Problem 1. Alice moves with uniform velocity $v$ relative to Bob. Alice sees a particle with velocity $v_A$ and acceleration $a_A$. Show that Bob measures the particle acceleration

$$a_{B,\parallel} = \frac{(1 - v^2/c^2)^{3/2}}{(1 + v \cdot v_A/c^2)^3} a_{A,\parallel},$$

$$a_{B,\perp} = \frac{(1 - v^2/c^2)^2}{(1 + v \cdot v_A/c^2)^3} \left[ a_{A,\perp} + v \times (a_A \times v_A)/c^2 \right],$$

where $a_{\parallel}$ and $a_{\perp}$ are the accelerations parallel and perpendicular to $v$, respectively.

Problem 2. Show that the following transformation forms a group with parameter $\alpha$:

$$T(\alpha) = \begin{cases} \tilde{x}^1 = \frac{x^1}{1 - \alpha x^1}, \\ \tilde{x}^2 = \frac{x^2}{1 - \alpha x^1}. \end{cases}$$

Problem 3. (a) Write the transformation laws for the following tensors under the Lorentz group: $A_\sigma^\nu \mu$, $A_\mu^\lambda \rho$, $A_{\mu
u}^\sigma \mu$. (b) If $a_{\mu\nu}$ is a skew-symmetric rank-two tensor, show that i) $b_{\mu\nu\sigma} = \partial_\sigma a_{\mu\nu} + \partial_\mu a_{\nu\sigma} + \partial_\nu a_{\sigma\mu}$ is a rank-three tensor, ii) $b_{\mu\nu\sigma}$ is skew-symmetric in all pairs of indices and iii) determine the number of independent components this tensor has.

Problem 4. Consider a symmetric rank-two contravariant tensor $T^{\mu\nu}$ in the coordinate system $x^\mu$ ($\mu, \nu = 0, 1, 2, 3$). (a) Show that $T$ is symmetric in all coordinate systems. (b) If $T$ satisfies the condition $\partial_\mu T^{\mu\nu} = 0$, show that

$$\frac{d^2}{dt^2} \int d^3x T^{00} x^i x^j = 2 \int d^3x T^{ij},$$

where $i, j = 1, 2, 3$ and $T^{\mu\nu}$ is assumed to be localized (boundary terms can be neglected).