PHYS 749 – FINAL (TAKE-HOME) EXAM

STRICT DEADLINE: TUESDAY DECEMBER 5, 2017, NOON CST

INSTRUCTIONS

Choose two out of the three problems to solve. Return your exam by the above deadline. **Late returns will not be accepted and you will fail the course.** Each problem fully solved is worth 40 points (40% of your final grade). Partial credit will be given for unfinished problems that contain correct results.

You need to show enough steps in your derivations to make the course instructor believe you did actually work out the solutions yourselves. **Solutions with derivations but no explanations and/or text comments will *not* be graded and *no* partial credit will be given for those.**

Use Carroll's conventions for the metric signature and the curvature tensor.

You are expected to abide by the University of Mississippi Student Academic Conduct and Discipline Policy (of which you may request to see a copy). In particular, you must work independently and keep your work confidential. Failure to do so, or any other form of academic misconduct related to this examination, may result in disciplinary actions which may include expulsion from the university.

I acknowledge that I have read and understood the above instructions.

______________________________ ____________________

signature date

Problem 1: Compute the gravitational waveform generated by a coconut-carrying African swallow flapping its wings. How does it compare to the gravitational waveform of an European swallow?

Problem 2: The metric of a spherically-symmetric space-time in four dimensions can be written as:

$$ds^2 = e^{2A(t,\tilde{r})}[-dt^2 + d\tilde{r}^2] + e^{2B(t,\tilde{r})}[d\theta^2 + \sin^2\theta\, d\varphi^2]\,.$$

1. Solve the coupled Einstein-Maxwell equations for the above metric. Show that the general solution is static, i.e., the Birkhoff theorem generalizes to a spherically-symmetric space-time when an EM field is present. Write the solution in the standard Reissner-Nordström form (see Carroll's book).

2. The metric depends on two constants which can be identified with the mass M and the charge Q. Assume $M^2 > Q^2$. Show that the metric possesses three distinct regions and can be written in the form

$$ds^2 = \frac{|\Delta|}{r^2} du dv + r^2[d\theta^2 + \sin^2\theta\, d\varphi^2]\,.$$

   Write Δ, u and v in terms of the mass, charge, and the coordinates r and t. *Be careful that these relations are not the same in the three distinct regions!* Draw the Penrose diagram blocks for the three regions.

3. Using a conformal transformation $\tan U = \pm e^{\pm\alpha u}$, $\tan V = e^{\alpha v}$ (*be careful about choosing the right signs for the different regions!*), show that the metric can be put in the maximally-extended form:

$$ds^2 = \frac{4}{\alpha^2}|r-r_+||r-r_-|\,\mathrm{cosec}\,(2U)\,\mathrm{cosec}\,(2V)\,dU dV + r^2[d\theta^2 + \sin^2\theta\, d\varphi^2]\,.$$

   where $r_\pm$ are the two roots of Δ and the coordinates U and V are defined implicitly by

$$\tan U \tan V \;=\; \pm e^{2\alpha r}\, |r - r_+|\, |r - r_-|^{-\beta}\,,$$

   where $\alpha = (r_+ - r_-)/2r_+^2$ and $\beta = r_-^2/r_+^2$.

Problem 3: Consider a two-component model of the Friedmann-Robertson-Walker universe with both radiation and (pressureless) matter. Use the best-fit values from the Planck mission to estimate the numerical values of the derived quantities.

1. Compute the redshift z, temperature, and time of the transition from a radiation- to matter-dominated evolution (radiation-to-matter transition era). You may assume $k = 0$.

2. Now add a cosmological constant to account for dark energy. Compute the redshift z of the matter-to-dark energy transition and the redshift z when the universe transitioned from decelerated expansion to accelerated expansion.

3. Neglecting the contribution of radiation, compute the present age of the universe and the age of the universe when the latter started accelerating.

4. Estimate the (luminosity) distance you need to observe supernovae type Ia to measure the Universe acceleration.