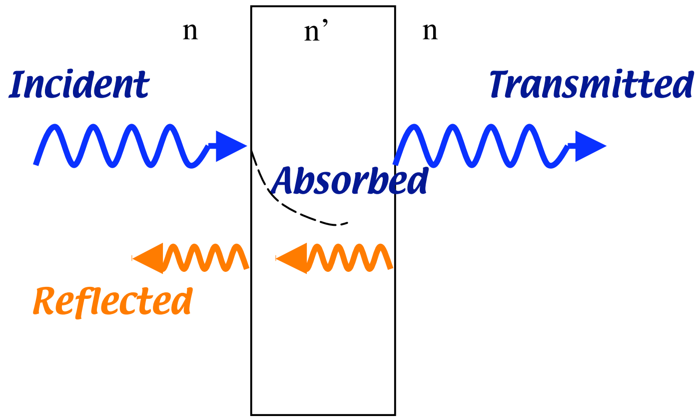
CHAPTER 7- TUNNELING PHENOMENA

CLASSICAL PICTURE



$$I = R + T + A$$

The incident power must be the sum of Reflected, Transmitted, and Absorbed Waves

EXAMPLE- Reflection in Glass

$R = (n' - n)^2 / (n' + n)^2$ *Reflection Coefficient*

$R_{GLASS} = (1.5-1.0)^2 / (1.5+1.0)^2 = 0.04$ *each interface*

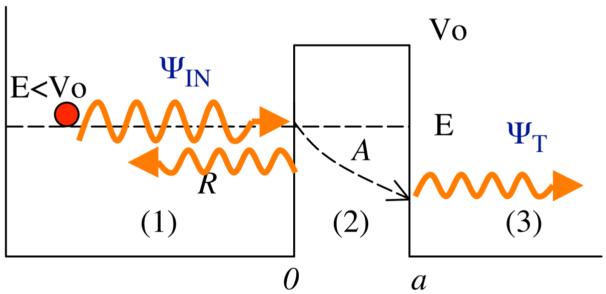
$T = I - R - A \sim 100\% - 8\% - 0\% = 92\%$

Here we assume the absorbed wave is negligible, not always true.

$$A = I \; e^{-x/xo}$$ xo = absorption length

BARRIER PENETRATION

Now consider a finite barrier of height Vo. Particles of energy E < Vo are incident.
We will only give a simplified argument for a general barrier solution here!



$\kappa = \sqrt{2m/\hbar^2 (Vo - E)}$
Wave Number

a = barrier thickness

The transmitted wave amplitude Ψ_T at x = a must be about

$\Psi_T = \Psi_{IN} \exp(-\kappa\, a)$

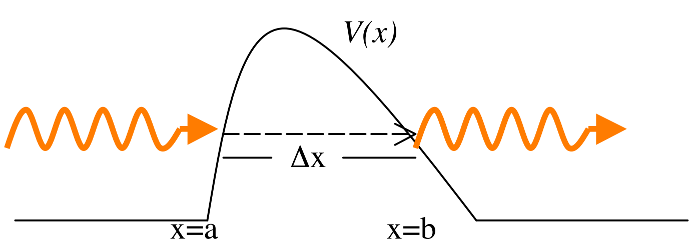
$\mathbb{T} = | \Psi_T |^2 / | \Psi_{IN} |^2 = \exp(-2\,\kappa\, a)$ *Transmission Probability*

It is clear that the energy difference *Vo-E* plays a major role in the probability of barrier penetration. For *E~Vo* and $\mathbb{T} = 0$.

We can calculate the transmission probability through a smoothly shaped barried *V(x)*

$\kappa(x) = \mathrm{sqrt}(\,2m(V(x)-E)/\hbar^2)$

$$\mathbb{T}(E) = \exp\left(-2\int_a^b \kappa(x)\,dx\right)$$



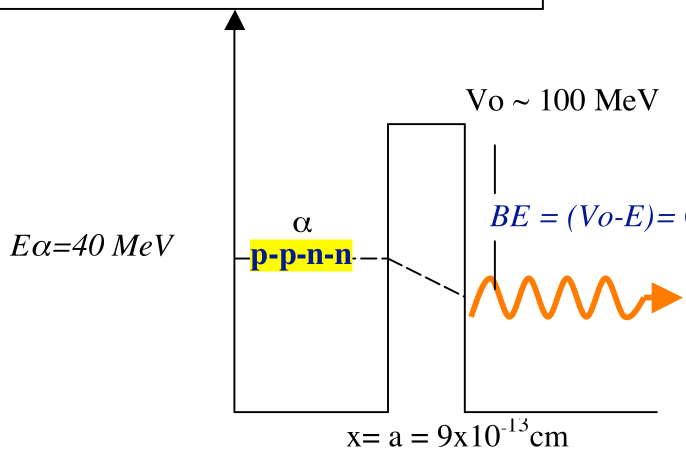
ALPHA PARTICLE $^{4}He_{2}$ ESCAPE PROBABILITY and DECAY

Alpha particle groups have strong binding and **p-p-n-n** combinations occur readily in any nucleus. Some nuclei are not stable to alpha decay. The nuclear binding energy is about 8 MeV per nucleon so there is little chance the alpha particle can escape over the top.

Scientist realize that the alphas can escape by barrier penetration. The α hits the barrier at a frequency f of about 10^{21}/s penetration! f =~v / d = 1/T

The tunneling rate is then

$R = f \bullet \mathbb{T}$	*Decay Rate*
τ=1/R	*Average Decay Time*



M_{α} ~ 3800 MeV/c^2

Consider an alpha particle trapped in a well Vo = 100MeV deep. The α energy is Eα=60 MeVs, thus bound by bound by ΔE=40 MeV. The barrier thickness a~9×10^{-13}cm = 9×10^{-6} nm. Find the alpha particle decay rate R and mean lifetime τ.

Vo – E = 100-40 MeV = 60 MeV Using $\hbar c$ = 197.3 eV-nm

κ = sqrt[$(2m/\hbar^2)$ |(E-*Vo*)|]

= sqrt[$(2m\,c^2 / \hbar^2 c^2)$|(*E-Vo*)|] = sqrt{ [2 (3800x10^6eV)/(197.32 eV-nm)^2] 60x10^6eV}

κ = 3.42e10^6 nm^{-1}

$2\,\kappa\,a$ = 2 (3.42e10^6 nm^{-1}) (9x10^{-6} nm) = 61.5

$\mathbb{T}$ = exp(-2 κ a) = exp(-61.5) = 1.77 x 10^{-27}

R = Rate of Escape = $f \bullet \mathbb{T}$ = (10^{21}/s) (1.77 x 10^{-27}) = 1.76x10^{-6} s^{-1}

τ = 1 / R = 5.66 x 10^{5} s = 0.18 yr average lifetime