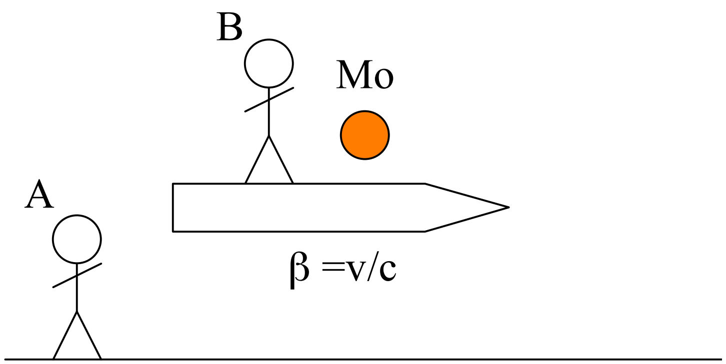
Chapter-2 Relativistic Kinematics- Part II



ENERGY

- The body Mo is at rest in Observer-B's frame.
- Mo is the bodies *rest mass.*

- Newton (Observer-A) would say the objects energy is
  $E = T = ½ Mo\ v^2$ T = *kinetic energy*

- But Einstein is sure that total energy of a body must include its *rest mass*!

  (1) $E = Mo\ c^2 + T$

- By making this assertion he is saying that $Mo\ c^2$ can be converted to energy! He does not tell us how? $Mo\ c^2$ = *rest energy*

- Using the Lorentz transformations he also concludes that Observer-A believes that the *rest energy* has increased by a factor of γ. Just like *time dilation* $\Delta t = \gamma\ \Delta to$!

  (2) $E = \gamma\ Mo\ c^2$ sometime written $E = M\ c^2$

- But remember $M = \gamma\ Mo$ the *relativistic mass.*

MOMENTUM

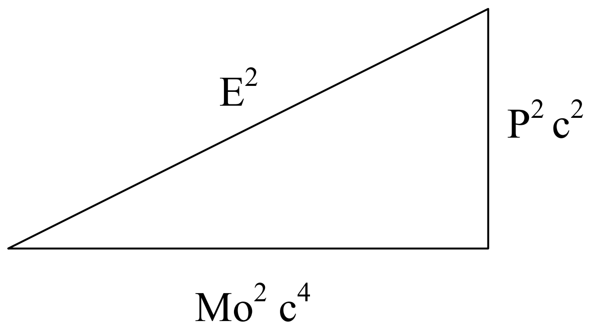
- Observer-A believes that momentum is still defined as $P = M\, v$!
- But $M = \gamma\, Mo$ must be the new relativistic mass which Einstein has defined.

(3) $P = (\gamma\, Mo)\, v$

MOMENTUM-ENERGY TRIANGLE

- One can show that

(4) $E^2 = P^2 c^2 + Mo^2 c^4$



SUMMARY

(1)	$E = Mo\, c^2 + T$
(2)	$E = \gamma\, Mo\, c^2$
(3)	$P = (\gamma\, Mo)\, v$
(4)	$E^2 = P^2 c^2 + Mo^2 c^4$

ELECTRON VOLT UNITS

In order to speed up calculations and reduce confusion scientist generally use *electon-volt* as the standard unit of energy in atomic and nuclear physics.

$1\ eV = (1.6x10^{-19}\ C)\ (1V) = 1.6x10^{-19}\ J$

$1\ eV = 1.6x10^{-19}\ J$

$E \rightarrow eV$

$P \rightarrow eV/c$

$M \rightarrow eV/c^2$

$Me = 0.511\ MeV/\ c^2$

$Mp = 938.3\ MeV/\ c^2$

$Mn = 939.6\ MeV/\ c^2$

EXAMPLE 2.3

An electron a speed v = 0.85c . What is its total energy and kinetic energy in electron-volts.

$\gamma = 1/sqrt(1\text{-}0.85\text{^}2) = 1.9$

$E = \gamma\, Mo\, c^2 = (1.9)(\ 0.511\ MeV/\ c^2\ )\ c^2$

$= \underline{0.97\ MeV}$

$T = E - Mo\ c^2 = 0.97 - 0.511 = \underline{0.46\ MeV}$

$$\boxed{\begin{aligned} E &= Mo\ c^2 + T \\ E &= \gamma\ Mo\ c^2 \\ P &= \gamma\ Mo\ v \\ E^2 &= P^2\, c^2 + Mo^2\, c^4 \end{aligned}}$$

EXAMPLE 2.4

$E = Mo\ c^2 + T$
$E = \gamma\ Mo\ c^2$
$P = \gamma\ Mo\ v$
$E^2 = P^2 c^2 + Mo^2 c^4$

The total energy of a proton is three time its rest energy.

(a) Find the proton's rest energy. $Mpc^2 = \underline{938.3\ \text{MeV}}$

(b) What is the proton's speed?

$P/E = v/c^2$ or $v/c = P/E\ c$
$E = 3 \times 938.3\ \text{MeV} = 2815\ \text{MeV}$
$P = \text{sqrt}(E^2 - Mo^2c^4) = \text{sqrt}(2815^2 - 938^2) = 2653\ \text{MeV/c}$
$v/c = (2653\ \text{MeV/c}) / (1876\text{MeV})c = \underline{0.94}$
$\underline{v = 0.94c}$

(c) Find the kinetic energy of the proton.
$T = E - Mo\ c^2 = 2815 - 938 = \underline{1876\ \text{MeV}}$

(d) What is the proton's momentum?
$P = \underline{2653\ \text{MeV/c}}$

MASS DEFICIT ***ΔM***

- Consider the reaction A --> B + C where A is at rest. $T_A=0$

- From the *Conservation of Energy*

$$E(A) = E(B) + E(C)$$

$$M_A c^2 + T_A = M_B c^2 + T_B + M_C c^2 + T_C$$

$$Q = T_A + T_B + T_C = M_A c^2 - M_B c^2 - M_C c^2$$

$Q = (M_A - M_B - M_C) c^2$ *Energy Deficit or Disintergration Energy*
Q-value=excess kinetic energy

$\Delta M = M_A - M_B - M_C$ *Mass Deficit*

- If $Q > 0$ then the reaction is *exothermic.*

Particles B and C share the excess energy and go flying apart.

- If $Q < 0$ the reaction is *endothermic*.

It will not proceed without adding energy of at least $>Q$.

ATOMIC MASS UNIT $^{A}X_{Z}$

The mass of an atom is often given in atomic mass units (u).

A = atomic number = N + Z = # neutrons +# protons

Z = # protons

$1\ u = M(^{12}C_6)\ /12. = 931.5\ MeV/c^2$

EXAMPLE 2.*7** ***Fission Reaction

$^{236}U_{92} \dashrightarrow\ ^{143}Cs_{55} +\ ^{90}Rb_{37} + 3\ ^{1}n_0$

$\Delta M = M(U) - M(Cs) - M(Rb) - M(3\ n) = 0.177537\ u$ *mass deficit*

$Q = \Delta M\ c^2 = 0.177537\ u \times 931.5\ MeV = 165.4\ MeV$ *released per U fission!*

1 kg of $^{236}U = 2.55 \times 10^{24}$ atoms

1 kg of $^{236}U \rightarrow 2.55 \times 10^{24}$ atoms x 165.4 MeV = 7,500 Mwh

100W light bulb for 8500 yrs !

BINDING ENERGY of STABLE NUCLEI $^{N+Z}X_Z$

- The mass of a stable nucleus is always less than the mass of its constituents.

This lesser amount is called the binding energy *BE*.

$$Mc^2 = \sum_{i=1}^{n} m_i c^2 - |BE|$$

- To separate the nucleus in to its constituents we must supply or overcome this binding energy.

$Q = M(^{A}X_Z) - Z\,Mp - N\,Mn - Z\,Me < 0 = -|BE|$

- The binding energy of a hydrogen atom is 13.6 eV.