PHYS 621 – HOMEWORK # 4 – DUE WEDNESDAY, 9/25/2009

Problem 1. The insulating floor of a laboratory is covered with thin flat circular metal tiles of radius a, held at finite potential. Assume that the surface of the laboratory is much larger than any measuring device.

(a) If a tile is held at constant potential $\phi = V$, while all the other tiles are grounded, find an integral expression for the potential at a generic point in the laboratory. **You must use the Green's function method.**

(b) Show that along the axis of the tile the potential is given by

$$\phi = V\left(1 - \frac{h}{\sqrt{a^2 + h^2}}\right),$$

where h is the height from the floor.

(c) Show that at large distances $\rho^2 + z^2 \gg a^2$ the potential is approximated by:

$$\phi = \frac{Va^2}{2} \frac{z}{(\rho^2 + z^2)^{3/2}} \left[1 - \frac{3a^2}{4(\rho^2 + z^2)} + \frac{5(3\rho^2 a^2 + a^4)}{8(\rho^2 + z^2)^2} + \dots\right].$$

Problem 2. Jackson problem 2.20 parts (a) and (b).

Problem 3. Show that the (three-dimensional) Green function for Dirichlet boundary conditions on a square two-dimensional region $0 \leq x \leq 1$, $0 \leq y \leq 1$ can be written

$$G(\mathbf{x}, \mathbf{x}') = 2 \sum_{n=1}^{\infty} g_n(y, y') \sin(n\pi x) \sin(n\pi x'),$$

where the functions g_n satisfy

$$\left(\frac{\partial^2}{\partial y'^2} - n^2\pi^2\right) g_n(y, y') = -4\pi\delta(y - y'),$$

and $g_n(y, 0) = 0$, $g_n(y, 1) = 0$.