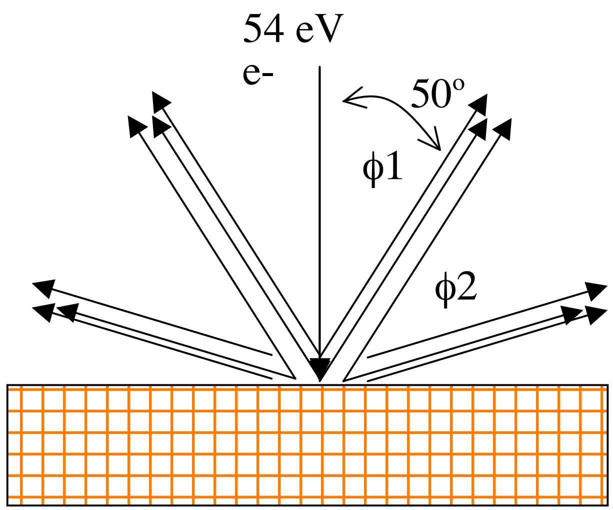
CHAPTER 5 Matter Waves

DeBroglie's hypothesis $\lambda = h / p$ allowed scientist to make predictions about the quantum nature of matter. $p = \gamma mv$ or $pc = \sqrt{E^2 - mc^4}$

Davisson and Germer performed an important experiment that directly displayed the wave nature of matter. They scatter an electron beam from a crystal and observed a wave interference phenomena.



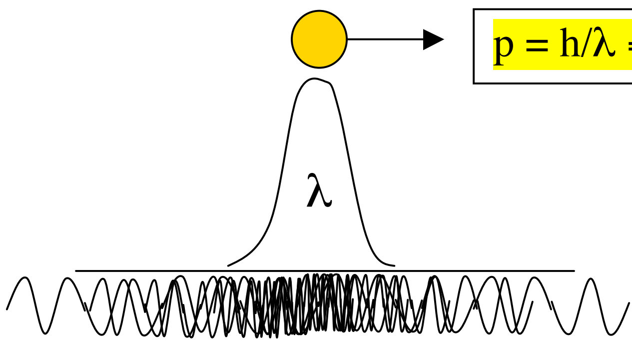
$d \sin \phi = n \lambda$ □=1,2,3,...

d = lattice spacing
n = *diffraction order*

T = 54 eV and T = 1/2 m v^2 = p^2 /2m
p = sqrt(2mT) = [2 (511 KeV/c^2) (54 eV) $]^{1/2}$ = 55.2e6 eV/c =7.43e3 eV/c
λ = h / p = (4.14e-16 eV-s) / 7.43e3 eV/c = 5.57e-19 s 3e8 m/s = 16.7 e-11 m = 0.167 nm
d = 0.167 nm/sin 50° = 0.167 nm / 0.77 = 0.22 nm *Very good agreement with X-ray diffraction measurements d=0.215 nm* !!

SUPERPOSITION of WAVES and WAVE GROUPS

DeBroglie's wave can be thought of as a *wave packet*. The packet has a size of $\sim\lambda$, but it is made of a superposition of traveling waves of different wavelengths.



$p = h/\lambda = \hbar k$

Fourier Wavelets
sin(kx-wt) or cos(kx-wt)

FOURIER SERIES
Fourier showed that any periiodic function may be expanded in a series of sine and cosine functions. These are called ***Fourier Series.***

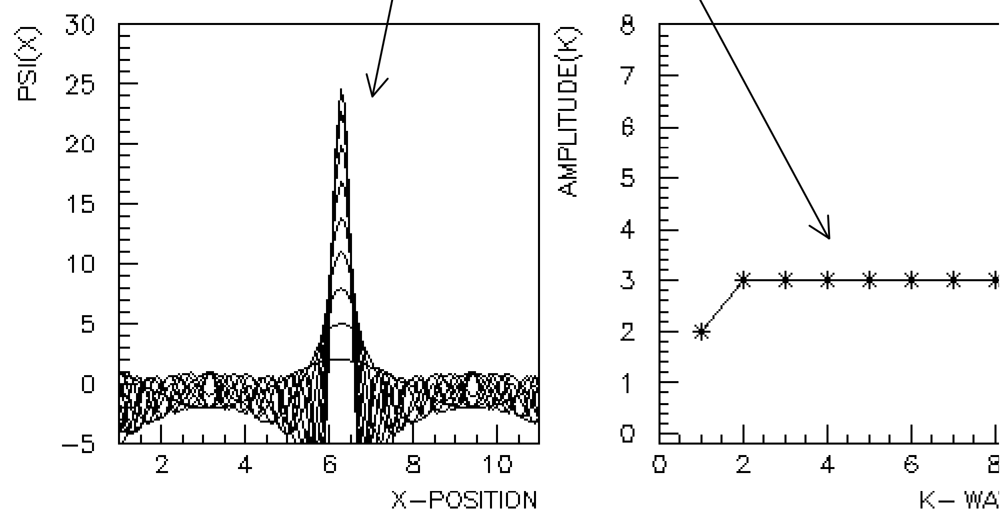
$$F(x) = \sum_k A_k \sin(kx) + B_k \cos(kx)$$

$$F(t) = \sum_\omega A_\omega \sin(\omega t) + B_\omega \cos(\omega\tau)$$

$p = h/\lambda = \hbar k$ $\quad k = 2\pi / \lambda$ *wave number*

Mathematically we can construct a group of wa called a wave packet by adding sine or cosine w of different amplitudes and wavelengths λ or frequencies ω .

$$\psi_X = \sum_k A_K \sin(kx)$$

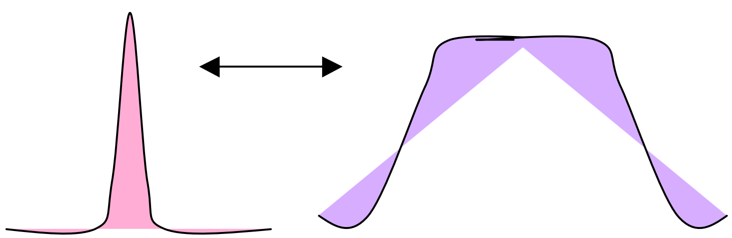


$\psi(x) = \mathbf{2} \sin(1\ x) + \mathbf{3} \sin(2\ x) + \mathbf{3} \sin(3\ x) + \mathbf{3} \sin(4\ x) + \mathbf{3} \sin(5\ x) + \mathbf{3} \sin(6\ x) + \mathbf{3} \sin(7\ x)$

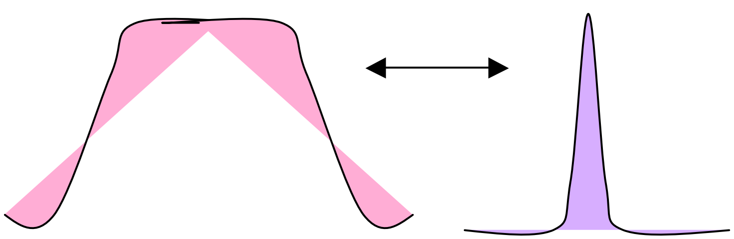
UNCERTAINTY RELATION for WAVES

In signal theory we know that a narrow wave packet in space can only be produced by a broad pack in *momentum or k-space*. And vice versa.

$\Psi(x)$ $\quad$ $A(\hbar k)$



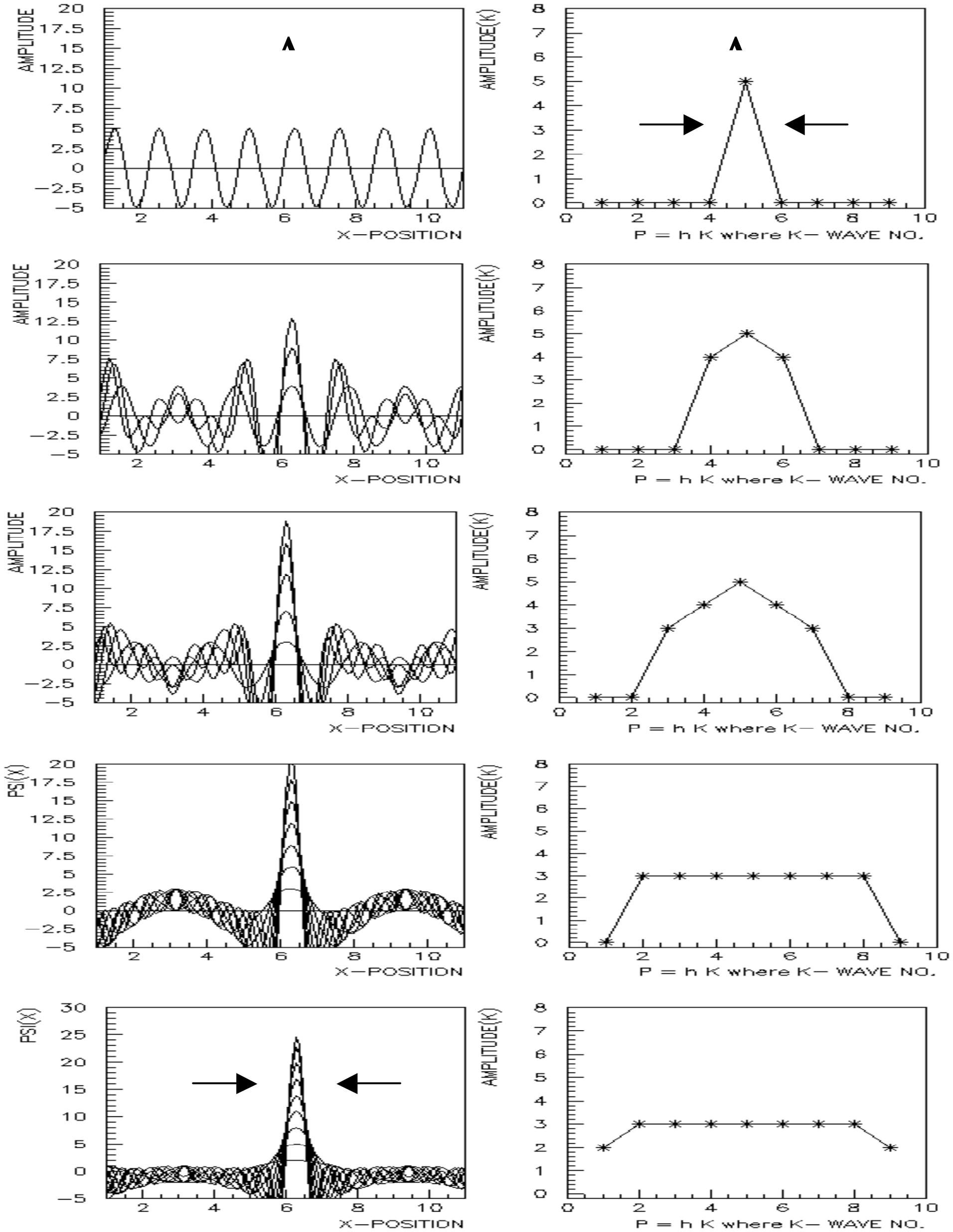
$\Psi(x)$ $\quad$ $A(\hbar k)$



$\Delta x\, \Delta p > \hbar/2$ Uncertainty in x p

$\Delta E\, \Delta t > \hbar/2$ Uncertainty in t E

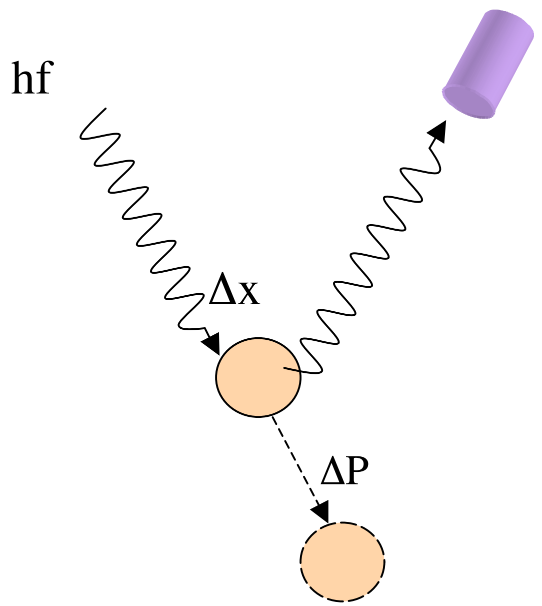
$\Psi(x)$ $\quad$ $A(\hbar k)$



UNCERTAINTY and QUANTUM MEASUREMENT

If I attempt a precise measure on the position of a particle I necessarily will disturb its momentum!

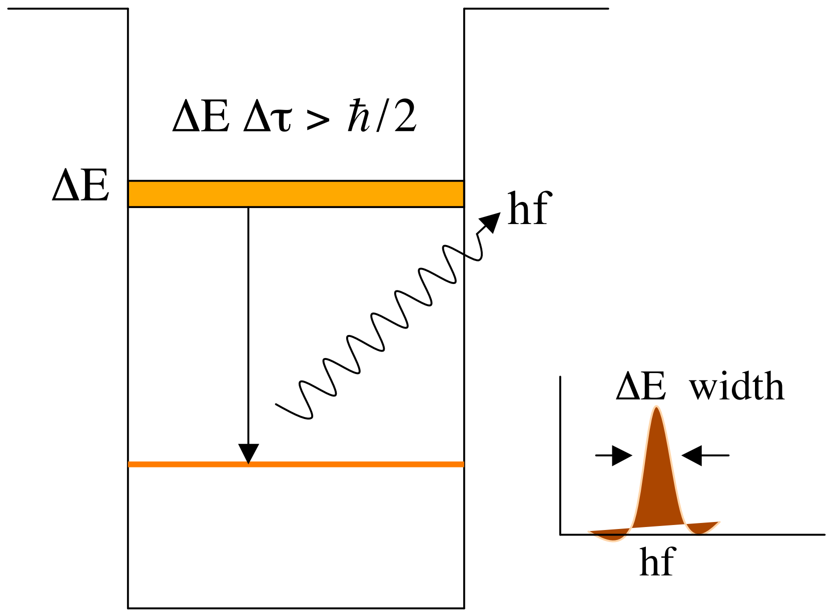
If I attempt a precise momentum measurement I disturb its position!



$\Delta x\ \Delta p > \hbar/2$

An excited state of energy width ΔE will take Δτ time to decay!

Δτ is called the *mean lifetime* of the state. (see problem 25)

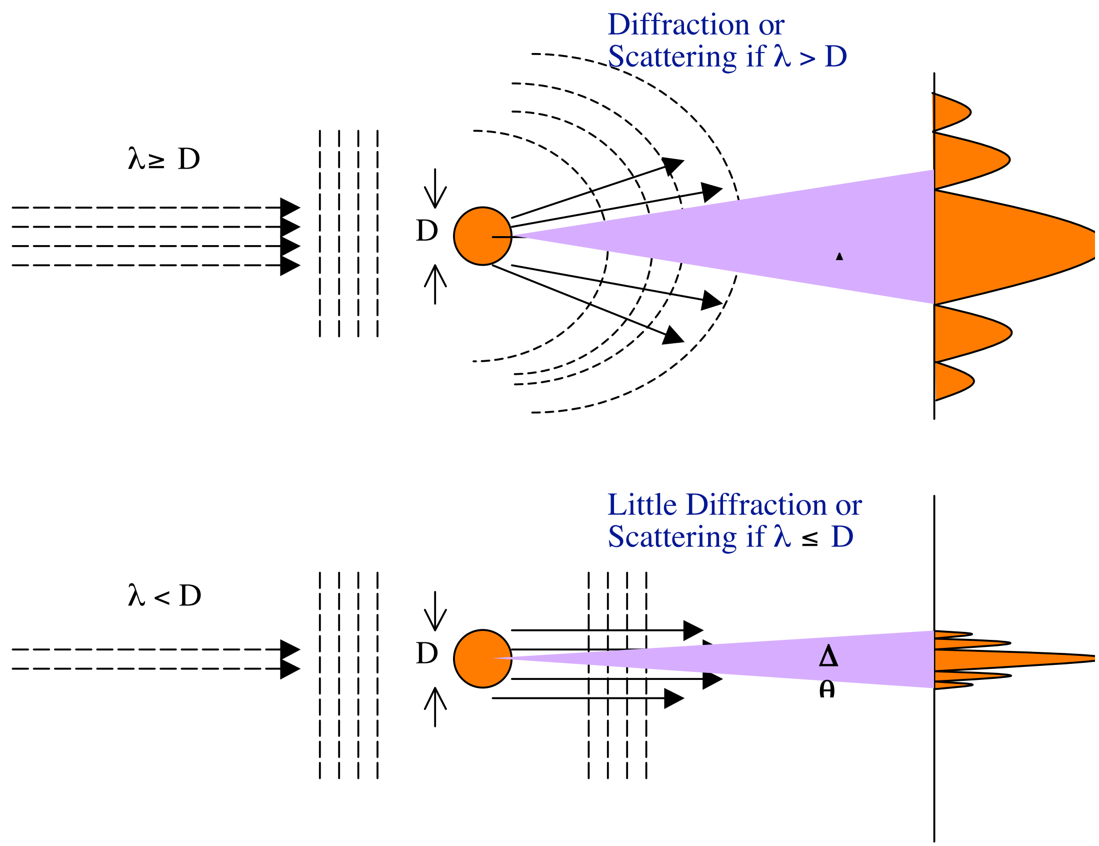


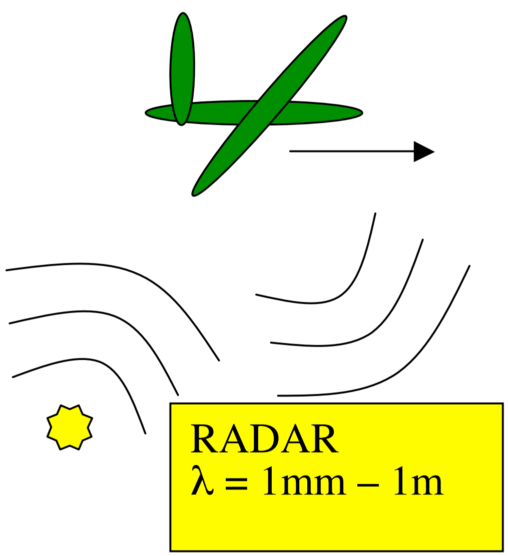
DIFFRACTION of WAVES

Consider a wave incident on a slit or object of size *D*.

If the incident wave has wavelength $\lambda \geq D$ the wave scattering occurs. This is call *diffraction*.

If the incident wave has wavelength $\lambda < D$ little scattering occurs and the wavepasses through the opening or around the object unscattered.





ELECTRON MICROSCOPY

Engineers Max Knoll and Ernst Ruska realized that particles were capable of producing much shorter wavelengths and could be used for observing features down to fractions of an Angstrom (*10-8cm)*

They developed the idea in Germany in 1931 and produced the first Transmission Electron Microscope (TEM). Later a scanning (SEM) microscope emerged.

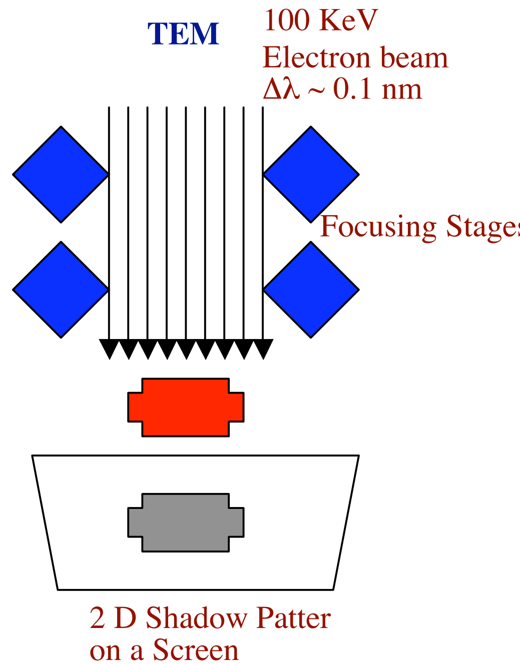
Consider a KE = 100KeV electron of mass m= 511 KeV/c2
Since KE $\approx$ mc^2 we **must us the** relativistic equations!

$p = [E^2 - m^2 c^4]^{1/2}$ $\quad$ $E^2 = p^2 c^2 + m^2 c^4$

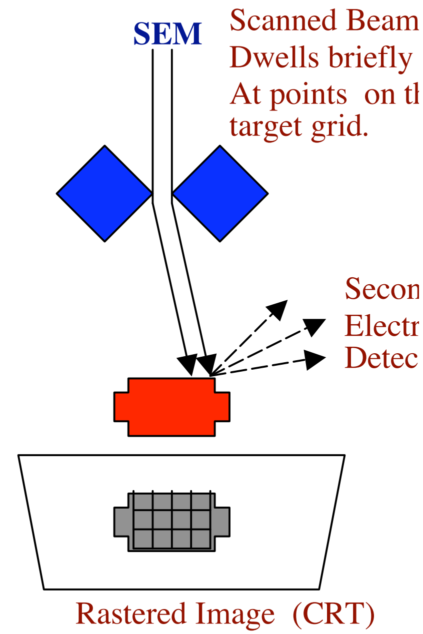
$= [611000^2 - 511000^2]^{1/2} = 335000$ eV/c

$\lambda = h / p = hc / pc = 1240 / 335000$ nm = $\underline{0.004\text{nm}}$

The practical resolution in a TEM is limited by electrostatic focusing issues of the electron beam to a resolution of about $\Delta\lambda \sim 0.1$ nm .



TRANSMISSION ELECTRON MICROSCOPE



SCANNING ELECTRON MISCROSCOPE

THE DOUBLE SLIT EXPERIMENT for PARTICLES

Consider a particle of mass *M* approaching a two slit apparatus. Of slit separation *D*.

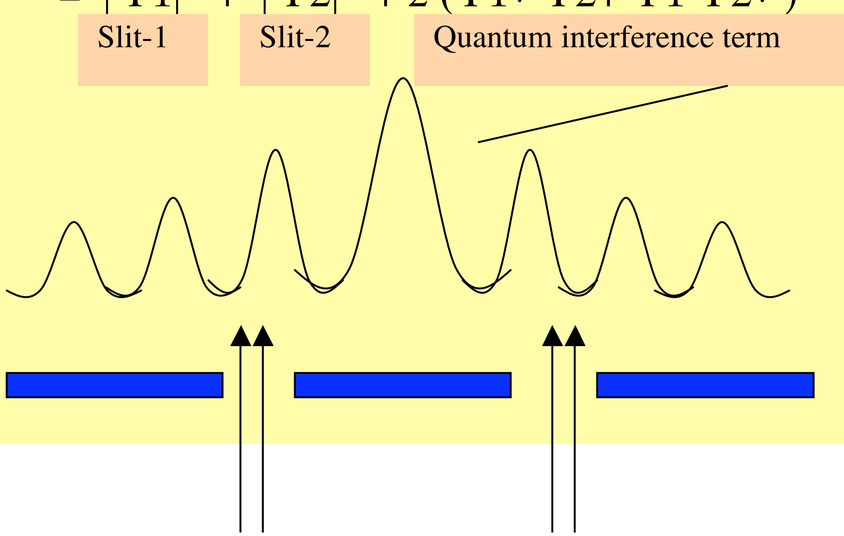
The matter wave can take either path-1 or path-2.

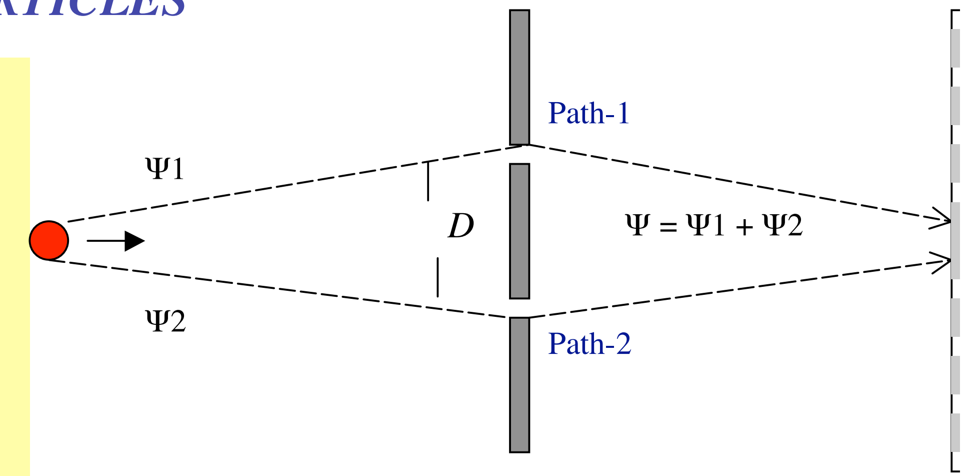
The wave at the screen must be a superposition of $\Psi 1$ and $\Psi 2$

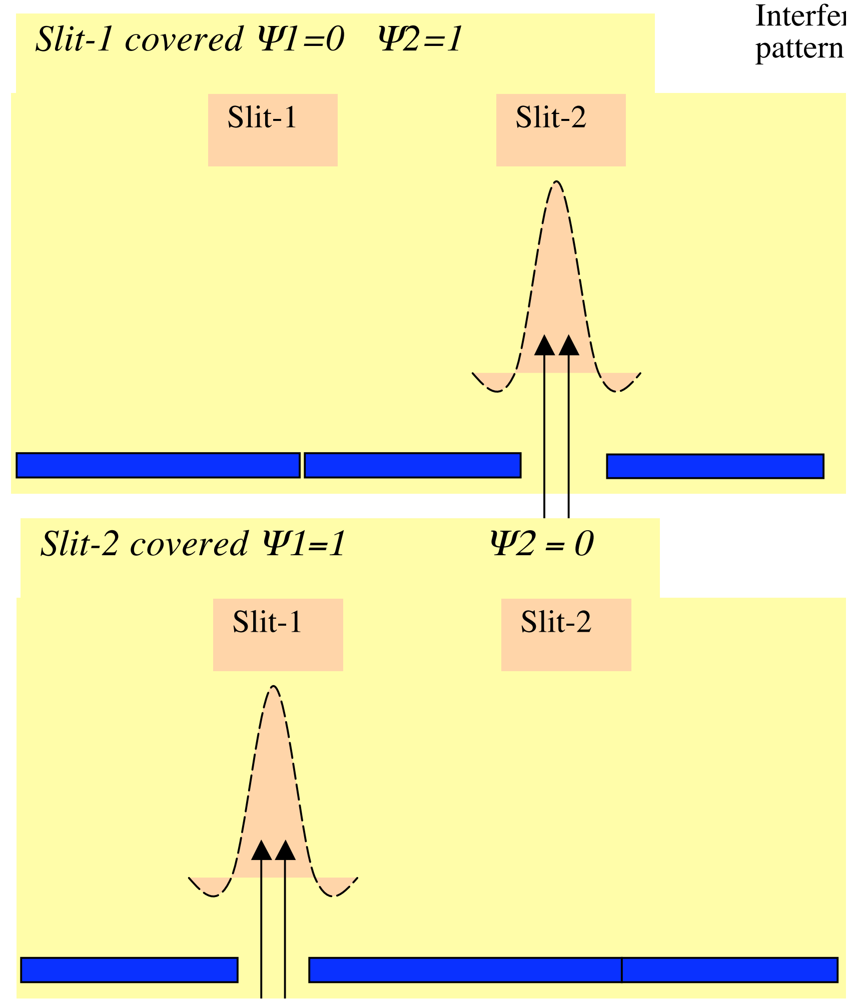
$$\Psi = \Psi 1 + \Psi 2$$

The quantum physicist new that $\Psi(x,t)$ could be complex. Since physical measurements should yield real answers they interpreted $|\Psi(x,t)|^2$ as the *Probability* of finding the particle *M* at position x on the screen.

$$P(x) = |\Psi|^2 = (\Psi 1 + \Psi 2)(\Psi 1^* + \Psi 2^*)$$
$$= |\Psi 1|^2 + |\Psi 2|^2 + 2(\Psi 1^* \Psi 2 + \Psi 1 \Psi 2^*)$$



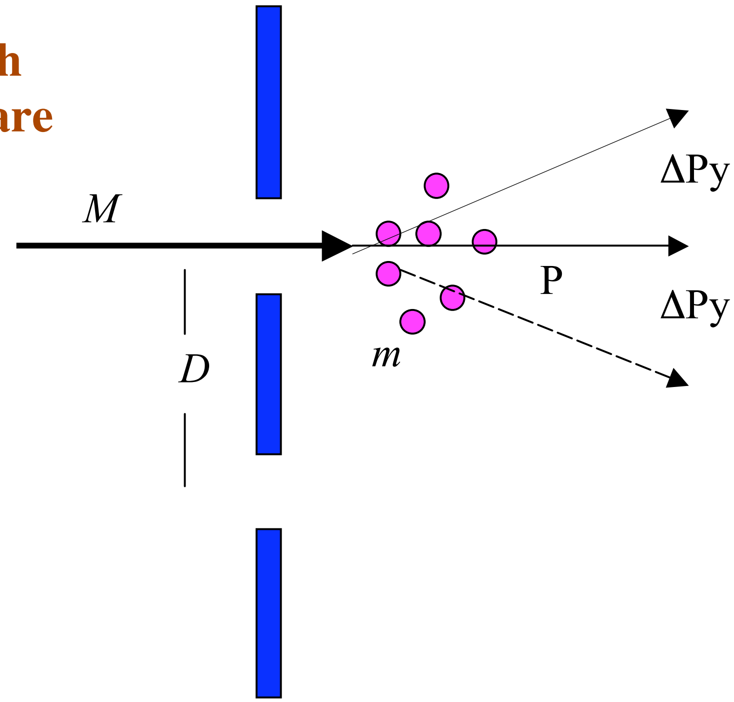




UNCERTAINTY INTEPRETATION

Consider quantum non-believer Albert devising an experiment to determine which slit a particle *M* traversed when both sits are open.

He will observe the deflection of an atom *m* and trace it back. The



ΔP D $\geq \hbar/2$

Tan (θ) ~ θ = ΔP/P

P $\Delta\theta$ D ~ $\hbar/2$

$\Delta\theta$ = (h/2π) 1/P D = (6.6e-16 eV-s)/ (100eV/c)(.01m) = 6.6e-16 radians

No apparatus can resolve an angle to this precision!!

ATOMS

- Each energy state in an atom represents a quantum state $\phi 1, \phi 2, \phi 3$, .. with energy E1, E2, E2, …

- The excited states have energy widths ΔE and the time Δt to make a transition is dictated by uncertainty principle

$$\Delta E\ \Delta t \sim \hbar/2.$$

- The atomic wave function Ψ is a superposition of energy states.

$$\Psi = a\,|\,\phi 1> \ + b\,|\,\phi 2> + c\,|\,\phi 2> + d\,|\,\phi 4 > + e\,|\,\phi 5> \ + f\,|\,\phi 6>$$

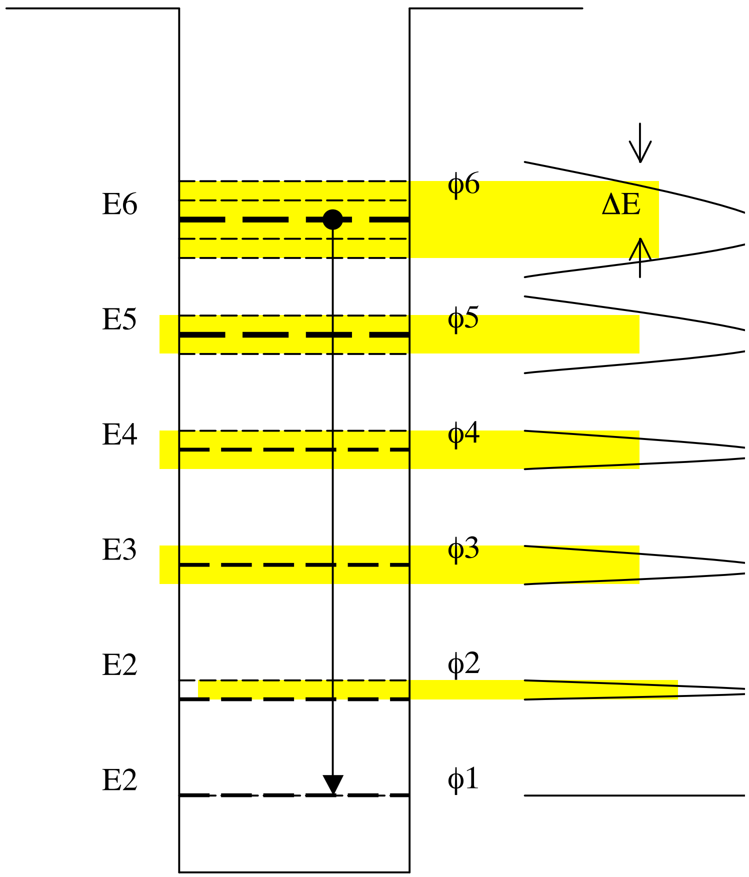
- **We don't know which state the atom is in. All we can calculate is the probability for being in state 1,2,3,4,5,6,**

P1 = $|a|^2$
P2 = $|b|^2$
P3 = $|c|^2$
P4 = $|d|^2$
P5 = $|e|^2$
P6 = $|f|^2$

Normalization Condition

$$\sum_{n=1}^{6} Pn = |a|^2 + |b|^2 + |c|^2 + |d|^2 = 1$$

Meaning the atom must be in at least one state!



QUIZ PROBLEM

Let $\Psi = N\ [\ 3\ |\ \phi 1> + 6\ |\ \phi 2 > + \sqrt{7}\ |\ \phi 3 >\ )\ ]$. N is called the overall normalization.

If $|\ a\ |^2 + |\ b\ |^2 + |\ c\ |^2 = 1$ find A and the probability for being in the n=2 state

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$a = 3N$
$b = 6N$
$c = \sqrt{7}\, N$

$(3N)^2 + (6N)^2 + (\sqrt{7}\, N)^2 = 1$
$(9 + 36 + 7)\, N^2 = 1$
$52\, N^2 = 1$
$\boldsymbol{N = 1} / \sqrt{52}$

P2 $= |\ b\ |^2 = (\ 6 / \sqrt{52}\ )^2 =$ **0.69**