PHYS 721 – HOMEWORK # 4 – DUE THURSDAY, OCTOBER 4, 2018

Problem 1. A sphere of radius a has charge uniformly distributed over its surface with charge density $Q/(4\pi a^2)$, except for a spherical cap at the north pole defined by the cone $\theta = \alpha$, which is kept at zero potential. Show that the potential outside the sphere is:

$$\phi = \frac{Q}{8\pi\epsilon_0 a} \sum_{l=0}^{\infty} \frac{P_{l+1}(\cos\alpha) - P_{l-1}(\cos\alpha)}{2l+1} \left(\frac{a}{r}\right)^{l+1} P_l(\cos\theta)\,,$$

where $P_{-1}(\cos\alpha)$ is defined to be equal to -1. Discuss the limiting form of the potential as the spherical cap becomes very small or very large.

Problem 2. A thin flat conducting disc of radius a is maintained at constant potential V. If the surface charge density is proportional to $1/\sqrt{a^2 - d^2}$, where d is the distance from the center of the disc:

a) Show that the potential for $r > a$ is:

$$\phi = \frac{2Va}{\pi r} \sum_{l=0}^{\infty} \frac{(-1)^l}{2l+1} \left(\frac{a}{r}\right)^{2l} P_{2l}(\cos\theta)\,;$$

b) Find the potential for $r < a$;

c) Find the capacitance of the disc.

Problem 3. A flat conducting ring of infinitesimal thickness, internal radius a, and external radius b is uniformly charged with total charge Q.

a) Write the three-dimensional charge distribution density in cylindrical coordinates;

b) Find the potential at a distance $r > b$ from the center of the ring.