PHYS 721 – HOMEWORK # 5 – DUE THURSDAY, OCTOBER 25, 2018

Problem 1. Show that the Green function for a flat two dimensional circular wedge of angle β and radius a is

$$G(\rho,\rho',\varphi,\varphi') = \sum_{n=1}^{\infty} \frac{4}{n} \rho_-^{n\pi/\beta} \left[\rho_+^{-n\pi/\beta} - \left(\frac{\rho_+}{a^2} \right)^{n\pi/\beta} \right] \sin(n\pi\varphi/\beta)\, \sin(n\pi\varphi'/\beta)\,,$$

where ρ and φ are the two-dimensional polar coordinates, and $\rho_- = \mathrm{Min}(\rho,\rho')$, $\rho_+ = \mathrm{Max}(\rho,\rho')$.

Problem 2. Jackson problem 3.7.

Problem 3. A hollow thin cylinder of length L and radius a has the end faces kept at zero potential. The potential on the side surface is $V(\varphi, z)$. Using cylindrical coordinates with the end faces at $z = 0$ and $z = L$, find a series solution for the potential inside. How does the series simplify for a constant potential V?

Problem 4. A flat conducting ring of infinitesimal thickness, internal radius a, and external radius b is uniformly charged with total charge Q.

(a) Write the three-dimensional charge distribution density in cylindrical coordinates;

(b) Find the potential outside the ring.

Problem 5. An infinite thin flat sheet of conducting material has a circular thin cut of radius a. The part of the conducting sheet inside the cut is kept at constant potential V, while the conducting sheet outside the cut is kept at zero potential.

a) Show that the potential at any point above the sheet is

$$\phi(\rho,\varphi,z) = \int_0^\infty dk e^{-kz} \left\{ \frac{1}{2} B_0 J_0(k\rho) + \sum_{n=1}^{\infty} J_n(k\rho)[A_n(k)\sin(n\varphi) + B_n \cos(n\varphi)] \right\},$$

where

$$\left. \begin{matrix} A_n(k) \\ B_n(k) \end{matrix} \right\} = \frac{kV}{\pi} \int_0^a d\rho\rho \int_0^{2\pi} d\varphi J_n(k\rho) \left\{ \begin{matrix} \sin(n\varphi) \\ \cos(n\varphi) \end{matrix} \right. .$$

b) Using the limit $\rho \to 0$ of the previous equation, find the potential above the center of the inner part of the conducting sheet.

c) Show that the potential above the cut is

$$\phi(\varphi, z) = Va \int_0^\infty dk e^{-kz} J_0(ka) J_1(ka)\,.$$