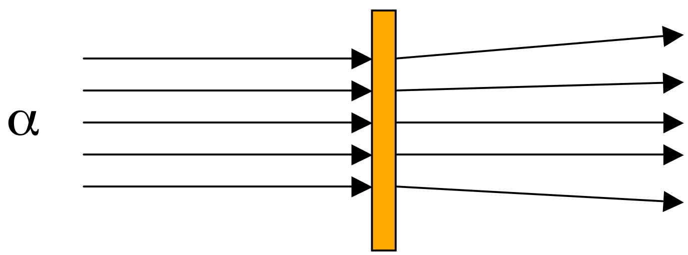
CHAPTER 4- PARTICLE/WAVE NATURE OF MATTER

RUTHERFORD ATOM

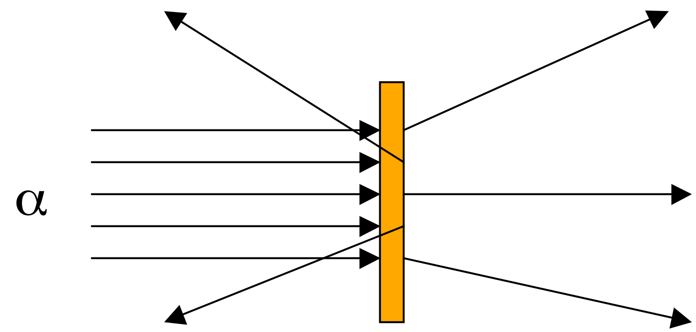
E. Rutherford discovered that the atom had a hard core we call the nucleus.

He scattered alpha particles ($^4He_2$) on a gold foil and noticed some α's were deflecting backwards. How could bb's reflect backwards from a tissue paper.

Expected minimal deflection



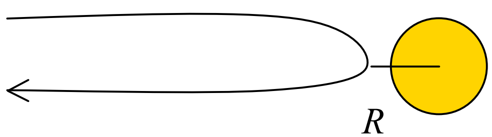
Observed large deflection



He could only conclude that he gold atoms must have a hard core.
He estimated the radius R of the nucleus by using conservation of energy.

$$KE\alpha + \overset{0}{PE\alpha} = \overset{0}{KE\alpha} + PE\alpha$$

Source *Turning point*



$KE\alpha = k(2e)(Ze)/R$

$R = k(2e)(Ze)/KE$

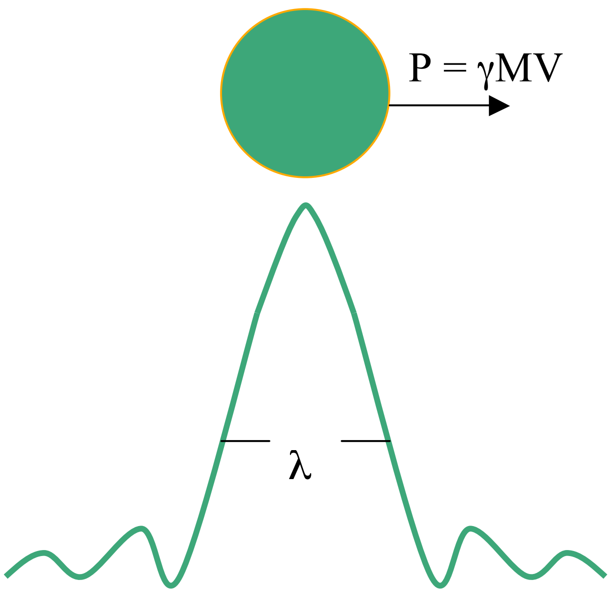
$R < 4.9e\text{-}15$ m

See example 4.5

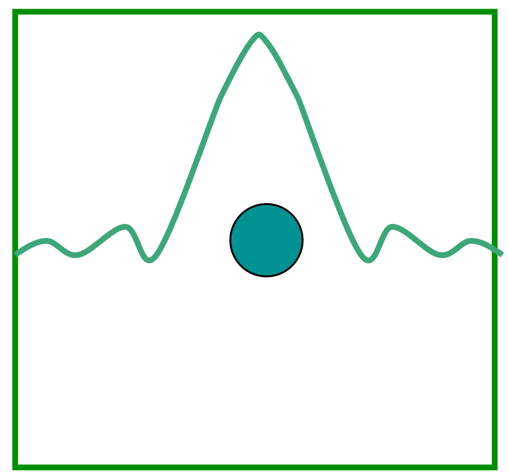
DeBROGLIE HYPOTHESIS

Louis DeBroglie speculated that if light could behave as a particle, then matter might behave as a wave?

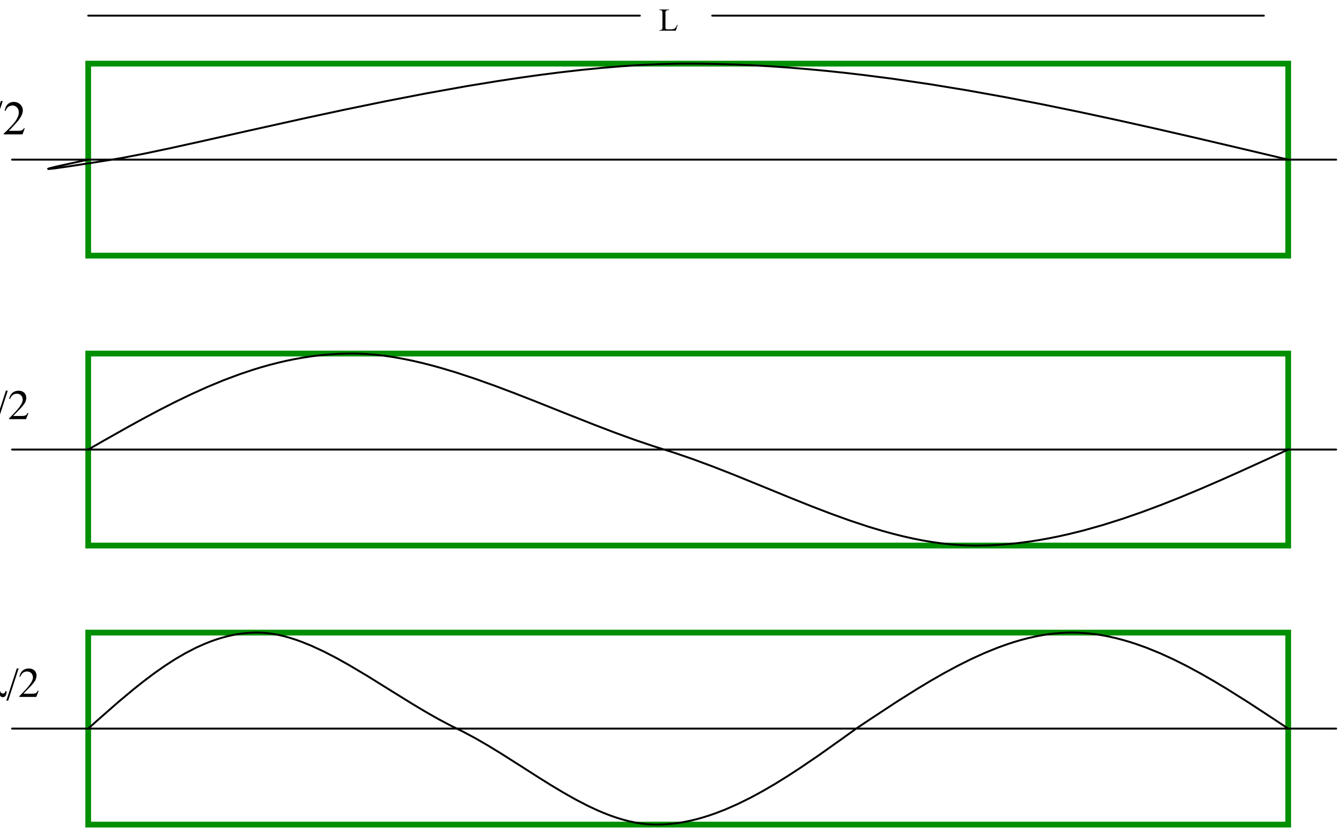
$\lambda = h / p$



Consider a particle of mass M trapped in an enclosure. Might only certain wavelenths be trapped just as in a musical instrument?



*Particle trapped in a **Box** is a good analogy to a **Nucleus** or **Atom** trapped in its orbit about the nucleus.*



$L = \lambda/2$

$L = 2\lambda/2$

$L = 3\lambda/2$

•

•

•

$L = n\,\lambda/2$

$\lambda = 2\,L/n$

$P = h/\lambda$

$P_n = nh/2L$ *Momentum of nth harmonic*

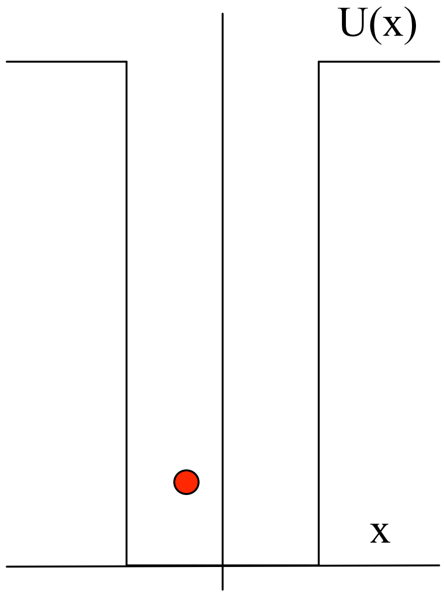
$E = p^2/2m$

$E_n = n^2h^2/8mL^2$ *Energy of nth harmonic*

$y(x) = A \sin(2\pi x/\lambda) = A \sin(k x)$ $k = 2\pi/\lambda$ = wave number

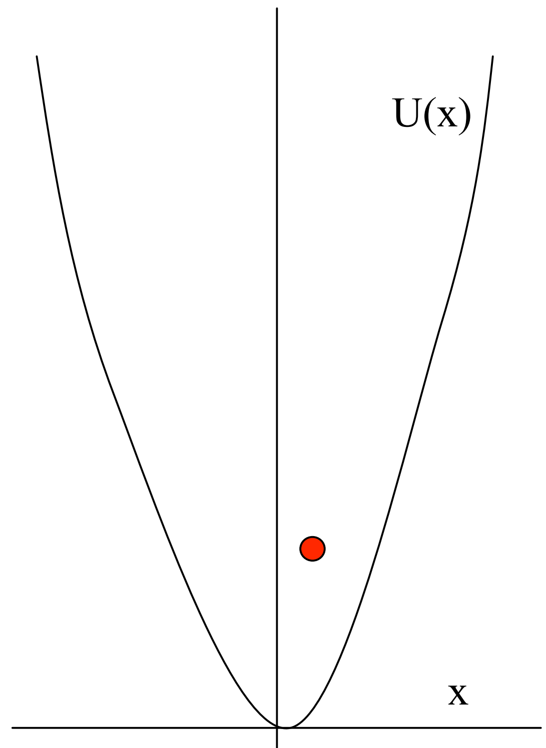
POTENTIAL ENERGY BARRIERs U(x)

The Box represents a Potenial Energy Barrier U(x). where $F(x) = - dU(x)/dx$



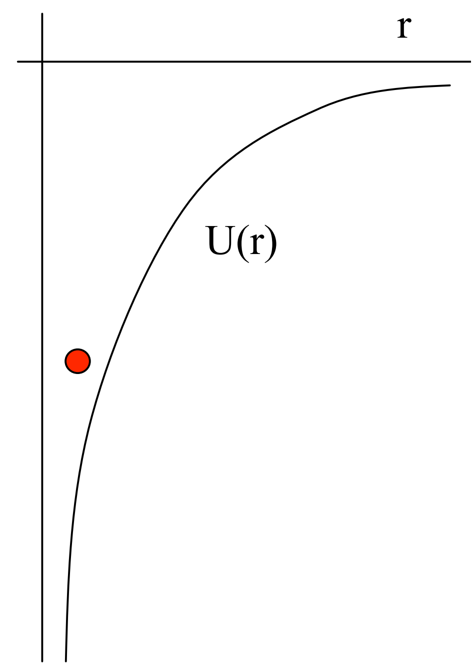
<u>1-D Square well</u>
U(x) = const
F(x) = 0 inside
F(x) = ∞ at walls

NUCLEUS



<u>Harmonic Oscillator</u>
$U(x) = 1/2k\ x^2$
$F(x) = -kx$

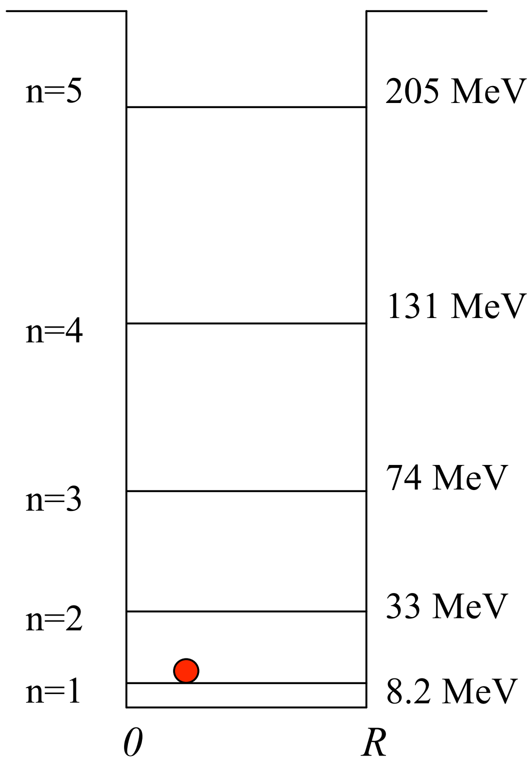
CRYSTAL



<u>Coulomb Potential</u>
$U(r) = k\ q1\ q2 / r$
$F(r) = -k\ q1\ q2 / r^2$

HYDROGEN ATOM

NEUTRONS, PROTONS TRAPPED IN A NUCLEAR WELL



The nucleus is made of neutrons and protons bound together by the nuclear force. The force F(x) felt by any one *nucleon* is roughly constant until the nucleon reaches the nuclear radius *R*.

The wave representing the n or p can only take on momentum given by the DeBroglie wave: $P = h / \lambda$.

$En = n^2h^2 8mR^2$ n=1,2,3,4,… *R=5.0e-15m=Nuclear Radius*

$En = n^2\ (4.136e\text{-}15\ eV\text{-}s)^2/8(939e6\ eV/c^2)\ (5.0e\text{-}15m)^2$

$En = n^2\ 8.2\ MeV$ *Energy Levels*

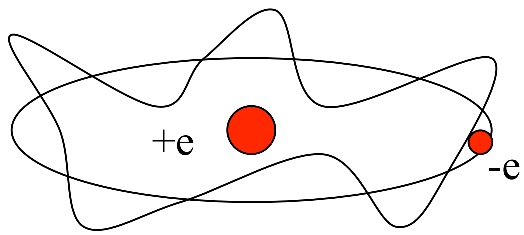
What is the momentum of a nucleon in the ground state?

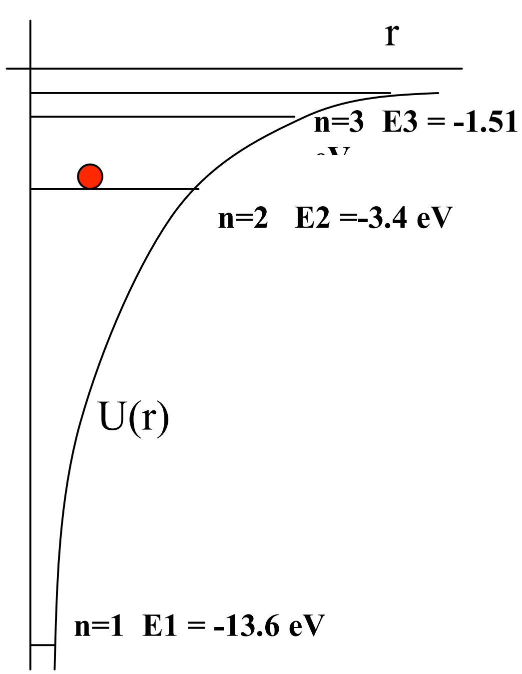
$E = 1/2\ m\ v^2 = P^2/2m$

$P_1 = sqrt(2mE_1) = sqrt\{2\ (939\ MeV/c^2)\ (9.2MeV)\}$

$P_1 = \underline{131\ MeV/c}$

THE BOHR ATOM - Quantum Solution of the Hydrogen Atom





<u>Coulomb Potential</u>

$U(r) = k\, q1\, q2 / r$

$F(r) = -k\, q1\, q2 / r^2$

The electron orbits about the nucleus bound by the Coulomb force.

The electron's potential energy is given by $U(r) = k\,(+Ze)\,(-e) / r$.
The electron's kinetic energy is given by $T = 1/2\, mv^2$

$2\pi r = n\lambda = nh/p$ — *Bohr's Hypothesis of constructive*

$mvr = n\, h/2\pi = n\hbar$ — $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ *Angular momentum*

$L = mvr = n\hbar$

$mv^2/r = ke^2/r^2$ — *centripetal force balance*

$m^2v^2r^2 = m\, ke^2 r$

$(n\hbar)^2 = m\, ke^2 r$

$r_n = (n\hbar)^2 / m\, ke^2$ — $n=1,2,3,\ldots$ *quantum radii*

$ao = h^2/m\, ke^2 = 0.0529\text{nm}$ — $n=1$ *Bohr radius*

$r_n = n^2\, ao$ — $r_n = (n^2 / Z)\, ao$ *hydrogen-like atom*

$1/2\, mv^2 = ke^2/2r$

$E = 1/2\, mv^2 - ke^2/r$ — *total energy (non-relativistic)*

$E = ke^2/2r - ke^2/r = -ke^2/2r$

$En = -ke^2/2r = -ke^2/2[(n\hbar)^2/m\, ke^2] = -(ke^2/2ao)\, 1/n^2$

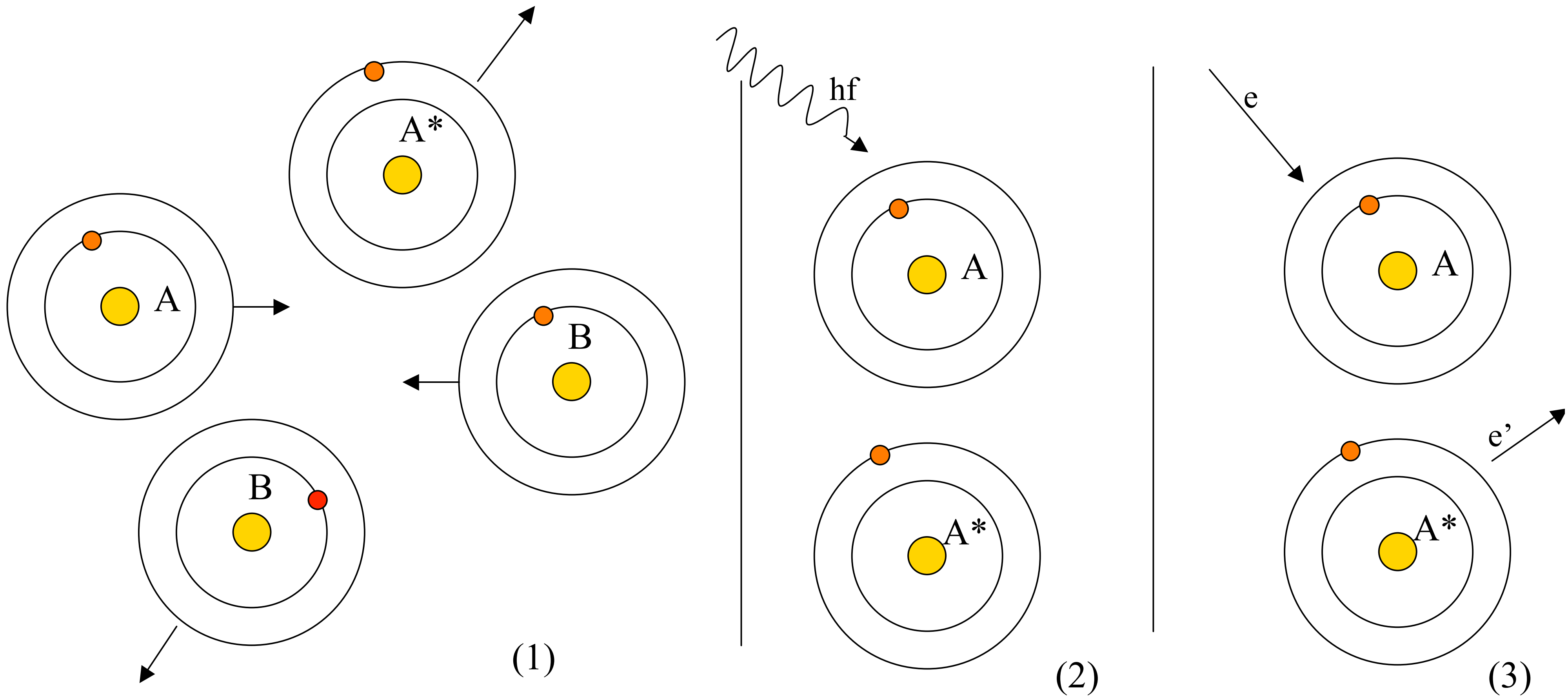
$En = -13.6 / n^2$ eV — $En = -13.6\, (Z^2/ n^2)$ eV *hydrogen-like atom*

ATOMIC TRANSITIONS

Electrons in the atom are only allowed to reside in the energy *eigenstates*. $E_n = -13.6 \text{ eV } Z^2/n^2$

The atom can be excited by

	excitation	de-excitation
(1) Collision with another atom. (A flame, heat)	$A + B \rightarrow A^* + B$	$A^* \rightarrow A + \gamma$
(2) Absorption of a photon (like the photoelectric effect).	$A + \gamma \rightarrow A^*$	$A^* \rightarrow A + \gamma$
(3) Absorption of energy from a electron (arc lamp, etc.)	$A + e \rightarrow A^* + e'$	$A^* \rightarrow A + \gamma$



De-EXCITATON

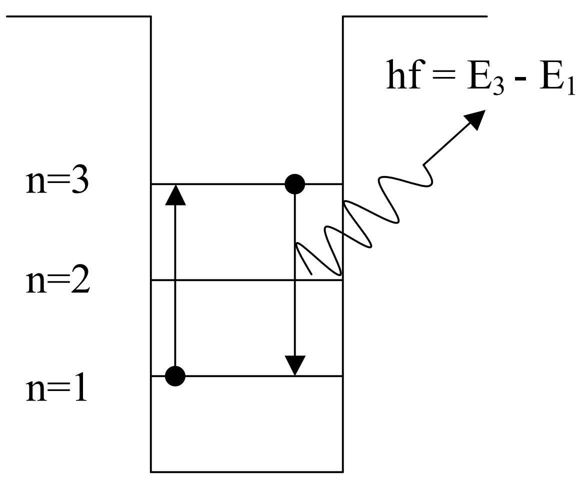
In each process of de-excitation $A^* \rightarrow A + \gamma$ from conservation of energy.

$E(A^*) = E(A) + E\gamma$

$E\gamma = E(A^*) - E(A)$

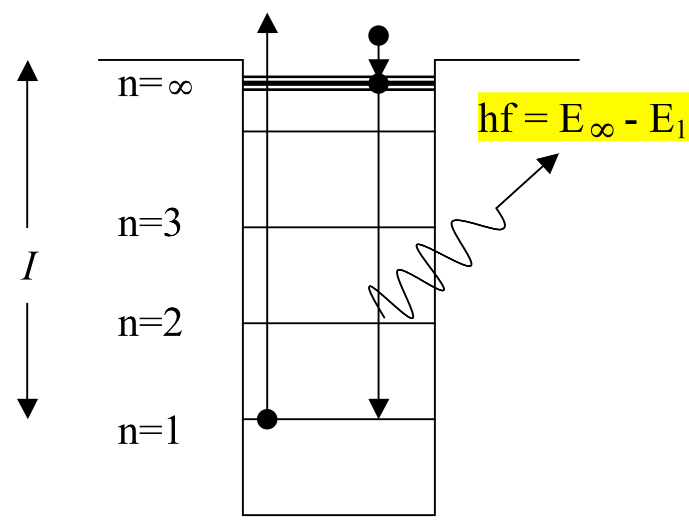
$E\gamma = Ef - Ei$

The de-excitsation created a photon of energy $hf = Ef - Ei$



IONIZATION

In the i*onization* process the electron is ejected from the atom forming an A+ ion. An energy *I (ionization energy)* must be supplied.
An electron soon falls back in to the ion at the upper level and then decays to the ground state.
A photon of energy $hf = E_{\infty} - E_1$ is emitted.

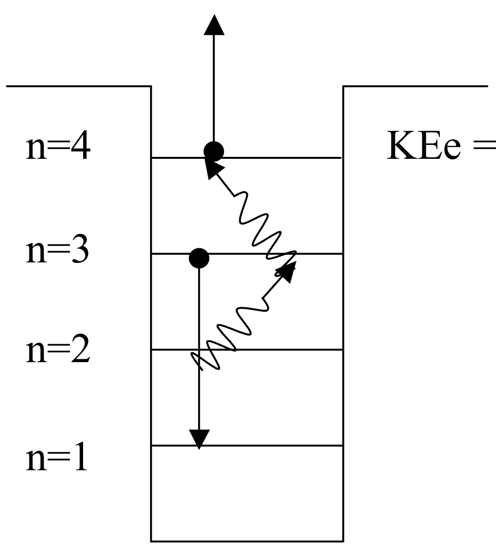


AUGER PROCESS (Internal Conversion)

An emitted photon can eject an electron in a higher energy state. This is called an *Auger electron*.

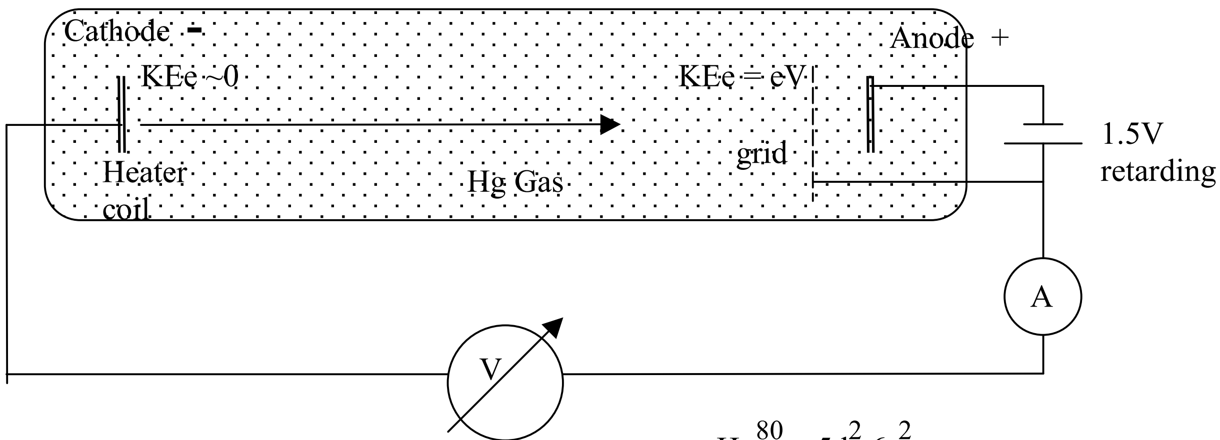
The kinetic energy of the electron is just the Photon's energy hf less the ionization energy from the upper state. KEe = hf – *I*

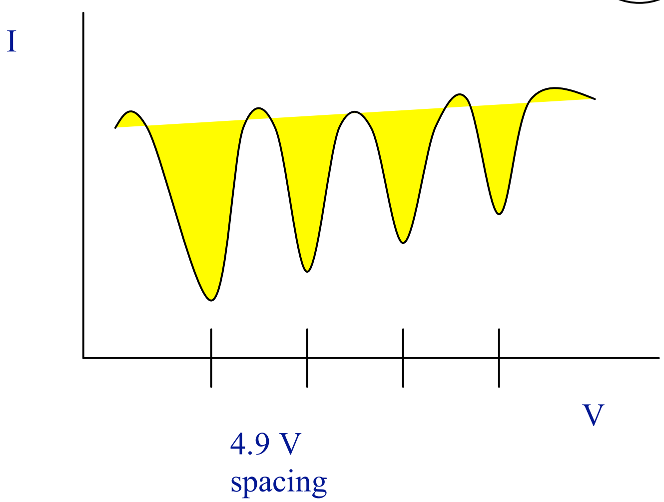
In the case below
hf = E3-E1 and I = E∞ - E4

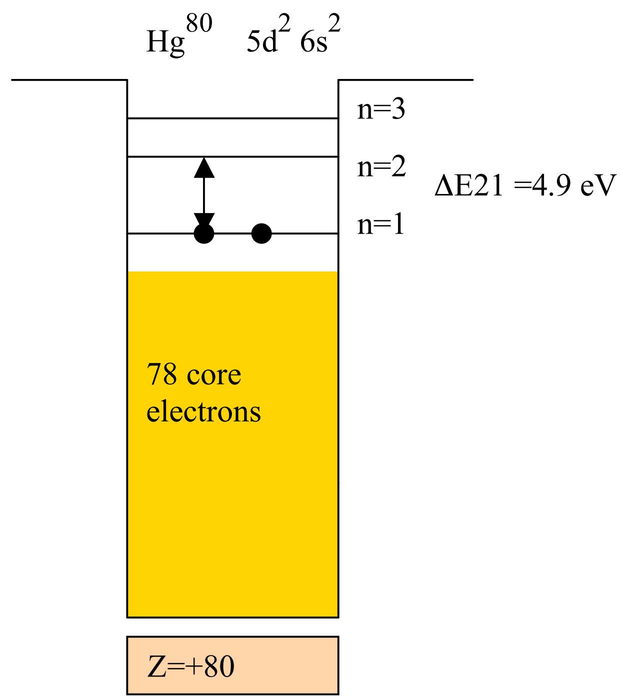


KEe = $(E_3 - E_1) - I$

FRANCK-HERTZ EXPERIMENT







<u>1914</u>
Franck and Hertz performed the fi experiment which directly revealed quantum energy levels of the atom.

A beam of electrons are accelerate through a potential V.

As electrons looses energy in collis with Hg atoms in the tube the anod current I drops. Thus a series of di as the acceleration and energy loss repeated over-and-over.

The dips are spaced at 4.9V or E =4.9eV intervals.

This corresponds to the ΔE21 transition in Hg.