PHYS 621 – HOMEWORK # 1 – DUE WEDNESDAY, 9/02/2009

Problem 1. Using the definition of the Dirac δ function, prove the following properties:

1) $\delta(ax) = \delta(x)/|a|$
2) $x\delta(x) = 0$
3) $x^n \delta^{(n)}(x) = (-1)^n\, n!\, \delta(x)$, where $\delta^{(n)}(x)$ is the n-th derivative of the Dirac δ distribution.
4) $\delta[y(x)] = \sum_i \delta(x - x_i)|dy/dx|^{-1}_{x=x_i}$, where x_i are the simple zeros of the function $y(x)$.

Problem 2. Show that the one-dimensional integral representation of the Dirac δ function

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dk\; e^{ikx}$$

can be written as the distribution $\lim_{n\to\infty} nG(nx)$, where (1) $G(x) = e^{-\pi x^2}$ and (2) $G(x) = \sin^2(x)/(\pi x^2)$.

Problem 3. Using Dirac delta functions in the appropriate coordinates, express the following charge distributions as three-dimensional charge densities $\rho(\mathbf{x})$.

a) In spherical coordinates, a charge Q uniformly distributed over a spherical shell of radius R.
b) In cylindrical coordinates, a charge λ per unit length uniformly distributed over a cylindrical surface of radius R.
c) In cylindrical coordinates, a charge Q uniformly distributed over a flat annulus of negligible thickness and radii R_1 and $R_2 > R_1$.
d) In Cartesian coordinates, a charge per unit length λ uniformly distributed on a square loop of wire with negligible radius and side L.