CHAPTER 3- QUANTUM THEORY OF LIGHT

The quantum theory of light was realized in the later 19\textsuperscript{th} and early 20\textsuperscript{th} century. Soon after Boltzman set the laws of dealing with large ensembles of atoms with so called “Statistical Mechanics” some inconsistencies were seen involving radiation from solids.

Wedgewood notes in 1789 that all objects in his ovens, regardless of chemical nature, size, shape, became red at the same temperature.

The radiation from the walls of the oven is absorbed by the object and re-emitted with the same color as the temperature of the oven and insides becomes the same (thermal equilibrium).

By the mid 1800’s it is known that glowing objects emit \textit{Continuous} light spectra.

Astronomers used Spectrographs to see both \textit{Emission} and \textit{Absorption} Spectra. The details of the spectra were not understood.
• **Continuous Spectra** caused by an incandescent light source or the sun.
• **Emission Spectra** caused by a fluorescent light bulb, spectal tube or heated gases.
• **Absorption Spectra** caused by absorption of sunlight in the solar atmosphere or earth’s atmosphere.
**STEPHAN’s LAW**

- **Josef Stephan** circa 1879 establishes from experiment, that the total power radiated per unit area from a near black body follows a universal law.

\[ E_{\text{radiated}} = \alpha \sigma T^4 \text{ (W/m}^2\text{)} \]

- \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4 \) is the Stephan-Boltzmann constant.
- \( \alpha \) = emissivity \((\alpha = 1 \text{ for a black body radiator.})\)

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How much power are you radiating into the room? Let your emissivity \( \alpha = 1 \)

\[ T_{\text{BODY}} = 37\text{C} + 273\text{K} = 310\text{K} \quad A = \text{surface area of your body} \sim 4/5 \text{ m}^2 \]

\[ e_{\text{TOTAL}} = (5.7 \times 10^{-8} \text{ W/m}^2\text{-K}^4 \]  
\[ e_{\text{TOTAL}} = 260\text{W/m}^2 \]

\[ P = e_{\text{TOTAL}} (4/5) = 200\text{W} \]
**WEIN’s DISPLACEMENT LAW** (see example 2.2)

- Wein measured the radiant spectra from hot objects and proposed a universal law called the *Wein Displacement Law*. It describes the position of the maximum wavelength $\lambda_{MAX}$ versus temperature, $T$.

$$\lambda_{MAX} = \frac{2.898 \times 10^{-3} \text{ m-K}}{T}$$
**BLACK BODY RADIATION**

- Any surface absorbs a fraction $\alpha$ and re-emits a fraction $\gamma$ of the radiant energy it receives. These quantities depend on the frequency $\nu$ of the radiation and on the surface temperature, and by energy conservation $\alpha(\nu,T) + \gamma(\nu,T) = 1$.

- Kirchoff showed in 1860 that when radiation is absorbed ($A$) on a surface the emission ($E$) is balanced at each frequency. $E(f,T) \sim A(f,T)$ or $E(f,T) = \alpha \ A(f,T)$

- Wein realized that a cavity with a small hole will absorb all radiation incident on the hole. Thus the hole (disk) becomes a black body and the blackbody emission spectrum can be measured for a cavity at temperature $T$.

- The blackbody spectrum could not be explained by conventional theory!
**LIGHT QUANTA - MAX PLANCK**

- Max Planck drove a universal law which describes the black-body radiation spectra!
- He was confounded because he had to assume that light energy came in *packets* with energy-

\[ E = hf \]

- *Particle Picture - Planck*

- *Wave Picture – Maxwell*

- Scientist of the day were very confused. Even Planck did not fully believe his result.

How many quanta does a 50W light bulb emit per second? Let \( \lambda = 300\text{nm} \) or \( f = \frac{c}{\lambda} = \frac{3\times 10^8 \text{ m/s}}{300 \times 10^{-9} \text{ m}} = 1 \times 10^{15} \text{ Hz} \)

50 W = 50 J/s = Nh f / s  \[ \Rightarrow \quad N/s = \frac{50}{hf} = \frac{50}{[(6.63 \times 10^{-34} \text{ J-s}) (1 \times 10^{15} \text{ s})]} = 7.5e19 \text{ quanta/s} \]
INTERACTIONS in MATTER

• How Do Electrons, Photons, Protons, Neutrons Interact in Materials?
• Tissue Damage, Shielding of Radiation, Dosimetry
• Probability to interact $= \frac{\rho}{A} N_A \sigma L$

Diagram:
- Photoelectric Effect
- Compton Scattering
- Bragg X-ray Scattering
- X-rays
- $L$
LIGHT QUANTA

• Planck, Einstein, others established that the energy of a light quanta is given by

\[ E = hf = \frac{hc}{\lambda} \]

\[ E(eV) = \frac{1240}{\lambda(nm)} \]

• Is this consistent with Einsteins idea \( E = mc^2 + T \)

• Yes, because \( m = 0 \) for light quanta.

• What is the momentum of a light quanta?

\[ E^2 = p^2 c^2 + m^2 c^4 \Rightarrow E = pc \]

\[ p = \frac{E}{c} \]

\[ E(eV) = \frac{1240}{\lambda(nm)} \]

\[ \lambda(nm) = \frac{1240}{E(eV)} \]

What is the energy of blue light of wavelength \( \lambda = 400nm \)?

\[ \Rightarrow 1240/400 \sim 3.1 \text{ eV} \]
PHOTOELECTRIC EFFECT – EINSTEIN

- It was long known that visible light could liberate charge from a metal’s surface. The reason was not clear. This was observed by Hertz. Later (1899) J.J. Thompson showed that these were negative charges (electrons).

1. Electrons were only emitted once light of a high enough frequency was used no matter how intense the light?
2. Once this critical frequency was reached the photoelectric current rose with intensity.
3. Different metal seem to have different frequency threshold values.

- Einstein proposes the solution involves the interaction of light with the charges in the metal, not by a wave phenomena but as a particles quanta-like billiard balls!

\[
KEe = hf - Wi \quad i=0,1,2,3
\]

\( KEe \) = kinetic energy of the ejected electron  
\( hf \) = photon energy  
\( Wi \) = Energy needed to escape the metal  
\( W = Wmin \) = work function

Since the photon can interact at different depths \( W1 < W2 < W3 \) the maximum kinetic \( KEmax \) energy occurs when the \( W = Wmin \)

\[
KEmax = hf - W
\]
**PHOTOELECTRIC EFFECT APPARATUS**

1. Photons of energy $E = hf$ hit a metallic surface.
2. Electrons of many different kinetic energies are ejected and travel to the collection anode.
3. A photo-current $I$ flows in the circuit.
4. A battery $V_s = V_{stop}$ can be used to stop the flow of current $I = 0$. At $I = 0$ \[ eV_s = KE_{\text{max}} = hf - W \]

\[ eV_s = hf - W \]
$V_s = -1.99 + 4.96 \times 10^{-15} f$

$W = 1.99 \text{ V}$ Work Function

Slope = $4.96 \times 10^{-15} = h/e$

$h = (1.6 \times 10^{-19}) (4.96 \times 10^{-15})$

$h = 7.9 \times 10^{-34} \text{ J-s}$
**X-RAYS**

- X-rays are light quanta on energy in the KeV range. X-rays were first produced by Roentgen (1895) by accident. He discovered that mysterious and very penetrating rays could be generated by directing a beam of electrons into a metal.

![Diagram of X-ray Spectrum]

- Continuous Spectrum - No X-rays are observed below a minimum wavelength.

\[ E_{\text{max.}} = \frac{hc}{\lambda_{\text{MIN}}} = E_e \]

- Discrete Spectra correspond to excited atomic transitions. *(explained later)*
**X-RAY PRODUCTION**

*Continuous X-ray* spectra are produced by de-accelerating electrons. As the electron looses energy in the metal through collisions with atoms X-rays are emitted.  \[ E_e = E_{x1} + E_{x2} + E_{x3} + E_{x4} + \ldots \]

The electron may loose all of its energy in one collision producing the maximum energy X-ray \( E_x \) equal to the electron beam energy \( E_e \).

\[ E_{x_{\text{MAX}}} = E_e \]

*Discrete X-ray Lines* are produced when the electron excites one or more atomic transitions.
**Electron Beam Computer Aided Tomography - EBCT**

An electron beam is quickly fired around a metallic ring creating X-rays. Detectors on the opposite side image the X-rays passing through the patients body.

In normal CT scanner the X-rays are produced in an X-ray tube. The tube is moved quickly around the body. Detectors on the opposite side form a computer image.

The EBCT is much faster and can image movement in the body in real time, heart, lungs, blood flow, etc.

The EBCT is still in a somewhat experimental stage.
COMPTON SCATTERING

• Another experiment which illustrated the photon nature of light was performed in 1923 by Compton.
• A beam of light of wavelength $\lambda_0$ was directed on to a material. Compton discovered that light of a different wavelength $\lambda > \lambda_0$ was emitted at different angles $\Theta$.
• Like in billiard–ball collisions he considered conservation of momentum. For the photon’s momentum he used $p = h/\lambda$ or $E=hc/\lambda$ from Planck and Einstein’s work.

\[ \lambda = \lambda_0 + \left(\frac{h}{m_e c}\right) [1-\cos(\theta)] \text{ nm} \]
\[ \lambda = \lambda_0 + 0.00243 [1-\cos(\theta)] \text{ nm} \]

\[ E_{\lambda_0} = \frac{hc}{\lambda_0} \]
\[ E_{\lambda} = \frac{hc}{\lambda} \]
\[ E_e = m_e c^2 \]
\[ E_e' = m_e c^2 + T_e' \]
\[ E_{\lambda_0} + E_e = E_{\lambda} + E_e' \]
\[ T_e' = E_{\lambda_0} - E_{\lambda} \]

scattered photon’s wavelength
initial photons energy
scattered photons energy
initial electron’s energy
scattered electron’s energy
conservation of energy
**Bragg Scattering Law**

- A crystal is an orderly arrangement of atoms.
- It can be though of as different sets of parallel planes.
- Light which scatters from the planes should show interference effects.

A 50KeV X-ray has wavelength $\lambda = (1240/50000)$ nm = 0.025 nm = 0.25 making an X-ray beam perfect for exploring atomic distance scales.