetization M per unit volume, M/V.



Electrons, protons and neutrons have intrinsic magnetic moments $\vec{\mu}_i$ which are related to the spins of the particle. The electron magnetic moment is $\mu_B = e\hbar/2m_e$ = the *Bohr Magneton*. Because protons and neutrons are heavier and made of quarks, their moments differ.

$\mu_{\rm p} = +2.7939 \mu_{\rm N}$	$g_P = +2.7939$ gyromagnetic ratio for the proton
$\mu_{\rm n} = -1.9135 \ \mu_{\rm N}$	$g_N = -1.9135$ gyromagnetic ratio for the neutron

Spin Precession

 $f = (1/h) \mu B$

The quantity $\mu_N = e\hbar/2M = 5.05 \times 10^{-37} \text{ J/T}$ is called the *nuclear magneton*.

The energy of a magnetic dipole in a magnetic field *B* is is given by $\mathbf{E} = -\mu \cdot \mathbf{B}$, *Where* $|\vec{\mu}| = g \mu_N$. In and external magnetic field *B* along the z-axis the magnetic moment $\vec{\mu}$ orients and precesses about the z-axis like a top. If a second *RF* field is applied with energy hf, $\vec{\mu}_i$ will absorb this energy and begin to flip up and down absorbing and releasing this energy at the resonant frequency given by hf = \mathbf{E}^{\uparrow} - \mathbf{E}_{\downarrow} . A resonance condition will occur when hf = $g \mu_N B - (-g \mu_N B) = 2 g \mu_N B$.



Example-1 Find the energy difference ΔE between the spin up and spin down state of a proton in a magnetic field of B = 4.4 T. Find the resonant frequency f. h $f = 2 \text{ g } \mu \text{ B} = 2 \text{ x } 2.793 \text{ x } (5.05 \text{ x } 10^{-27} \text{ J/T}) (4.4\text{T}) = 1.24 \text{ x } 10^{-25} \text{ J} = 7.7 \text{ x } 10^{-7} \text{ eV}$ $f = \Delta E/h = 1.24 \text{ x } 10^{-25} \text{ J} / 6.626 \text{ x } 10^{-34} \text{ J-s} = 1.87 \text{ MHz}$



MRI

E&M waves from the 66.4 MHz oscillations are detected by pick up coils. An x-y image can be formed The B(z) field is set with a constant field Bo + trim coil fied dB. Thus the B(z) varies for each x-y slices of the patient's body. The RF frequency f is modulated to look at slices such that the resonance condition hf = 2µ B. is strictly satisfied. Only tissue in the 4.4T x-y slice is detected.





Pickup coils sense the de-exitation of the proton nuclear moments and in software an image is formed of the slice. The MRI operator has the capability to select a slice in X, Y, an Z.

187MHz excitation signal

pickup deexcitation

BINDING ENERGY

Binding energy is defined as the energy Q necessary to separate all the nucleons in the atom from each other. Instead of Mp, M(Hydrogen) is used.

$\frac{^{N+Z}X_{Z} + Q}{BE} = Z M(H) + N M(n) - M(^{A}X_{Z})$							
HYDROGEN DEUTERIUM TRITIUM HELIUM IRON	P P-N P-N-N P-P-N-N	BE = 2.2 MeV BE = 8.5 MeV BE = 27.5 MeV	BE/A =1.1 MeV/N BE/A = 2.7 MeV/N BE/A = 6.9 MeV/N BE/A = 7.5 MeV/N				



EXCHANGE FORCE MODEL

One model for the nuclear force is that the proton and neutron are exchanging a A field of π mesons. This is called the Yukawa model.



Some problems arise just as in the Bohr Model!

How can conservation of energy by applied. After many exchanges the nucleons would gain enough energy to fly apart?

VIRTUAL PARTICLES

Answer: Under the QM uncertainty principle energy conservation *can be violated* if it happens quickly enough !!! The pi-meson is referred to as a *virtual particle*, a particle that can violate energy conservation for a short time!



Yukawa estimated the mass of the force carrier, the pi meson mass as

$$M_{\pi}c^2 = \hbar c/d = \hbar c / 1 \text{fm} = 197.32 \text{ MeV} / 1.5 \text{ fm} = 131 \text{ MeV}/c^2$$

A particle of mass ~ 110 MeV was discovered in the cosmic rays and thought to be the pi-meson force carrier. Yukawa recieved the Nobel Prize in 1949 for predicting the pi-meson and his work in meson field theory.

SEMI-EMPIRICAL MASS FORMULA

The nuclear binding energy curve can be represented by a formula involving a number of terms. It is called *semi-empirical* because the terms give the general trend of the binding energy but not a concise calculation.

$BE = c1 A + c2 A^{2/3}$	$- c3 Z(Z-1)/A^{1/3} - c$	$4 (N-Z)^2/2$	<mark>4</mark>	
c1 = 15.7 MeV	+ Nuclear Volume		BE1 ~ Volume ~ $\mathbb{R}^3 \rightarrow \mathbb{A}$	
c2 = 17.8 MeV c3 = 0.71 MeV c4 = 23.6 MeV	 Surface Energy T Coulomb Repulsi (N-Z) asymmetry 	erm on Term term	BE2 ~ Area BE3 ~ q1 c BE4 ~ (N-	$ \sim R^2 \rightarrow A^{2/3} $ $ A^{2/3} $ $ A^{2/3} $
Energy per unit volume.	Energy per unit surface area.	Could Reput	omb sion.	Neutron Proton Asymmetry.
Core nucleons most bound.	Surface Nucleons less bound.	Z(Z-1 Repel avera distar) pairs ling at ge ace R.	Neutrons space Protons apart- Reduction.
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INDEPENDENT PARTICLE MODEL – SHELL MODEL

- In this model the nucleons sit in a square well whose depth and width depends N and Z of the nucleus. Proton levels are slightly elevated with respect to neutrons due to the Coulomb replulsion term.
- In solving this problem the energy levels of a single nucleon is written with the PE term corresponding to the mean potential due to all other nucleons- "Mean Field Approach".
- The magic numbers correspond to filled shells just as in the atom's periodic table.



- Protons are slightly less bound than neutrons due to the Coulomb repulsion.
- Alpha particles p-p-n-n are particularly stable in the nuclei and often stick together, eg. alpha decay.

OUR UNIVERSE

Helium formed in the early universe from free protons and neutrons as the temperature cooled. The ratio of helium-to-hydrogen is an important clue to the universe's apparent high degree of uniformity and Big Bang Theory.

Heavier atoms formed in the interior of stars through He burning. The earth and living things are composed of star material!

The molten interior of the earth is heated by radiaoctive isotopes present during the formation of the earth.

RADIOACTIVE DECAY

In the creation of the Universe Atomic Nuclei were under constant bombardment from other particles as p, n, β , α , γ 's. We say the nuclei are *activated* or *excited*. Upon activation some radioactive nuclei decay promptly within milliseconds or can remain radioactive for millions of years.



MEAN LIFE - τ and **DECAY RATE** $\lambda = 1/\tau$

The radioactive decay of a nuclei follows an exponential decay law with mean lifetime τ . The number of N(t) nuclei in a sample at time t is given by

 $N(t) = N(0) \exp(-\lambda t).$

The number having decayed is then $N_D(t) = N(0) - N(t) = N(0) [1 - exp(-\lambda t)]$

The decay rate

 $\mathbf{R}(t) = |\mathbf{dN}/\mathbf{dt}| = \mathbf{N}(0) \lambda \exp(-\lambda t)$

Decay Constant = $\lambda = .693/T_{1/2}$

UNIT OF ACTIVITY

The unit of radioactive decay is given by the number of decays per second coming from 1 gram of radium, called the curie (Ci):

 $1 \text{ Ci} = 3.7 \text{ x} 10^{10} \text{ decays/s}$

A more convenient unit the Becquerel (Bq) is commonly used 1 Bq = 1 decay/s and 1 Ci = 3.7×10^{10} Bq.

HALF LIFE

The time at which the decay rate drops by a factor of 2 is called the *half life*.

$$\begin{split} R(t=T_{1/2}) &= R(0)/2 = R(0) \exp(-\lambda T_{1/2}) & Derivation \, !\\ \exp(-\lambda T_{1/2}) &= 1/2\\ -\lambda T_{1/2} &= \ln(1/2) \\ \end{split}$$

$$\begin{split} T_{1/2} &= 0.693 \, / \, \lambda = \, 0.693 \, \tau \\ N(t) &= N(0) \, (1/2)^{1/T_{1/2}} \end{split}$$

NUMBER OF NUCLEI PER GRAM

To find the number of atoms in a gram of matter we find the number of Moles of substance and use Avogadro's number.

No = M(g) /A (gram/mole) x N_A = Number of atoms

Example-2 35 mg of pure ¹¹C₆ has a half life of 20.4 min. Determine the number of nuclei in the sample t = 10 hrs. N(0) = (0.035g/11.01g/mole) $N_A = 3.2x10^{-3}$ (6.02 x 10⁻²³) = **1.92** x **10**⁻²¹ atoms N(t) = N(0) exp(- λ t) $\lambda = 0.693/T_{1/2} = 3.4 \times 10^{-2}$ min N(t) = N(0) exp(- λ t) = 1.92 x 10⁻²¹ exp[-(.034)(600)] = 2.63 x 10¹² atoms OR by half life N(t) = N(0) (1/2)^{-1/T1/2} t / T1/2 = 600min/20.4min= 29.41 N(t) = 1.92 x 10⁻²¹ (1/2)^{-29.41} = 1.92 x 10⁻²¹ 1.4 x 10⁻⁹ N(t) = 2.63 x 10⁻¹² atoms **DECAY PROCESSES** : α , β , γ **DECAYS**



ALPHA DECAY

An alpha particle tunnels through the nuclear barrier with decay rate $\lambda = R = 1/\tau$ The_*Parent* X decays to *Daughter* Y plus an alpha particle.



BETA DECAY

Beta decay is a 3-body decay process in which a neutron in the nucleus decays to a proton, electron, and anti-neutrino. (n \rightarrow p + e + v). $M_v = 0!!$



GAMMA DECAY

An excited nucleus decays to a more stable form through emission of a gamma ray. Very similar to atomic transitions!



Gamma decay is often closely associated with beta decay.

CARBON DATING

• Cosmic Rays create neutrons in the upper atmosphere for billions of years, creating a constant fraction of Carbon-14 to Carbon-12

$$f(0) = N(^{14}C)/N(^{12}C) = 1.3 \times 10^{-12}.$$

- The C forms in to molecules including CO₂ which plants breath. $n + {}^{14}N_7 -> {}^{14}C_6 + p$
- Thus all living plants and animals acquire ${}^{14}C$ in their bodies.
- Upon death ¹⁴C is not replenished and beta decays with a lifetme of $\tau = 8270$ yrs.

$$^{14}C_{6} \rightarrow ^{14}N_{7} + {}^{0}\beta_{1} + v$$
 T $_{1/2} = 5730 \text{ yrs}$

• By measuring the ratio of ¹⁴C/¹²C in a present day we can estimate the age of the sample.

$$N(t=today) = N(t=death) \ x \ e^{-t/\tau}$$

R(t=today) = R(t=death) x \ e^{-t/\tau}





Example 13-11:

A 25g of charcoal is found in an ancient city. 250 ^{14}C decays of ^{14}C per minute are measured. How old is the charcoal. $T_{1/2} = 5370$ y

$$\lambda = = 0.693/T_{1/2} = (0.693/5370) \text{ yrs} = 3.8 \text{ x } 10^{-12} \text{ s}^{-1}$$

$$N(C^{12}) = (25/12) mol N_A = 1.26 \times 10^{24} nuclei$$

$$N_0(C^{14}) = 1.3 \times 10^{-12} N(C^{12}) = 1.6 \times 10^{12} nuclei$$

$$R_0(C^{14}) = \lambda N_0(C^{14}) = 6.13 \ decays/s = 370 \ decays/min$$

 $(250/370) = e^{-\lambda t} \rightarrow \underline{t} = 3200 \text{ yrs}$

NATURAL RADIOACTIVE SERIES

The four naturally occuring radioactive series are:

$${}^{238}U_{92} \rightarrow -> -> {}^{206}Pb_{82} \qquad Uranium$$

$${}^{235}U_{92} \rightarrow -> -> {}^{207}Pb_{82} \qquad Actinium$$

$${}^{232}Th_{90} \rightarrow -> -> {}^{208}Pb_{82} \qquad Thorium$$

$${}^{237}Np_{93} \rightarrow -> -> {}^{209}Bi_{83} \qquad Neptunium$$

These decay sequences occur through multiple α and β emissions down to Pb and Bi.



FIG. 2.7 Plot of N(=A-Z) versus Z for the Uranium series and the Actinium Series designated by A = 4n + 2 and A = 4n + 3, respectively. The numbers below the half-lives are the maximum energies in Mev.



FIG. 2.6 Plot of N(=A-Z) versus Z for the Thorium Series and the Neptunium Series designated by A = 4n and A = 4n + 1, respectively. The numbers below the half-lives are the maximum energies in Mev.