PHYS 721 – HOMEWORK # 4 – DUE THURSDAY, SEPTEMBER 28, 2017

Problem 1. Carroll problem 2.9

Problem 2. Consider an element $f(x^i)$ of $\Lambda^0 V$, with $V = I\!R^3$ (0-form in $I\!R^3$).

(a) Show that df is a 1-form with components equal to $\nabla_i f$ (gradient of f);
(b) Show that $d^2 f = 0$;
(c) Show that $d^2 f = 0$ is equivalent to $\nabla \times (\nabla f) = 0$ (curl(grad f) = 0).

Problem 3. Consider the 2−form in $V = I\!R^3$:

$$\varphi = \varphi_1 \, dx^2 \wedge dx^3 + \varphi_2 \, dx^3 \wedge dx^1 + \varphi_3 \, dx^1 \wedge dx^2 \, .$$

Show that $d\varphi$ is a 3−form with component equal to the divergence of vector field φ_i.

Problem 4. Compute explicitly df and $d^2 f$ for $f = \arctan(x^1/x^2) \in \Lambda^0 I\!R^2$. Show that $d^2 f = 0$ but the 2×2 matrix $T_{ij} = \partial^2 f / \partial x^i \partial x^j$ is non-zero.

Problem 5. Show that Stoke's theorem:

$$\int_k d\varphi = \int_{\partial K} \varphi$$

coincides with usual Stoke's theorem of differential calculus (see, e.g., Jackson) if φ is a 1-form in $I\!R^3$.