PHYS 621 – HOMEWORK # 2 – DUE WEDNESDAY, 9/9/2009

Problem 1. The Green's function in two dimensions is defined by the equation:

$$\nabla^2 G(\mathbf{x}, \mathbf{x}') = -2\pi\delta^2(\mathbf{x} - \mathbf{x}')\,,$$

where ∇^2 is the Laplacian in two dimensions.

a) Find the Green's function $G(\mathbf{x}, \mathbf{x}')$.
b) Using the Green's function found in a), write the solution of the Poisson equation $\nabla^2\phi = -\rho(\mathbf{x})/\varepsilon_0$ in two dimensions with no boundary conditions.

Useful formula:

$$\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\,.$$

Problem 2. Show by derivation (not by substitution!) that the Green's function for the one-dimensional operator

$$L = -\left(\frac{d^2}{dx^2} + k^2\right), \qquad LG(x, x') = -\delta(x - x')\,,$$

with boundary conditions $G(0, x') = (dG/dx)(1, x') = 0$ is

$$G(x, x') = -\frac{\sin(kx)\cos[k(1 - x')]}{k\cos k}\theta(x' - x) - \frac{\cos[k(1 - x)]\sin(kx')}{k\cos k}\theta(x - x')\,.$$

Problem 3. Show that for a region $\mathcal{V}$ bounded by conductors (not necessarily grounded) the following relation holds:

$$\int_{\mathcal{V}} [\rho_1\phi_2 - \rho_2\phi_1]dV = \sum_n [Q_2V_1 - Q_1V_2]_n\,,$$

where Q and V represent the net charge and voltage on each conductor, the sum is over all conductors and the subscripts 1 and 2 refer to two different cases of charge distribution and resulting potential with the same geometry for the conductors. Use the above result to prove that the potential of a neutral conducting sphere of radius a due to a point charge at distance $d > a$ is $V = q/4\pi\epsilon_0 d$. [*Hint: Use the stated situation as case 1 and a suitable different "easy-to-solve" configuration as case 2.*]