CHAPTER 6- QUANTUM MECHANICS in ONE DIMENSION

BORN PROBABILITY INTERPRETATION

Scientists led by Max Born (~1921) interpreted the quantum matter wave $\Psi(x,t)$ with a probability.

$P(\Delta x,t) = \int_{x1}^{x2} |\Psi(x,t)|^2\, dx$ = *probability of finding a particle between x1 and x2 at time t.*

$P(\Delta x,t) = \int_{-\infty}^{+\infty} |\Psi(x,t)|^2\, dx = 1$ *the probability to be anywhere must be 1 (Normalization Condition)*

Wave Vectors $\vec{\Psi}$

$$|\Psi\rangle = a\,|\phi 1\rangle + b\,|\phi 2\rangle + c\,|\phi 3\rangle + d\,|\phi 4\rangle$$

$|\Psi|^2 = \;= a^*a + b^*b + c^*c + d^*d = 1$ *Normalization Condition*

Let $\Psi = (1/\sqrt{3})\,|\varphi 1\rangle + (1/\sqrt{4})\,|\varphi 2\rangle + c\,|\varphi 3\rangle$

From the normalization $|\Psi|^2 = 1$ find c and the probability to be in state #3.

$1/3 + 1/4 + c^2 = 1$ or $c = \sqrt{1 - 1/3 - 1/4} = \sqrt{5/12}$

$P3 = |c|^2 = 5/12$

SCHRODINGER EQUATION

Erwin Shrodinger used *Conservation of Energy* and thDeBroglie and Einstein Equations to Create the *Schrodinger Equation.*

Einstein Equation

$E = \hbar\,\omega$

DeBroglie Equation

$p = \hbar\,\kappa$

Schrodinger Equation

$$\frac{-\hbar^2}{2m} d^2/dx^{-}\Psi(x,t)^{-} + V(x)\Psi(x,t)^{-} = i\hbar\frac{\partial}{\partial t}\Psi(x,t)$$

Kinetic Energy *Potenial Energy* *Total Energy*

$T + V = E$ *Conservation of Energy*

$$d^2/dx^2\Psi(x,t)^{-} + \frac{2m}{\hbar^2}(E - V)\Psi(x,t) = 0$$

$\Psi'' + 2m/\hbar^2\,(E-V)\,\Psi = 0$ $k = \sqrt{2m/\hbar^2(E-V)}$ *Wave #*

$\Psi'' + k^2\,\Psi = 0$ *Harmonic Equation*

FREE PARTICLE $V(x) = 0$

$\Psi'' + k^2 \Psi = 0$

$k = \sqrt{2mE/\hbar^2}$

$E = \hbar^2 k^2 / 2m$

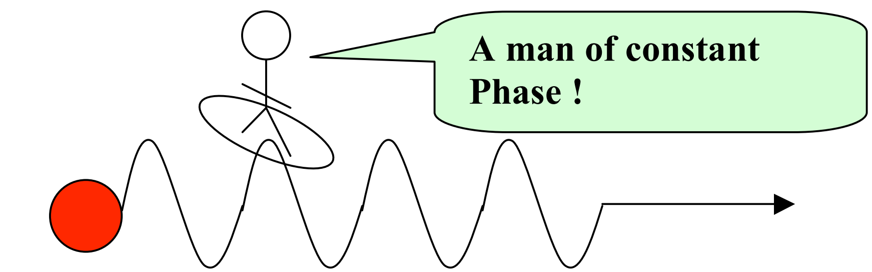
$\Psi(x,t) = \exp[\pm i\,(k\,x - \omega\,t)]$

Wave solution where $kx - \omega t = phase$

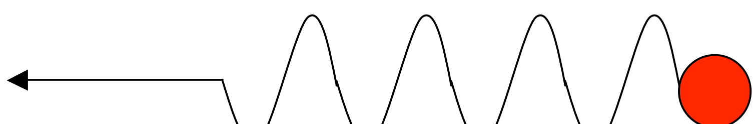
- A free particle is represented by a sinusoidal wave traveling to the right or left with velocity $v = \omega / k$.
- For a particle of light with zero mass $\omega / k = c$, *the speed of light.*
- Any point on the wave is a point of constant phase o $kx - \omega t = \phi$ = phase

$k\,dx - \omega\,dt = 0$

$v = dx/dt = \omega / k$ *phase velocity*



$\Psi = = \exp\{ +i(kx-\omega t) \}$



$\Psi = = \exp\{ -i(kx-\omega t) \}$

Show that $\Psi'' + k^2 \Psi = 0$

$\Psi = \exp[+i\,(k\,x - \omega\,t)]$

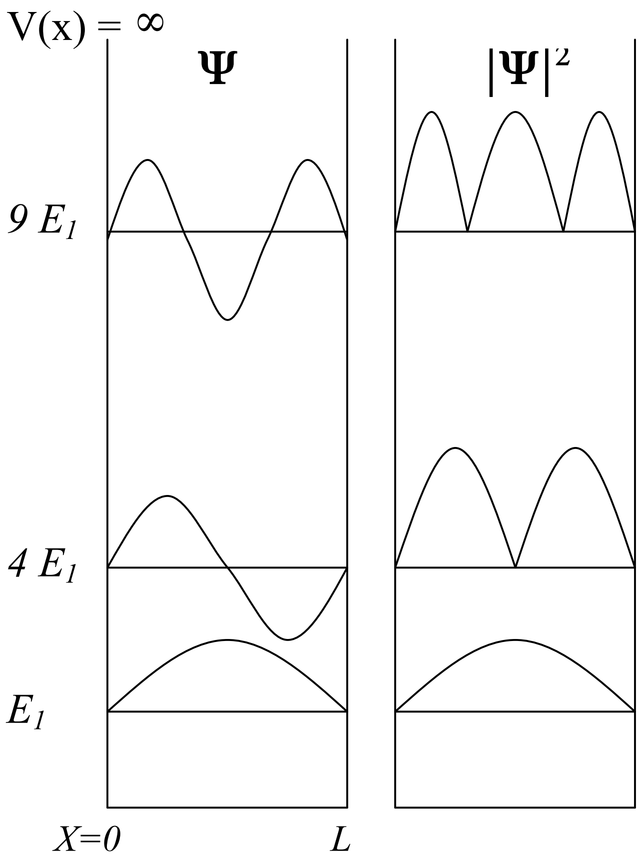
$\Psi' = ik \exp[+i\,(k\,x - \omega\,t)]$

$\Psi'' = -k^2 \exp[+i\,(k\,x - \omega\,t)]$

$\Psi'' = -k^2\,\Psi$

$\Psi'' + k^2 \Psi = 0$ *QED*

INFINITE WELL (V(x) = 0 Inside V(x) = ∞ Outside)



$-h^2/2m\ \Psi'' = E\ \Psi$

$\Psi'' + k^2\ \Psi = 0$ *Harmonic Equation*

$\Psi = A \sin(k x)$

The wave must vanish at x = 0 → $\Psi(x) = 0$

The wave must vanish at x = L → $\Psi(L) = 0$ $kL = n\pi$

$k = n\ \pi / L$

$E = \hbar^2 k^2 / 2m$

$E = \hbar^2 (n\ \pi / L)^2 / 2m$ $\psi_1 = A \sin(k_n x)$

$E_1 = \hbar^2 (1\ \pi / L)^2 / 2m$ $\psi_1 = A \sin(\pi / L\ x)$

$E_2 = \hbar^2 (2\ \pi / L)^2 / 2m$ $\psi_2 = A \sin(2\pi / L\ x)$

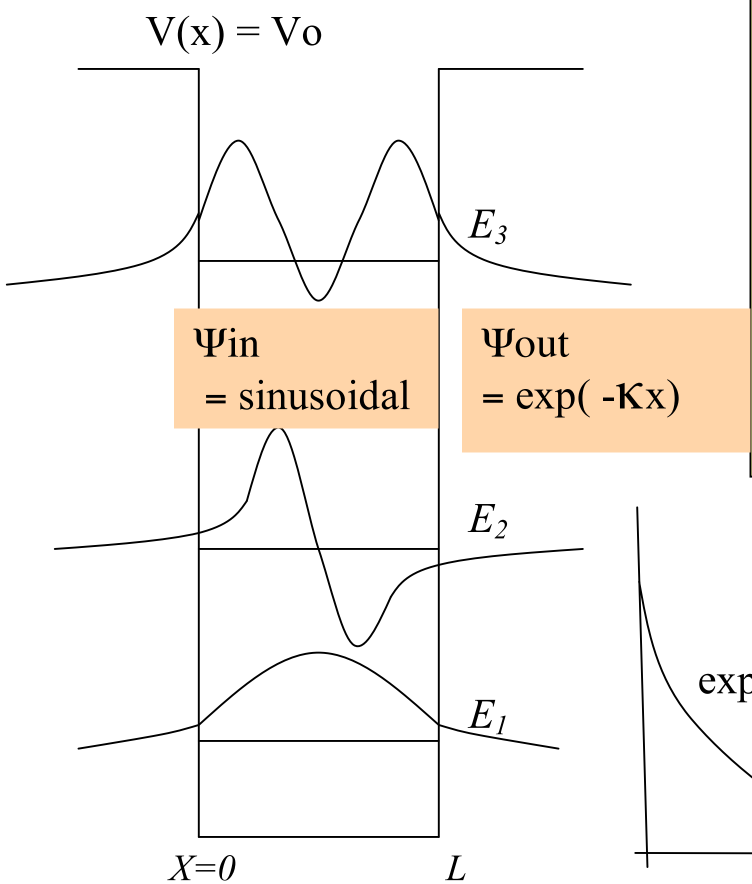
$E_3 = \hbar^2 (3\ \pi / L)^2 / 2m$ $\psi_3 = A \sin(3\pi / L\ x)$

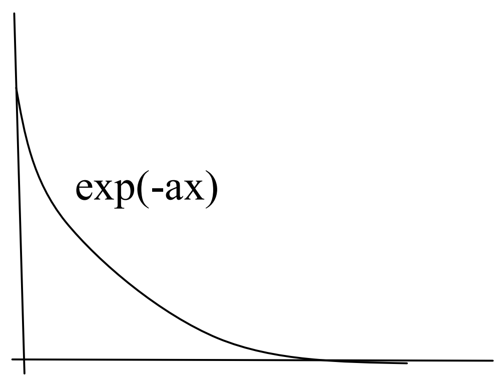
FINITE WELL V(x) = Vo

- If the potential barrier has a finite in height Vo the the wave Ψ may leak out of the barrier.
- Ψin = sinusoidal wave as before. $\Psi in = A \sin(kx + \phi)$
- Ψout = exponentially damped wave $\Psi out = B \exp(-\kappa x)$

  $\kappa = [\ 2m/h^2\ |(En\text{-}Vo)|\ ]^{1/2}$ *damping constant*

- Prob (leak out) $= |\Psi out|^2 = B^2 \exp(-2\kappa x)$





EXPECTATION VALUES

Since the quantum theory gives a probabilistic interpretation through the wave function $|\Psi|^2$ we can work out what we might expect to see upon making a measurement on an ensemble of particles in a quantum state Ψ . This is call the *Expectation Value*.

Consider that we want to make a measurement of the energy E of a system.

Let $\Psi = a\,\psi1 + b\,\psi2 + c\,\psi3 + d\,\psi4$ *a superposition of 4 states*

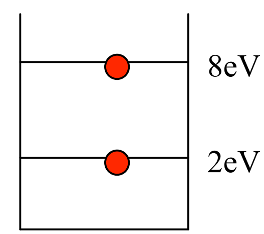
$<E>$ = expectation value of E (similar to the average value of E)

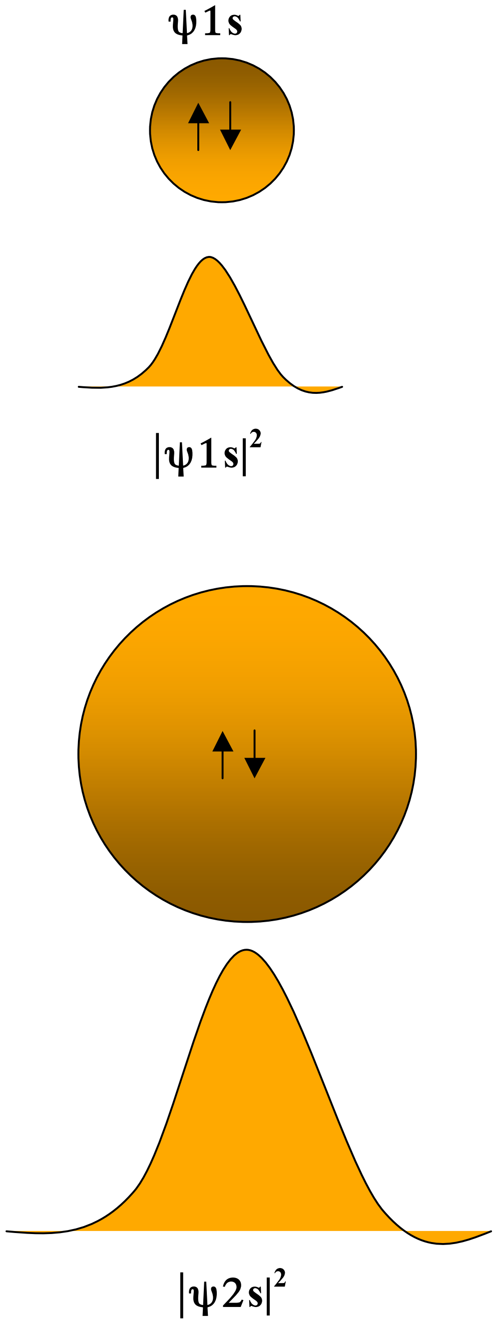
$$<E> = E1 \times P1 + E2 \times P2 + E3 \times P3 + E4 \times P4$$
$$<E> = E1\,|a|^2 + E2\,|b|^2 + E3\,|c|^2 + E4\,|d|^2$$
$$<p> = (\hbar\,k1)\,|a|^2 + (\hbar\,k2)\,|b|^2 + (\hbar\,k1)\,|c|^2 + (\hbar\,k1)\,|d|^2$$

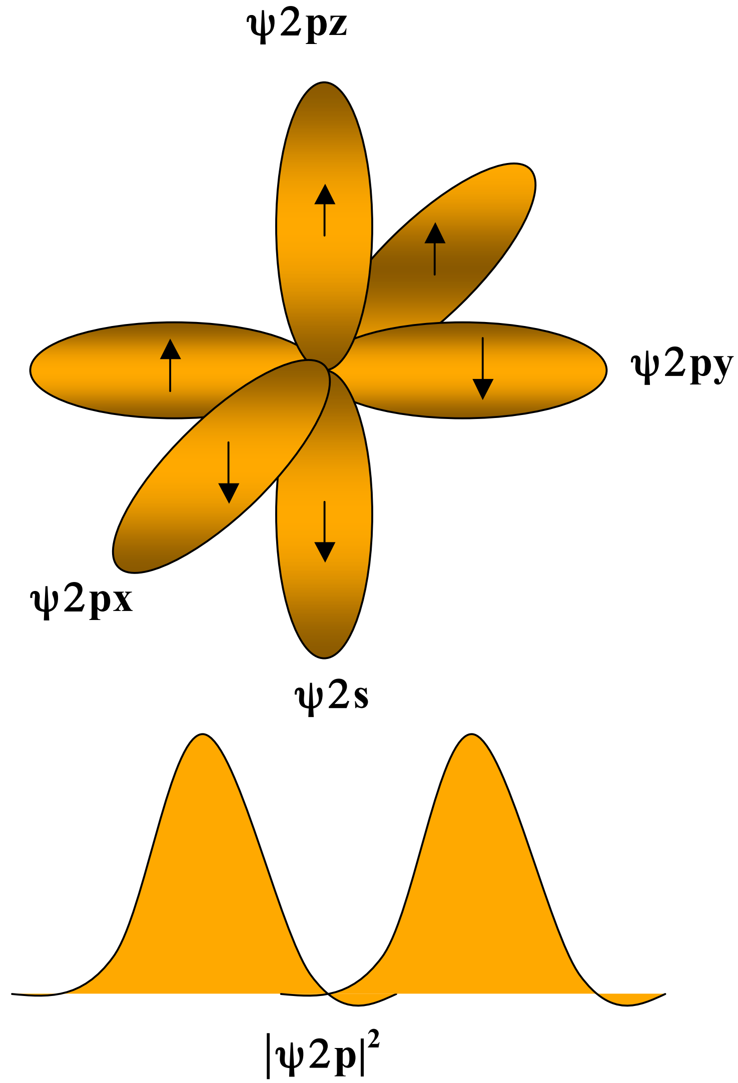
$$<Q> = q1\,|a|^2 + q2\,|b|^2 + q3\,|c|^2 + q4\,|d|^2$$
$$<Q> = \sum_{i=1,n} q_i P_i$$

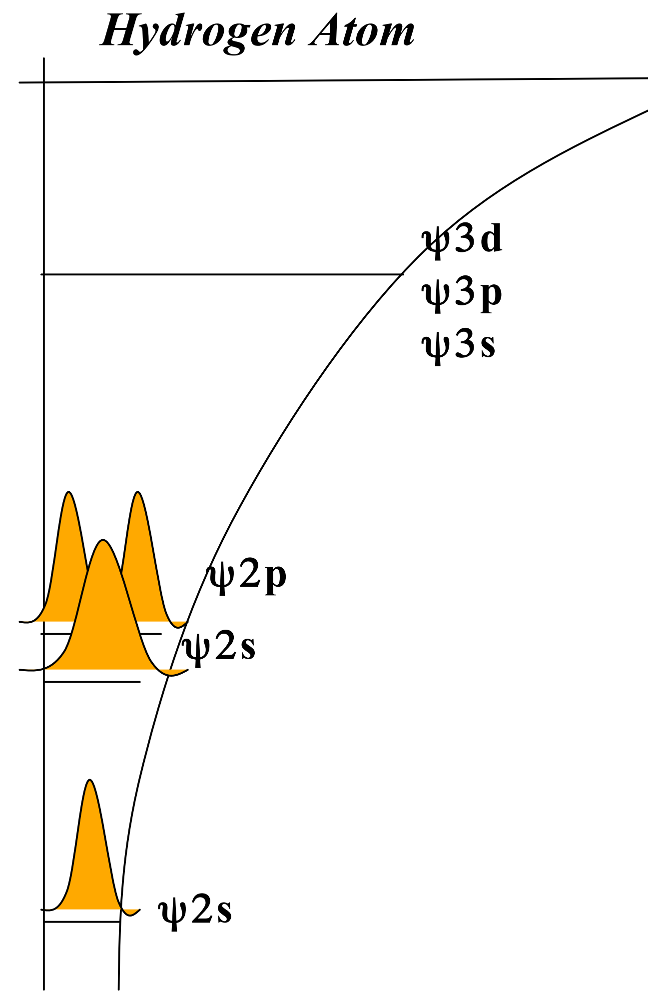
Consider particle in the Infinite Square Well with 50% probability to be in the n=1 and 50% probability to be in n=2 state were E1 = 2eV. If you make a measurement of E on an ensemble of particles in this state what would you expect to get?

E1 = 2 eV P1=1/2
E2 =□8 eV P2=1/2 ----> $<E> =$ 1/2 (2eV) + 1/2(4 x 2eV) = 5 eV









$\Psi\text{atom} = +a\,\psi 1s + b\,\psi 2s + c\,\psi 2px + d\,\psi 2py + e\,\psi 2pz$

Wave Superposition