ANNOUNCEMENTS

- Homework #8, due today!

  Conceptual questions: Chapter 8, #8 and #14
  Problems: Chapter 8, #62, #68

- 5-minute quiz on Chapter 8 Friday, October 19

- Study Chapter 9, sections 9.1-9.3 before Monday

- Second in-class test: Friday, November 2

LINEAR MOMENTUM AND COLLISIONS

The concepts of impulse, momentum, and center of mass are crucial for a major-league baseball player to successfully get a hit. If he misjudges these quantities, he might break his bat instead. (credit: modification of work by “Cathy T”/Flickr)

FIGURE 9.2

The velocity and momentum vectors for the ball are in the same direction. The mass of the ball is about 0.5 kg, so the momentum vector is about half the length of the velocity vector because momentum is velocity time mass. (credit: modification of work by Ben Sutherland)

MOMENTUM

The momentum *p* of an object is the product of its mass and its velocity:

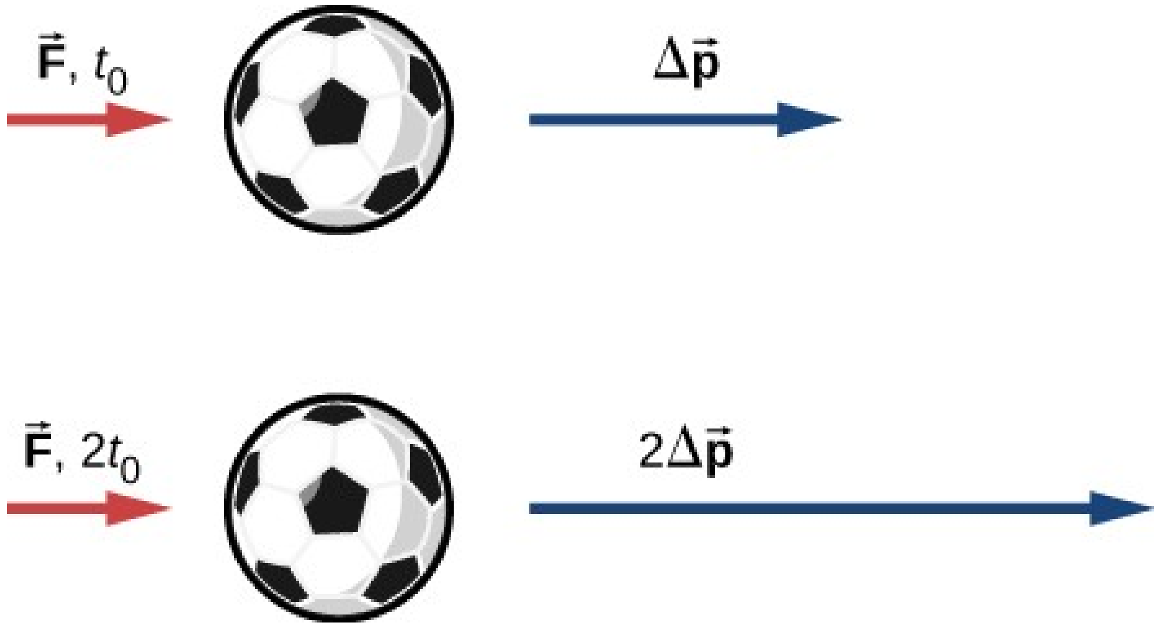
$$\vec{\mathbf{p}} = m\vec{\mathbf{v}}.$$

FIGURE 9.3

This supertanker transports a huge mass of oil; as a consequence, it takes a long time for a force to change its (comparatively small) velocity. (credit: modification of work by “the_tahoe_guy”/Flickr)

IMPULSE

We have defined momentum to be the product of mass and velocity. Therefore, if an object's velocity should change (due to the application of a force on the object), then necessarily, its momentum changes as well. This indicates a connection between momentum and force. The purpose of this section is to explore and describe that connection.



The change in momentum of an object is proportional to the length of time during which the force is applied. If a force is exerted on the lower ball for twice as long as on the upper ball, then the change in the momentum of the lower ball is twice that of the upper ball.

IMPULSE

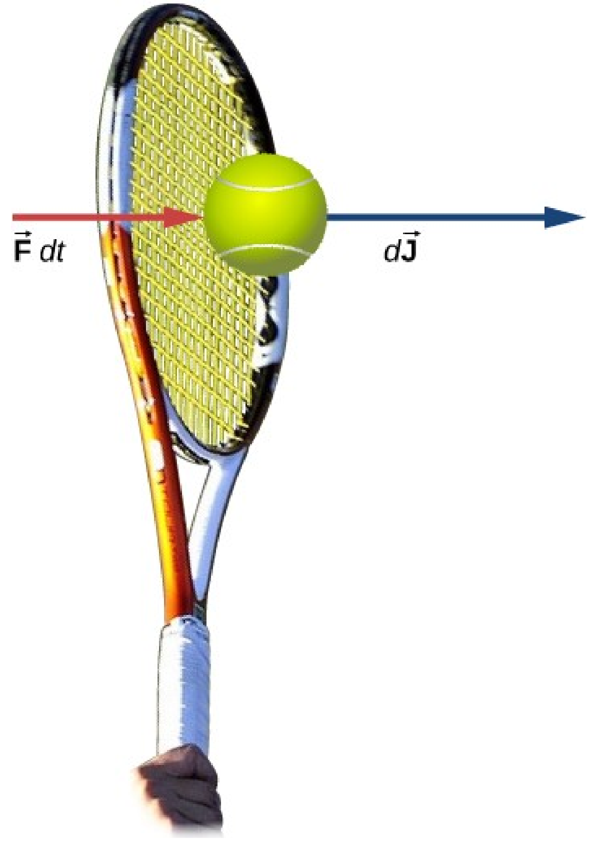
Let $\vec{\mathbf{F}}(t)$ be the force applied to an object over some differential time interval *dt* (Figure 9.6). The resulting impulse on the object is defined as

$$d\vec{\mathbf{J}} \equiv \vec{\mathbf{F}}(t)dt. \tag{9.2}$$

A force applied by a tennis racquet to a tennis ball over a time interval generates an impulse acting on the ball.

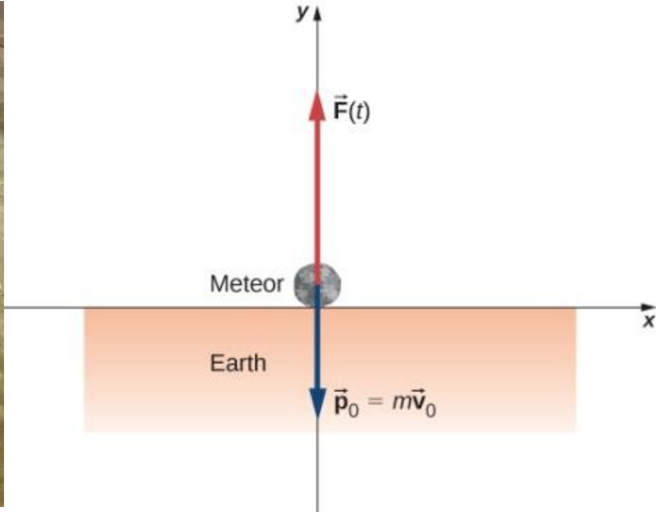
$$\vec{\mathbf{J}} = \int_{t_i}^{t_f} d\vec{\mathbf{J}} \quad \text{or} \quad \vec{\mathbf{J}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}(t)dt.$$

$$\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{ave}}\Delta t. \longrightarrow \boxed{\vec{\mathbf{J}} = m\Delta\vec{\mathbf{v}}.}$$



The Arizona Meteor Crater

Approximately 50,000 years ago, a large (radius of 25 m) iron-nickel meteorite collided with Earth at an estimated speed of 1.28×10^4 m/s in what is now the northern Arizona desert, in the United States. The impact produced a crater that is still visible today (Figure 9.7); it is approximately 1200 m (three-quarters of a mile) in diameter, 170 m deep, and has a rim that rises 45 m above the surrounding desert plain. Iron-nickel meteorites typically have a density of $\rho = 7970 \text{ kg/m}^3$. Use impulse considerations to estimate the average force and the maximum force that the meteor applied to Earth during the impact.



The average force during the impact is related to the impulse by

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{\vec{\mathbf{J}}}{\Delta t}.$$

From Equation 9.6, $\vec{\mathbf{J}} = m\Delta\vec{\mathbf{v}}$, so we have

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{m\Delta\vec{\mathbf{v}}}{\Delta t}.$$

The mass is equal to the product of the meteor's density and its volume:

$$m = \rho V.$$

If we assume (guess) that the meteor was roughly spherical, we have

$$V = \frac{4}{3}\pi R^3.$$

Thus we obtain

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{\rho V\Delta\vec{\mathbf{v}}}{\Delta t} = \frac{\rho\left(\frac{4}{3}\pi R^3\right)\left(\vec{\mathbf{v}}_{\text{f}} - \vec{\mathbf{v}}_{\text{i}}\right)}{\Delta t}.$$

The problem says the velocity at impact was -1.28×10^4 m/s$\hat{\mathbf{j}}$ (the final velocity is zero); also, we guess that the primary impact lasted about $t_{\text{max}} = 2$ s. Substituting these values gives

$$\begin{aligned}\vec{\mathbf{F}}_{\text{ave}} &= \frac{\left(7970\ \frac{\text{kg}}{\text{m}^3}\right)\left[\frac{4}{3}\pi(25\ \text{m})^3\right]\left[0\ \frac{\text{m}}{\text{s}} - \left(-1.28 \times 10^4\ \frac{\text{m}}{\text{s}}\hat{\mathbf{j}}\right)\right]}{2\ \text{s}}. \\ &= +\left(3.33 \times 10^{12}\ \text{N}\right)\hat{\mathbf{j}}\end{aligned}$$

This is the average force applied during the collision. Notice that this force vector points in the same direction as the change of velocity vector $\Delta\vec{\mathbf{v}}$.

The Benefits of Impulse

A car traveling at 27 m/s collides with a building. The collision with the building causes the car to come to a stop in approximately 1 second. The driver, who weighs 860 N, is protected by a combination of a variable-tension seatbelt and an airbag (Figure 9.9). (In effect, the driver collides with the seatbelt and airbag and *not* with the building.) The airbag and seatbelt slow his velocity, such that he comes to a stop in approximately 2.5 s.

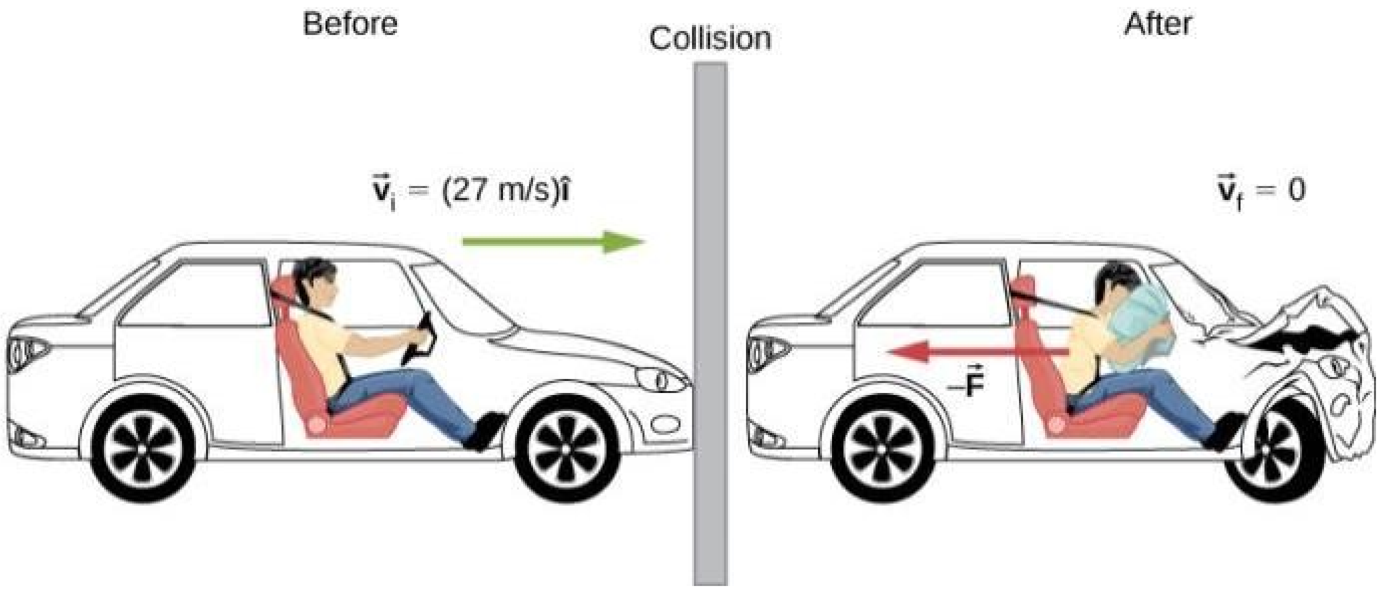
a. What average force does the driver experience during the collision?
b. Without the seatbelt and airbag, his collision time (with the steering wheel) would have been approximately 0.20 s. What force would he experience in this case?

$$\vec{\mathbf{J}} = \vec{\mathbf{F}}\Delta t$$

$$\vec{\mathbf{J}} = m\Delta\vec{\mathbf{v}}.$$

$$\vec{\mathbf{F}}\Delta t = m\Delta\vec{\mathbf{v}}.$$

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}.$$



The motion of a car and its driver at the instant before and the instant after colliding with the wall. The restrained driver experiences a large backward force from the seatbelt and airbag, which causes his velocity to decrease to zero. (The forward force from the seatback is much smaller than the backward force, so we neglect it in the solution.)

IMPULSE-MOMENTUM THEOREM

An impulse applied to a system changes the system's momentum, and that change of momentum is exactly equal to the impulse that was applied:

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}. \tag{9.7}$$

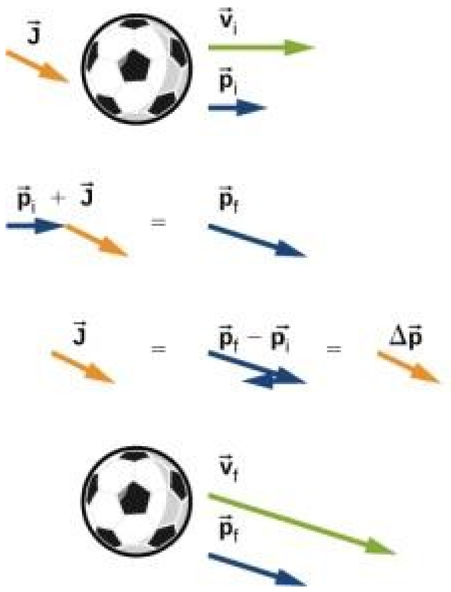


Illustration of impulse-momentum theorem.

(a) A ball with initial velocity $\vec{\mathbf{v}}_0$ and momentum $\vec{\mathbf{p}}_0$ receives an impulse $\vec{\mathbf{j}}$.

(b) This impulse is added vectorially to the initial momentum.

(c) Thus, the impulse equals the change in momentum, $\vec{\mathbf{j}} = \Delta\vec{\mathbf{p}}$.

(d) After the impulse, the ball moves off with its new momentum $\vec{\mathbf{p}}_f$.

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}.$$

The iPhone Drop

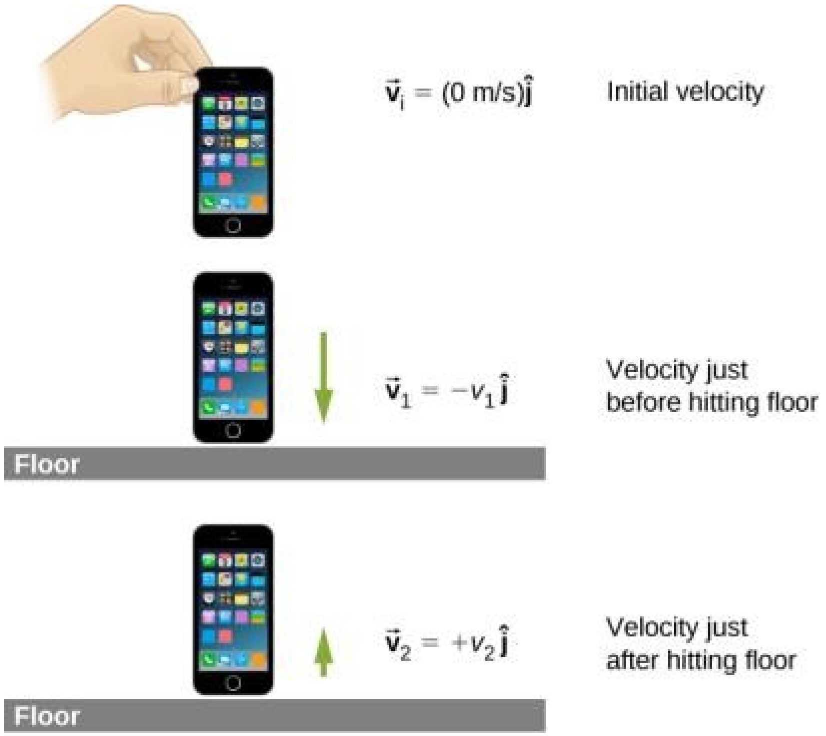
Apple released its iPhone 6 Plus in November 2014. According to many reports, it was originally supposed to have a screen made from sapphire, but that was changed at the last minute for a hardened glass screen. Reportedly, this was because the sapphire screen cracked when the phone was dropped. What force did the iPhone 6 Plus experience as a result of being dropped? assume the phone doesn't bounce at all when it hits the floor (or at least, the bounce height is negligible).

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{\vec{\mathbf{J}}}{\Delta t}.$$

$$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$$

$$\vec{\mathbf{F}}_{\text{ave}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}.$$

$$\Delta\vec{\mathbf{p}} = m\Delta\vec{\mathbf{v}}.$$



(a) The initial velocity of the phone is zero, just after the person drops it.

(b) Just before the phone hits the floor, its velocity is $\vec{\mathbf{v}}_1$, which is unknown at the moment, except for its direction, which is downward $(-\hat{\mathbf{j}})$.

(c) After bouncing off the floor, the phone has a velocity $\vec{\mathbf{v}}_2$, which is also unknown, except for its direction, which is upward $(+\hat{\mathbf{j}})$.

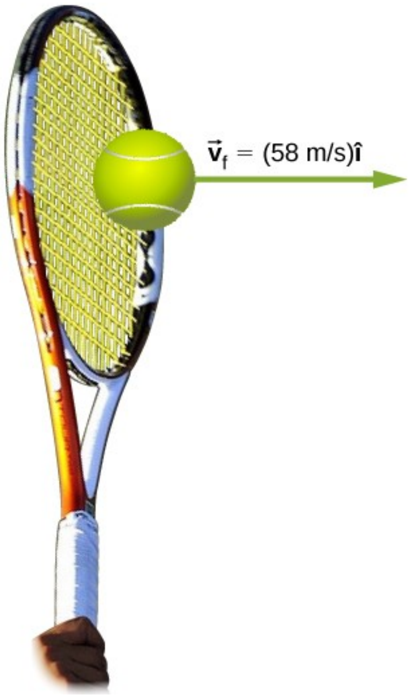
NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM

The net external force on a system is equal to the rate of change of the momentum of that system caused by the force:

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}.$$

Calculating Force: Venus Williams' Tennis Serve

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet? Assume that the ball's speed just after impact is 58 m/s, as shown in Figure 9.13, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms.



To determine the change in momentum, insert the values for the initial and final velocities into the equation above:

$$\begin{aligned} \Delta p &= m(v_f - v_i) \\ &= (0.057\text{ kg})(58\text{ m/s} - 0\text{ m/s}) \\ &= 3.3\,\frac{\text{kg}\cdot\text{m}}{\text{s}}. \end{aligned}$$

Now the magnitude of the net external force can be determined by using

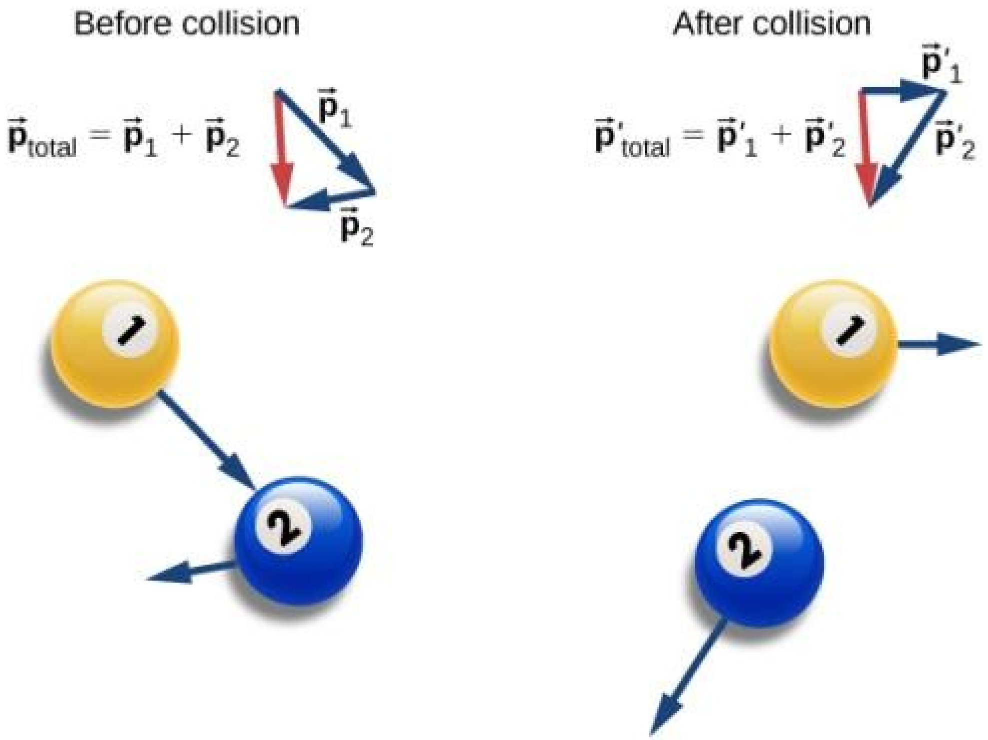
$$F = \frac{\Delta p}{\Delta t} = \frac{3.3\,\frac{\text{kg}\cdot\text{m}}{\text{s}}}{5.0 \times 10^{-3}\text{ s}} = 6.6 \times 10^{2}\text{ N}.$$

where we have retained only two significant figures in the final step.

LAW OF CONSERVATION OF MOMENTUM

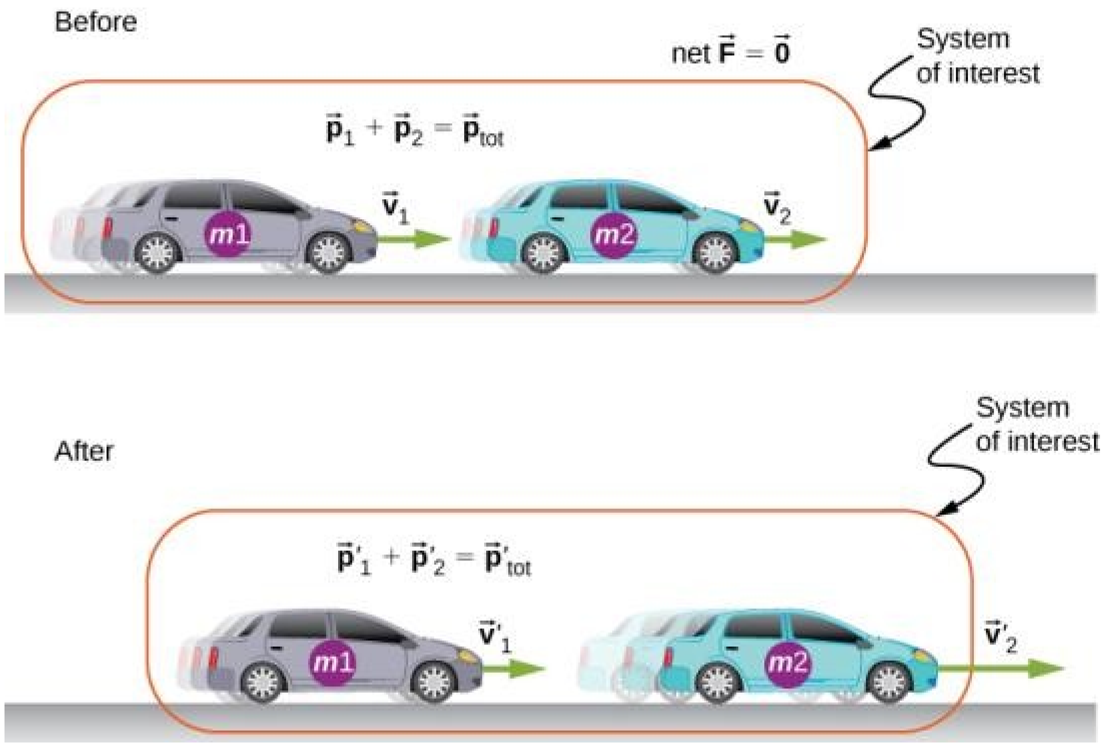
The total momentum of a closed system is conserved:

$$\sum_{j=1}^{N} \vec{\mathbf{p}}_{\mathrm{j}} = \text{constant}.$$



Before the collision, the two billiard balls travel with momenta $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_3$. The total momentum of the system is the sum of these, as shown by the red vector labeled $\vec{\mathbf{p}}_{\text{total}}$ on the left. After the collision, the two billiard balls travel with different momenta $\vec{\mathbf{p}}'_1$ and $\vec{\mathbf{p}}'_3$. The total momentum, however, has not changed, as shown by the red vector arrow $\vec{\mathbf{p}}'_{\text{total}}$ on the right.

FIGURE 9.15

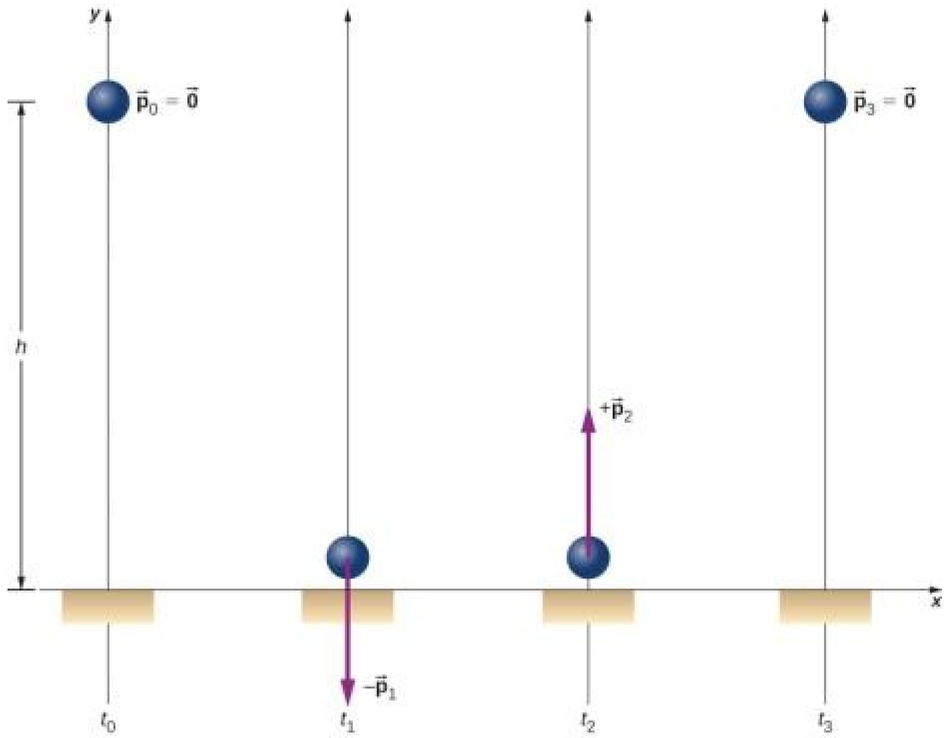


The two cars together form the system that is to be analyzed. It is important to remember that the contents (the mass) of the system do not change before, during, or after the objects in the system interact.

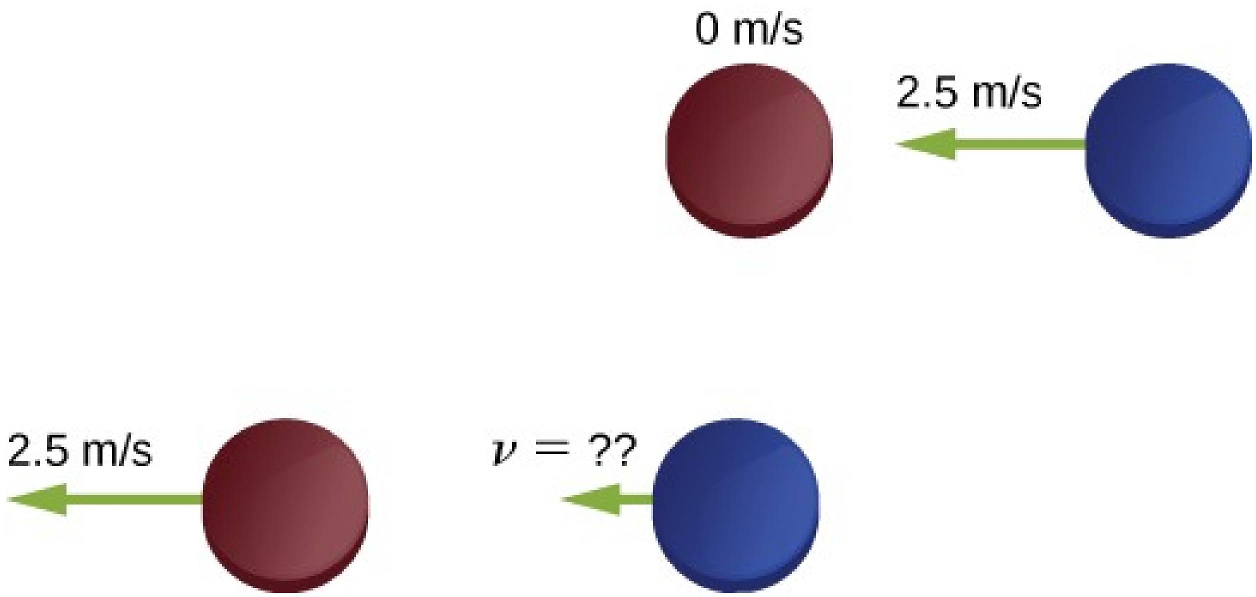
A Bouncing Superball

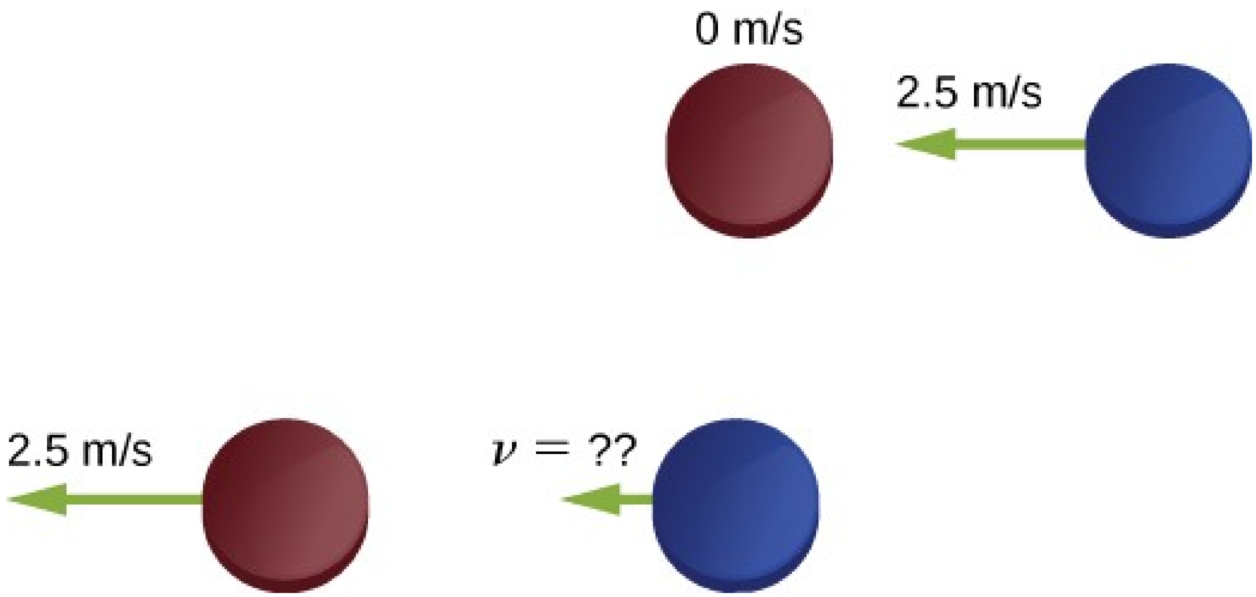
A superball of mass 0.25 kg is dropped from rest from a height of $h = 1.50\ \text{m}$ above the floor. It bounces with no loss of energy and returns to its initial height (Figure 9.17).

a. What is the superball's change of momentum during its bounce on the floor?
b. What was Earth's change of momentum due to the ball colliding with the floor?
c. What was Earth's change of velocity as a result of this collision?



A superball is dropped to the floor (t_0), hits the floor (t_1), bounces (t_2), and returns to its initial height (t_3).


0 m/s
2.5 m/s
2.5 m/s
v = ??



Define the +*x*-direction to point to the right. Conservation of momentum then reads

$$\begin{aligned} \vec{\mathbf{p}}_\mathbf{f} &= \vec{\mathbf{p}}_\mathbf{i} \\ mv_{r_f}\hat{\mathbf{i}} + mv_{b_f}\hat{\mathbf{i}} &= mv_{r_i}\hat{\mathbf{i}} - mv_{b_i}\hat{\mathbf{i}}. \end{aligned}$$

Before the collision, the momentum of the system is entirely and only in the blue puck. Thus,

$$\begin{aligned} mv_{r_f}\hat{\mathbf{i}} + mv_{b_f}\hat{\mathbf{i}} &= -mv_{b_i}\hat{\mathbf{i}} \\ v_{r_f}\hat{\mathbf{i}} + v_{b_f}\hat{\mathbf{i}} &= -v_{b_i}\hat{\mathbf{i}}. \end{aligned}$$

(Remember that the masses of the pucks are equal.) Substituting numbers:

$$\begin{aligned} -(2.5\ \text{m/s})\hat{\mathbf{i}} + \vec{\mathbf{v}}_{b_f} &= -(2.5\ \text{m/s})\hat{\mathbf{i}} \\ \vec{\mathbf{v}}_{b_f} &= 0. \end{aligned}$$

MOVING A COMET

An artist's rendering of *Philae* landing on a comet. (credit: modification of work by "DLR German Aerospace Center"/Flickr)