PHYS 721 – HOMEWORK # 2 – DUE THURSDAY, 9/13/2018

Problem 1. The Green's function in n dimensions is defined by the equation:

$$\nabla^2 G(\mathbf{x}, \mathbf{x}') = -2\pi \delta^n(\mathbf{x} - \mathbf{x}'),$$

where ∇^2 is the Laplacian in n dimensions.

a) Find the Green's function $G(\mathbf{x}, \mathbf{x}')$ for a generic $n \geq 2$.
b) Using the Green's function found in a), write the solution of the Poisson equation $\nabla^2 \phi = -\rho(\mathbf{x})/\varepsilon_0$ in $n \geq 2$ dimensions with no boundary conditions.

Problem 2. The insulating floor of a laboratory is covered with thin flat circular metal tiles of radius a, held at finite potential. Assume that the surface of the laboratory is much larger than any measuring device.

(a) Write down the appropriate Green function for the laboratory;
(b) If a tile is held at constant potential $\phi = V$, while all the other tiles are grounded, find an integral expression for the potential at a generic point in the laboratory; [use cylindrical coordinates ρ, φ, z]
(c) Derive an expression for the potential along the axis of the tile as a function of V, a and the height from the floor.
(d) Show that at large distances $\rho^2 + z^2 \gg a^2$ the potential is approximated by:

$$\phi = \frac{Va^2}{2} \frac{z}{(\rho^2 + z^2)^{3/2}} \left[1 - \frac{3a^2}{4(\rho^2 + z^2)} + \frac{5(3\rho^2 a^2 + a^4)}{8(\rho^2 + z^2)^2} + \dots \right].$$

Problem 3. A grounded conductor has the shape of an infinite plane except for a hemispherical bulge of radius a. A charge q is placed above the center of the bulge, at a distance d from the plane (or $d - a$ from the top of the bulge). (a) Find the potential in the region above the conductor. (b) Find the total induced charge on the conductor. (c) Find the Dirichlet Green's function.