

ANNOUNCEMENTS

- Homework #9, due Wednesday, October 24

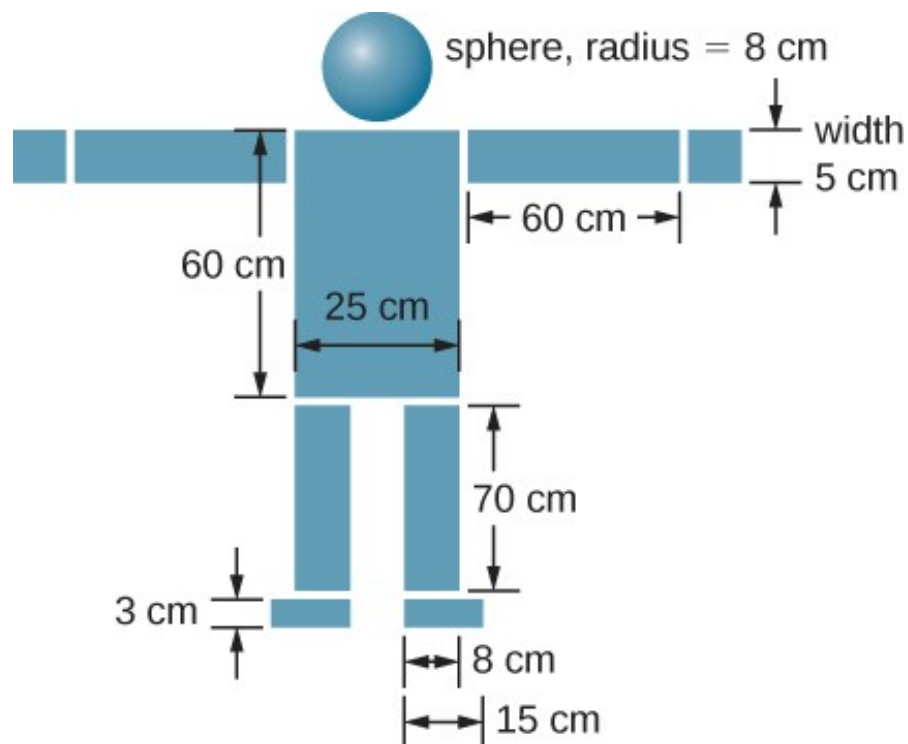
Conceptual questions: Chapter 9, #4 and #10

Problems: Chapter 9, #30, #54

- 5-minute quiz on Chapter 9 Friday, October 26
- Study all Chapter 9, sections 9.1-9.6 by Wednesday
- Second in-class test: Friday, November 2

EXERCISE

Find the center of mass of the structure given in the figure below. Assume a uniform thickness of 20 cm, and a uniform density of 1 g/cm^3 .



Position of the center of mass for a system
of particles

$$\vec{r}_{\text{CM}} \equiv \frac{1}{M} \sum_{j=1}^N m_j \vec{r}_j$$

KEY EQUATIONS

Definition of momentum	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
Impulse	$\vec{\mathbf{J}} \equiv \int_{t_i}^{t_f} \vec{\mathbf{F}}(t)dt$ or $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{ave}}\Delta t$
Impulse-momentum theorem	$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$
Average force from momentum	$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$
Instantaneous force from momentum (Newton's second law)	$\vec{\mathbf{F}}(t) = \frac{d\vec{\mathbf{p}}}{dt}$
Conservation of momentum	$\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0$ or $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$
Generalized conservation of momentum	$\sum_{j=1}^N \vec{\mathbf{p}}_j = \text{constant}$
Conservation of momentum in two dimensions	$p_{f,x} = p_{1,i,x} + p_{2,i,x}$ $p_{f,y} = p_{1,i,y} + p_{2,i,y}$
External forces	$\vec{\mathbf{F}}_{\text{ext}} = \sum_{j=1}^N \frac{d\vec{\mathbf{p}}_j}{dt}$

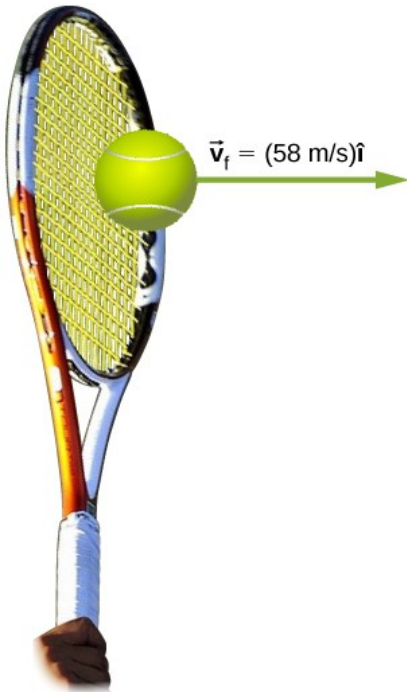
NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM

The net external force on a system is equal to the rate of change of the momentum of that system caused by the force:

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}.$$

Calculating Force: Venus Williams' Tennis Serve

During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet? Assume that the ball's speed just after impact is 58 m/s, as shown in [Figure 9.13](#), that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms.



To determine the change in momentum, insert the values for the initial and final velocities into the equation above:

$$\begin{aligned}\Delta p &= m(v_f - v_i) \\ &= (0.057 \text{ kg})(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.3 \frac{\text{kg}\cdot\text{m}}{\text{s}}.\end{aligned}$$

Now the magnitude of the net external force can be determined by using

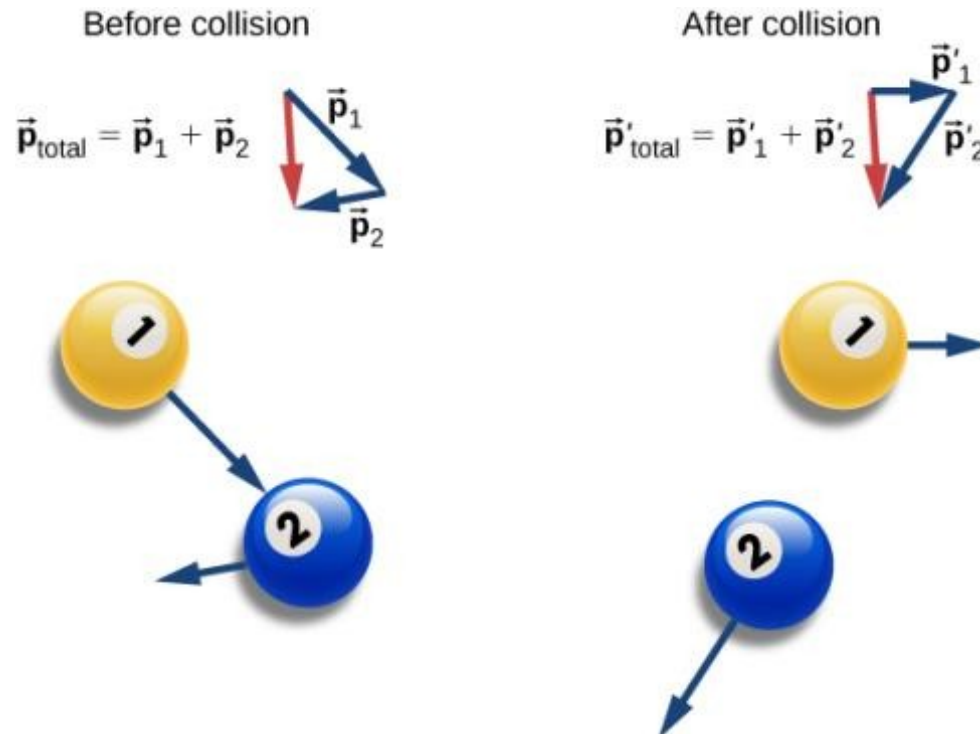
$$F = \frac{\Delta p}{\Delta t} = \frac{3.3 \frac{\text{kg}\cdot\text{m}}{\text{s}}}{5.0 \times 10^{-3} \text{ s}} = 6.6 \times 10^2 \text{ N}.$$

where we have retained only two significant figures in the final step.

LAW OF CONSERVATION OF MOMENTUM

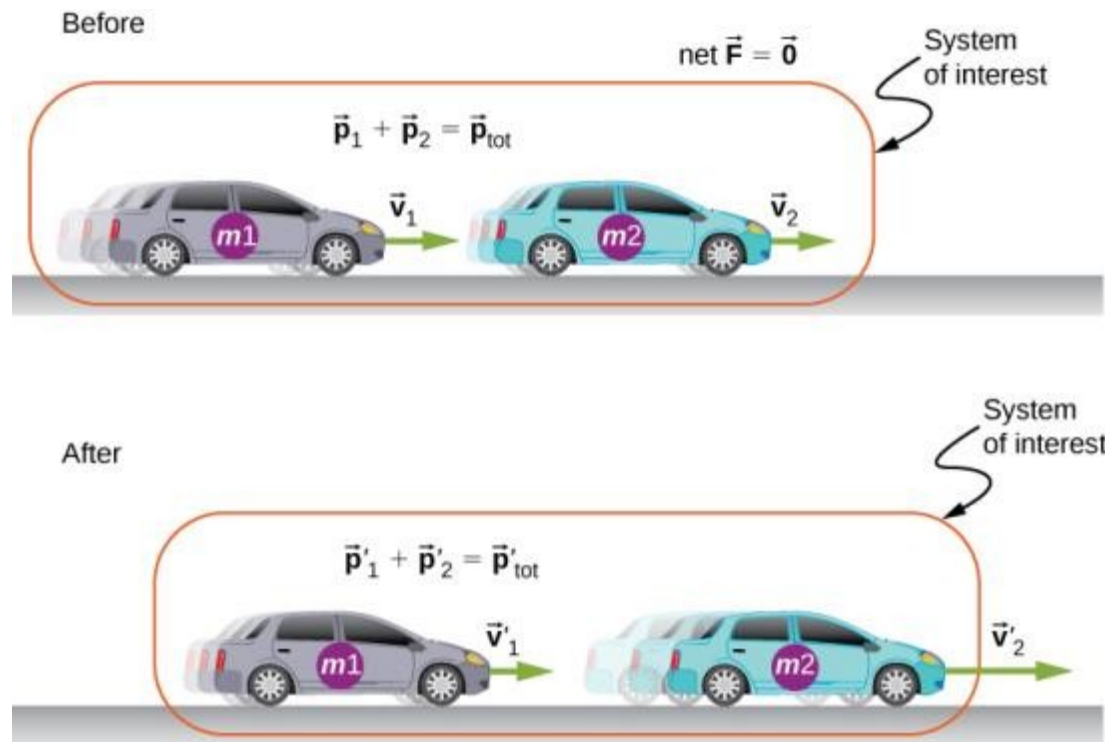
The total momentum of a closed system is conserved:

$$\sum_{j=1}^N \vec{p}_j = \text{constant.}$$



Before the collision, the two billiard balls travel with momenta \vec{p}_1 and \vec{p}_2 . The total momentum of the system is the sum of these, as shown by the red vector labeled \vec{p}_{total} on the left. After the collision, the two billiard balls travel with different momenta \vec{p}'_1 and \vec{p}'_2 . The total momentum, however, has not changed, as shown by the red vector arrow \vec{p}'_{total} on the right.

EXAMPLE

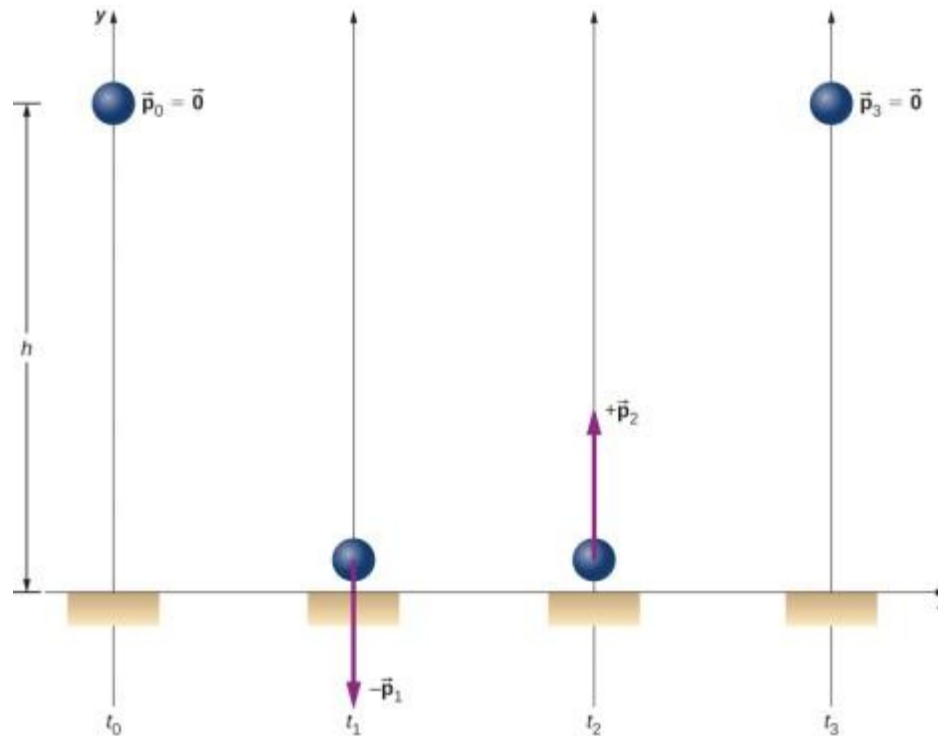


The two cars together form the system that is to be analyzed. It is important to remember that the contents (the mass) of the system do not change before, during, or after the objects in the system interact.

A Bouncing Superball

A superball of mass 0.25 kg is dropped from rest from a height of $h = 1.50$ m above the floor. It bounces with no loss of energy and returns to its initial height ([Figure 9.17](#)).

- What is the superball's change of momentum during its bounce on the floor?
- What was Earth's change of momentum due to the ball colliding with the floor?
- What was Earth's change of velocity as a result of this collision?



A superball is dropped to the floor (t_0), hits the floor (t_1), bounces (t_2), and returns to its initial height (t_3).

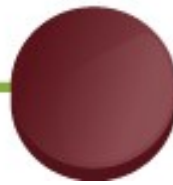
0 m/s



2.5 m/s

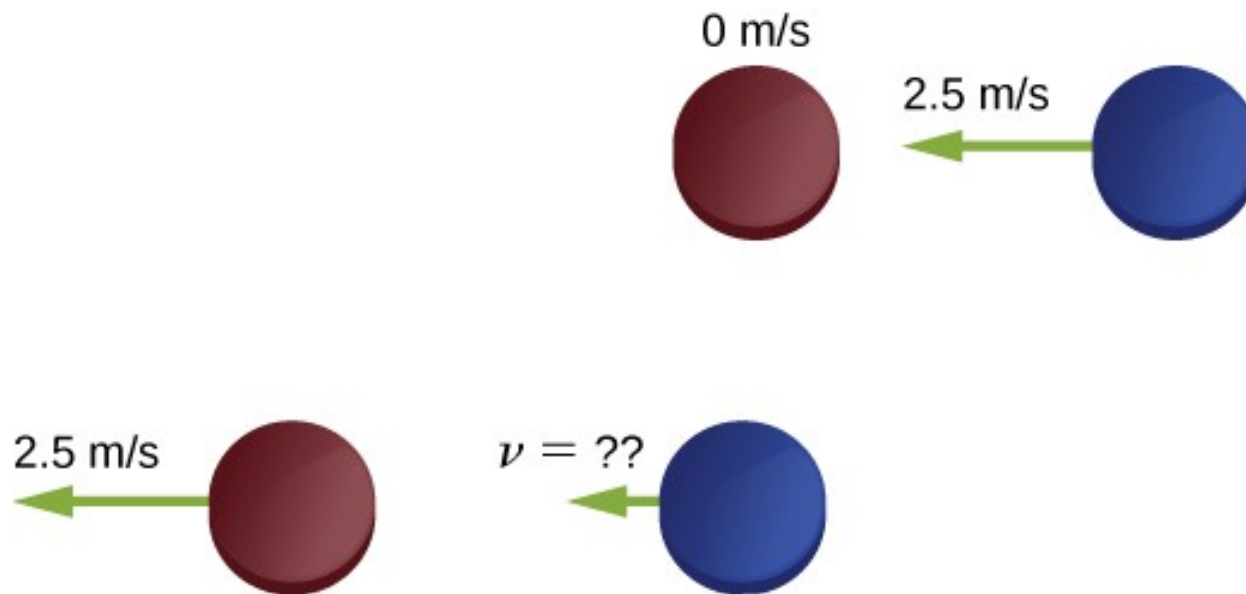


2.5 m/s



$v = ??$





Define the +x-direction to point to the right. Conservation of momentum then reads

$$\begin{aligned}\vec{\mathbf{p}}_{\text{f}} &= \vec{\mathbf{p}}_{\text{i}} \\ mv_{\text{rf}}\hat{\mathbf{i}} + mv_{\text{bf}}\hat{\mathbf{i}} &= mv_{\text{ri}}\hat{\mathbf{i}} - mv_{\text{bi}}\hat{\mathbf{i}}.\end{aligned}$$

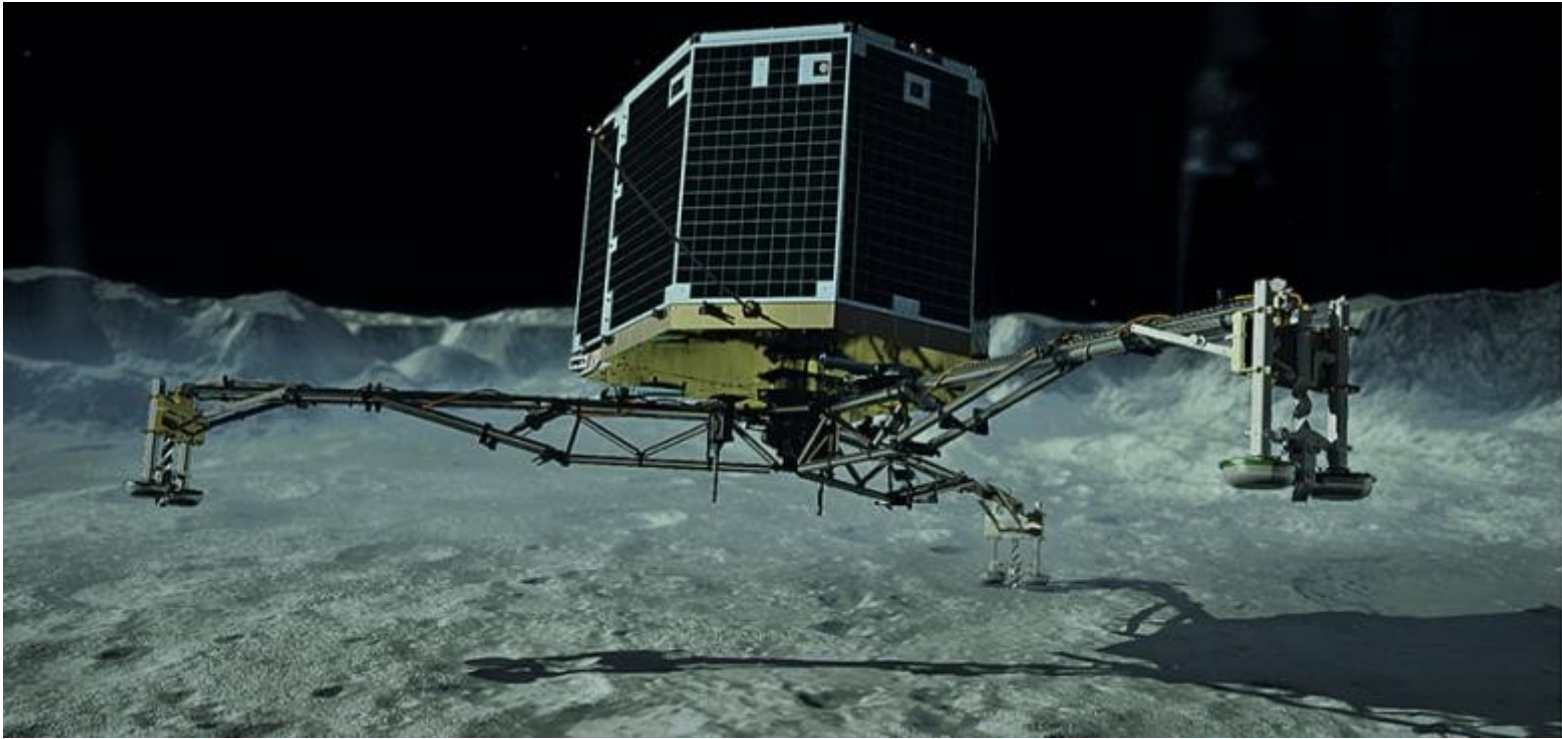
Before the collision, the momentum of the system is entirely and only in the blue puck. Thus,

$$\begin{aligned}mv_{\text{rf}}\hat{\mathbf{i}} + mv_{\text{bf}}\hat{\mathbf{i}} &= -mv_{\text{bi}}\hat{\mathbf{i}} \\ v_{\text{rf}}\hat{\mathbf{i}} + v_{\text{bf}}\hat{\mathbf{i}} &= -v_{\text{bi}}\hat{\mathbf{i}}.\end{aligned}$$

(Remember that the masses of the pucks are equal.) Substituting numbers:

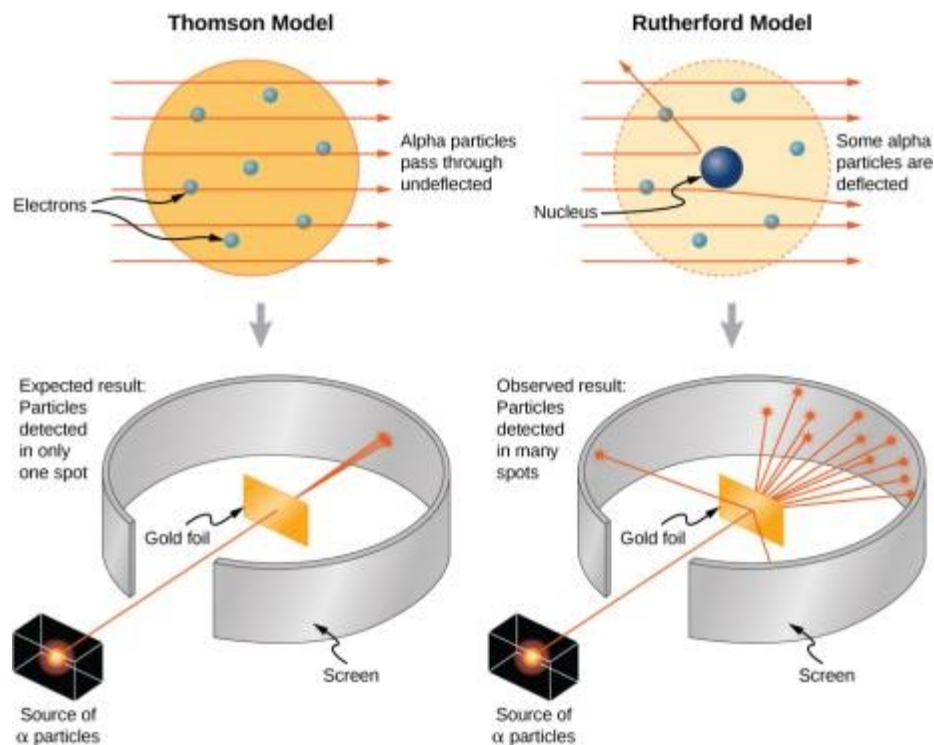
$$\begin{aligned}-(2.5 \text{ m/s})\hat{\mathbf{i}} + \vec{\mathbf{v}}_{\text{bf}} &= -(2.5 \text{ m/s})\hat{\mathbf{i}} \\ \vec{\mathbf{v}}_{\text{bf}} &= 0.\end{aligned}$$

MOVING A COMET



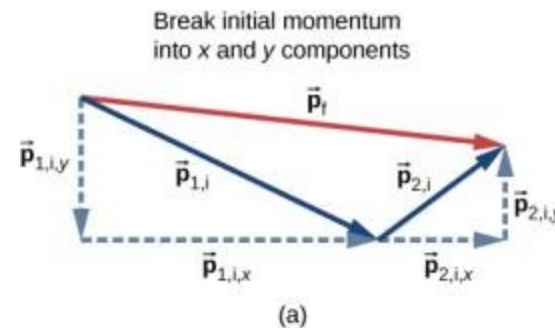
An artist's rendering of *Philae* landing on a comet. (credit: modification of work by “DLR German Aerospace Center”/Flickr)

COLLISIONS

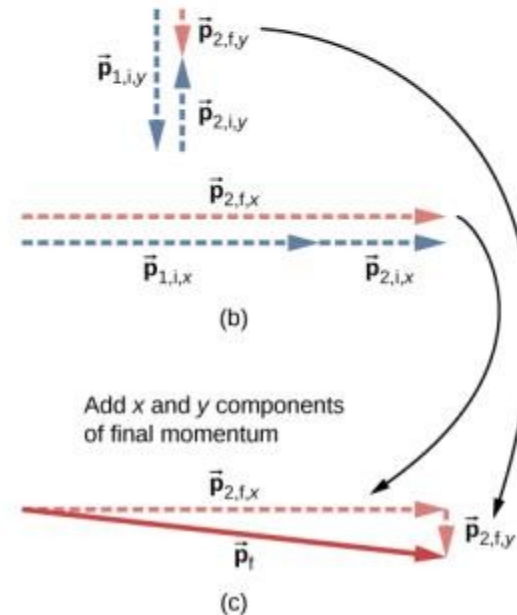


The Thomson and Rutherford models of the atom. The Thomson model predicted that nearly all of the incident alpha-particles would be scattered and at small angles. Rutherford and Geiger found that nearly none of the alpha particles were scattered, but those few that were deflected did so through very large angles. The results of Rutherford's experiments were inconsistent with the Thomson model. Rutherford used conservation of momentum and energy to develop a new, and better model of the atom—the nuclear model.

- (a) For two-dimensional momentum problems, break the initial momentum vectors into their x - and y -components.
- (b) Add the x - and y -components together separately. This gives you the x - and y -components of the final momentum, which are shown as red dashed vectors.
- (c) Adding these components together gives the final momentum.



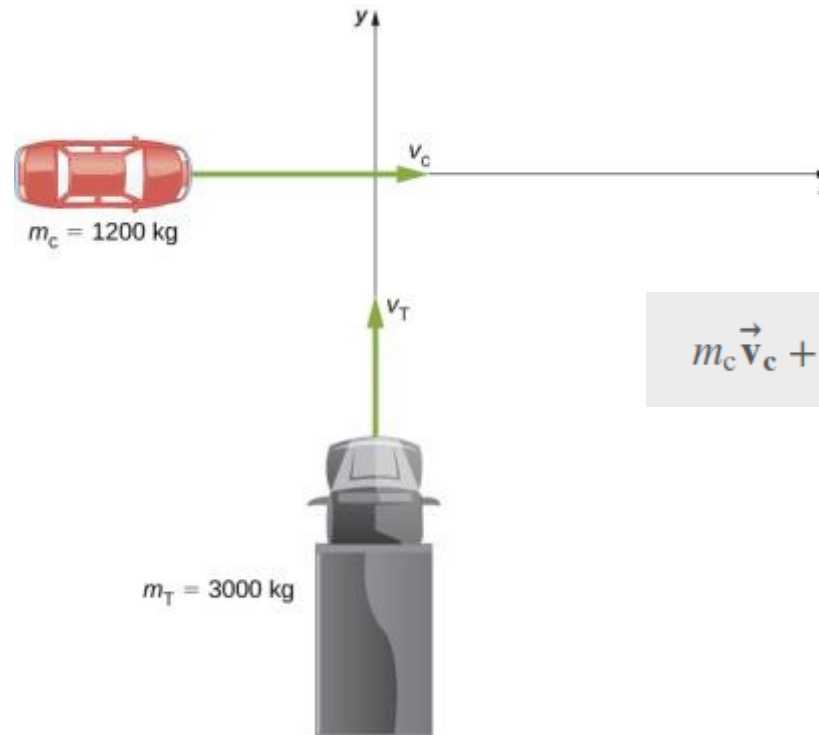
Add x and y components to obtain x and y components of final momentum



EXAMPLE

Traffic Collision

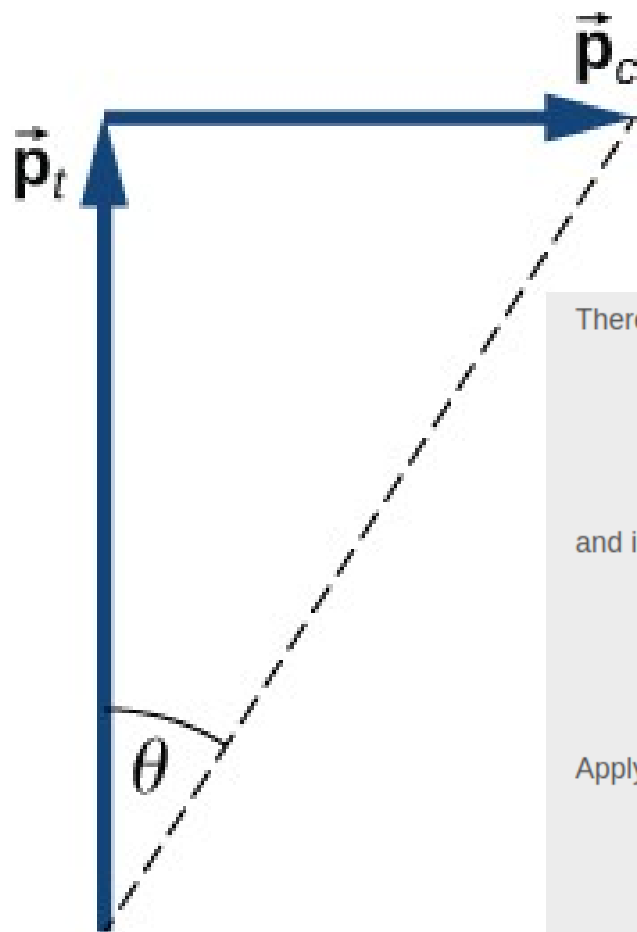
A small car of mass 1200 kg traveling east at 60 km/hr collides at an intersection with a truck of mass 3000 kg that is traveling due north at 40 km/hr ([Figure 9.23](#)). The two vehicles are locked together. What is the velocity of the combined wreckage?



$$m_c \vec{v}_c + m_T \vec{v}_T = (m_c + m_T) \vec{v}_w.$$

A large truck moving north is about to collide with a small car moving east. The final momentum vector has both x- and y-components.

Graphical addition of momentum vectors. Notice that, although the car's velocity is larger than the truck's, its momentum is smaller.



Therefore, in the x-direction:

$$m_c v_c = (m_c + m_T) v_{w,x}$$

$$v_{w,x} = \left(\frac{m_c}{m_c + m_T} \right) v_c$$

and in the y-direction:

$$m_T v_T = (m_c + m_T) v_{w,y}$$

$$v_{w,y} = \left(\frac{m_T}{m_c + m_T} \right) v_T$$

Applying the Pythagorean theorem gives

$$|\vec{v}_w| = \sqrt{\left[\left(\frac{m_c}{m_c + m_T} \right) v_c \right]^2 + \left[\left(\frac{m_T}{m_c + m_T} \right) v_T \right]^2}$$

$$= \sqrt{\left[\left(\frac{1200 \text{ kg}}{4200 \text{ kg}} \right) (16.67 \text{ m/s}) \right]^2 + \left[\left(\frac{3000 \text{ kg}}{4200 \text{ kg}} \right) (11.1 \text{ m/s}) \right]^2}$$

$$= \sqrt{\left(4.76 \text{ m/s} \right)^2 + \left(7.93 \text{ m/s} \right)^2}$$

$$= 9.25 \text{ m/s} \approx 33.3 \text{ km/hr}.$$

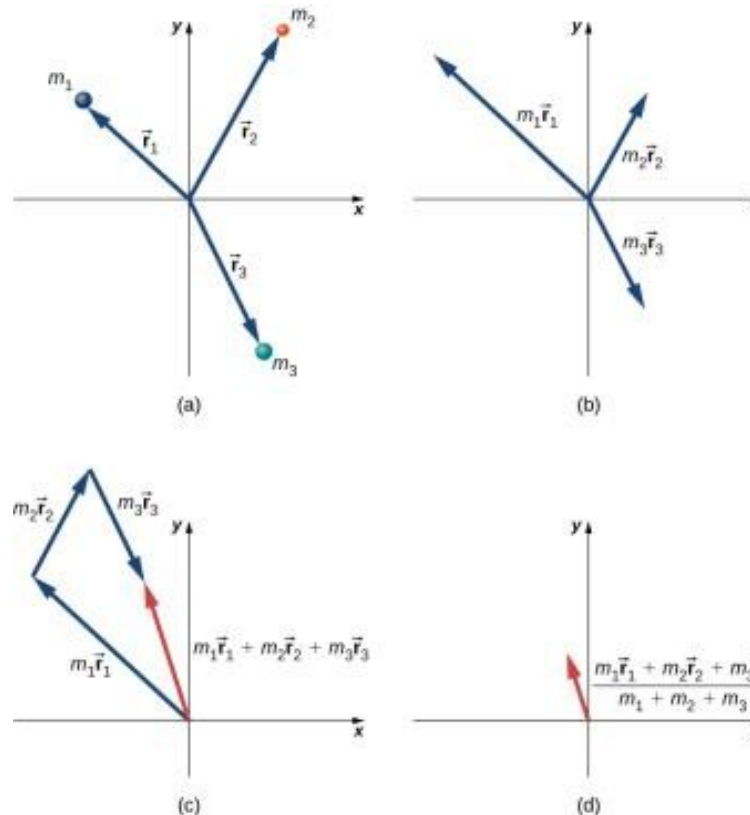
As for its direction, using the angle shown in the figure,

$$\theta = \tan^{-1} \left(\frac{v_{w,x}}{v_{w,y}} \right) = \tan^{-1} \left(\frac{7.93 \text{ m/s}}{4.76 \text{ m/s}} \right) = 59^\circ.$$

CENTER OF MASS

$$M = \sum_{j=1}^N m_j.$$

$$\vec{r}_{\text{CM}} \equiv \frac{1}{M} \sum_{j=1}^N m_j \vec{r}_j.$$



$$r_{\text{CM},x} = \frac{1}{M} \sum_{j=1}^N m_j x_j$$

$$r_{\text{CM},y} = \frac{1}{M} \sum_{j=1}^N m_j y_j$$

$$r_{\text{CM},z} = \frac{1}{M} \sum_{j=1}^N m_j z_j$$

$$\vec{r}_{\text{CM}} = r_{\text{CM},x}\hat{i} + r_{\text{CM},y}\hat{j} + r_{\text{CM},z}\hat{k}$$

$$r_{\text{CM}} = |\vec{r}_{\text{CM}}| = (r_{\text{CM},x}^2 + r_{\text{CM},y}^2 + r_{\text{CM},z}^2)^{1/2}.$$

Finding the center of mass of a system of three different particles.

- Position vectors are created for each object.
- The position vectors are multiplied by the mass of the corresponding object.
- The scaled vectors from part (b) are added together.
- The final vector is divided by the total mass. This vector points to the center of mass of the system. Note that no mass is actually present at the center of mass of this system.

$$\vec{v}_{\text{CM},f} = \vec{v}_{\text{CM},i}.$$

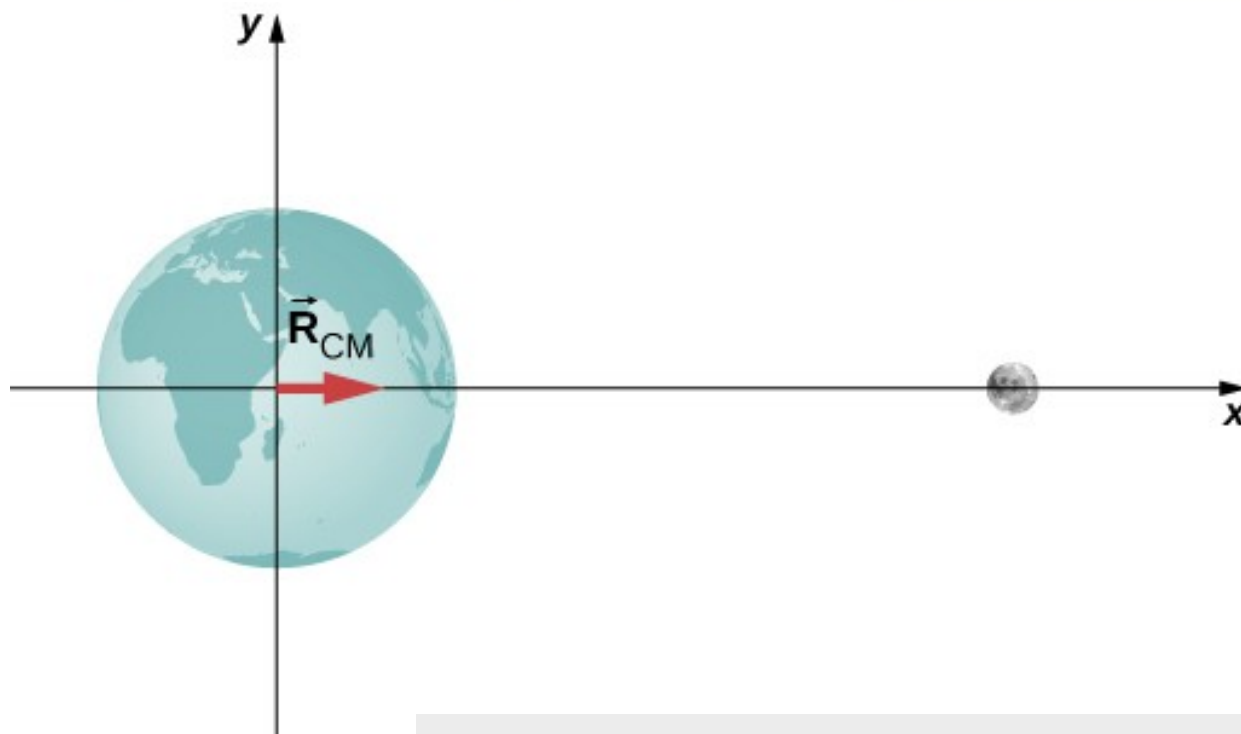
(9.37)

That is to say, *in the absence of an external force, the velocity of the center of mass never changes.*

$$\vec{F} = \frac{d\vec{p}_{\text{CM}}}{dt}.$$



These exploding fireworks are a vivid example of conservation of momentum and the motion of the center of mass.



$$R = \frac{m_e r_e + m_m r_m}{m_e + m_m}.$$

From [Appendix D](#),

$$m_e = 5.97 \times 10^{24} \text{ kg}$$

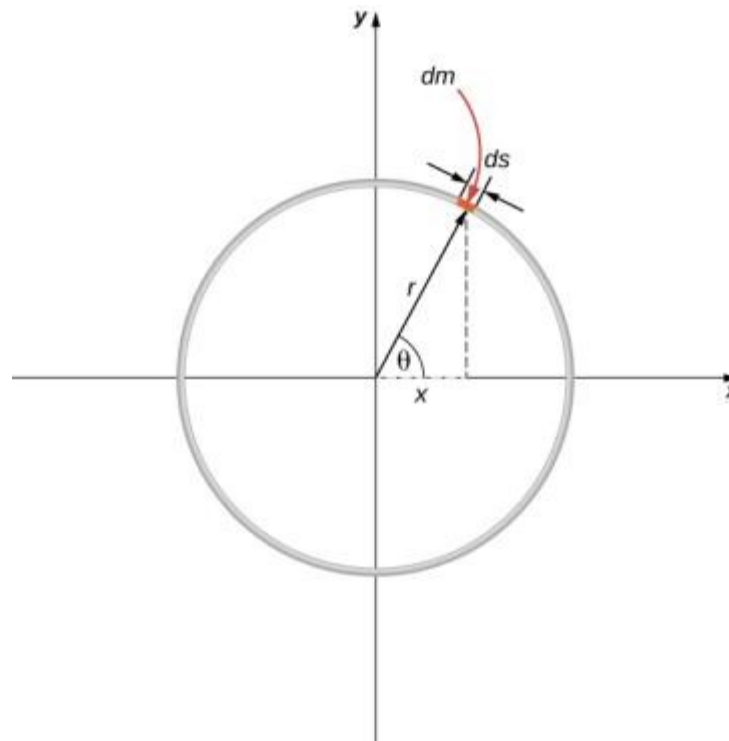
$$m_m = 7.36 \times 10^{22} \text{ kg}$$

$$r_m = 3.82 \times 10^8 \text{ m}.$$

We defined the center of Earth as the origin, so $r_e = 0 \text{ m}$. Inserting these into the equation for R gives

$$\begin{aligned} R &= \frac{(5.97 \times 10^{24} \text{ kg})(0 \text{ m}) + (7.36 \times 10^{22} \text{ kg})(3.82 \times 10^8 \text{ m})}{5.97 \times 10^{24} \text{ kg} + 7.36 \times 10^{22} \text{ kg}} \\ &= 4.64 \times 10^6 \text{ m}. \end{aligned}$$

CENTER OF MASS OF EXTENDED OBJECTS



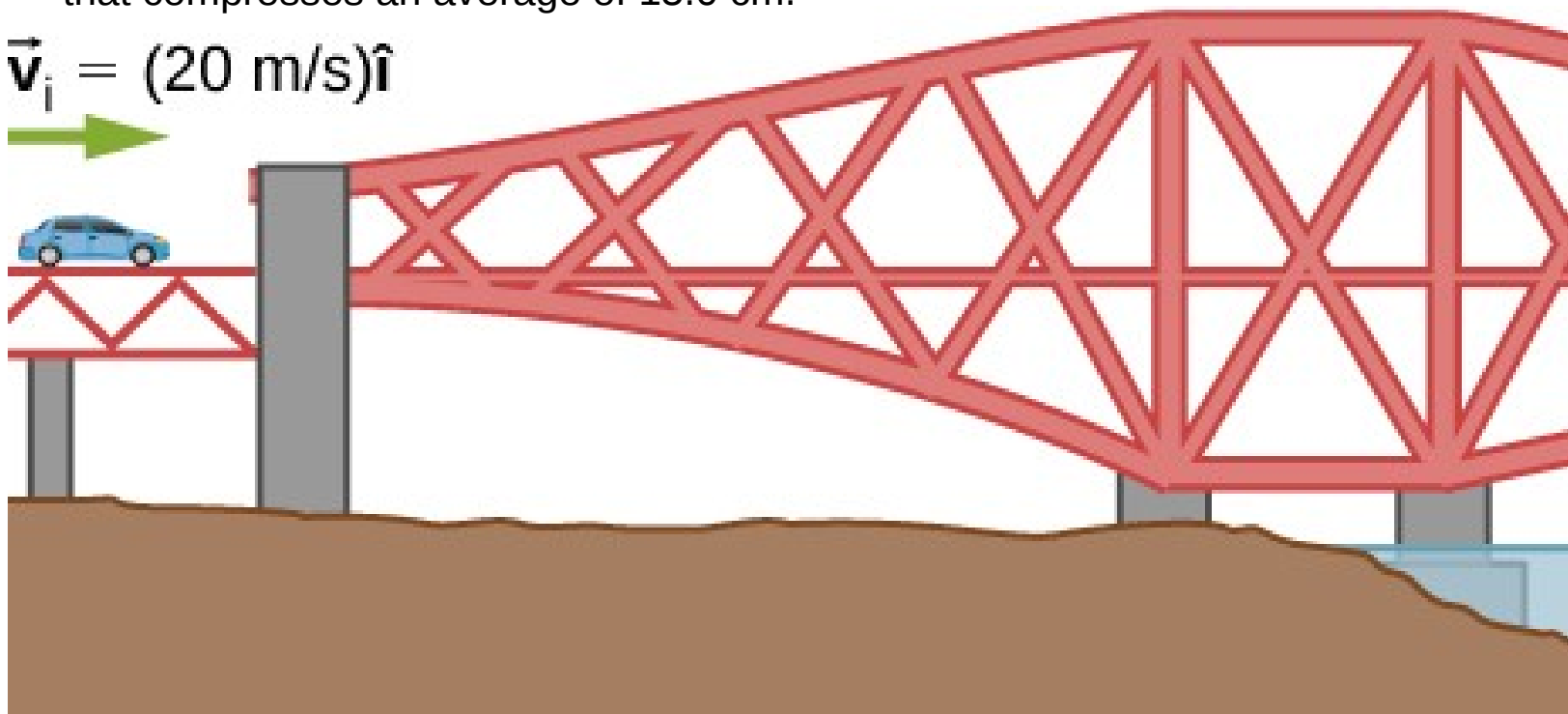
Finding the center of mass of a uniform hoop. We express the coordinates of a differential piece of the hoop, and then integrate around the hoop.

EXERCISE 25

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment

- Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm.
- Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.

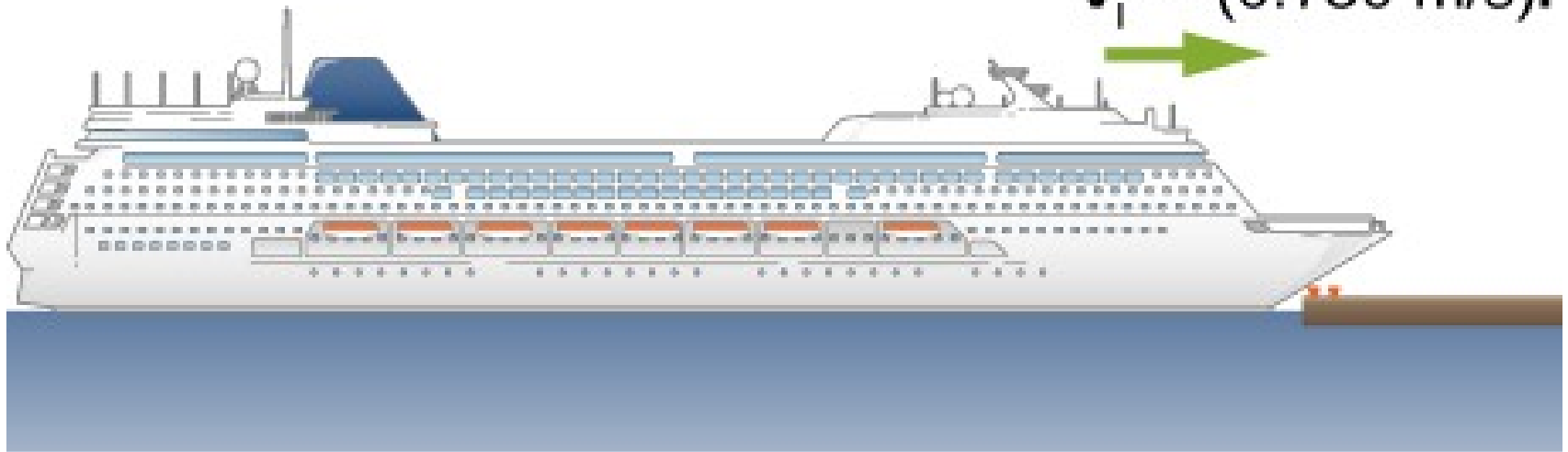
$$\vec{v}_i = (20 \text{ m/s})\hat{i}$$



EXERCISE

A cruise ship with a mass of 1.00×10^7 kg strikes a pier at a speed of 0.750 m/s. It comes to rest after traveling 6.00 m, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (*Hint*: First calculate the time it took to bring the ship to rest, assuming a constant force.)

$$\vec{v}_i = (0.750 \text{ m/s})\hat{i}$$



Impulse-momentum theorem

$$\vec{J} = \Delta \vec{p}$$

Average force from momentum

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Instantaneous force from momentum

$$\vec{F}(t) = \frac{d\vec{p}}{dt}$$

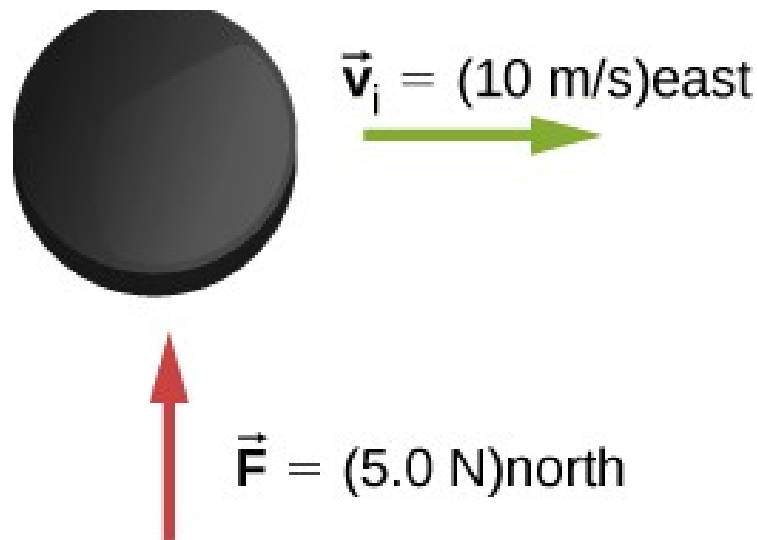
(Newton's second law)

Conservation of momentum

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \text{ or } \vec{p}_1 + \vec{p}_2 = \text{constant}$$

EXERCISE

A hockey puck of mass 150 g is sliding due east on a frictionless table with a speed of 10 m/s. Suddenly, a constant force of magnitude 5 N and direction due north is applied to the puck for 1.5 s. Find the north and east components of the momentum at the end of the 1.5-s interval.



Impulse-momentum theorem

$$\vec{J} = \Delta \vec{p}$$

Average force from momentum

$$\vec{F} = \frac{\Delta \vec{p}}{\Delta t}$$

Instantaneous force from momentum

$$\vec{F}(t) = \frac{d\vec{p}}{dt}$$

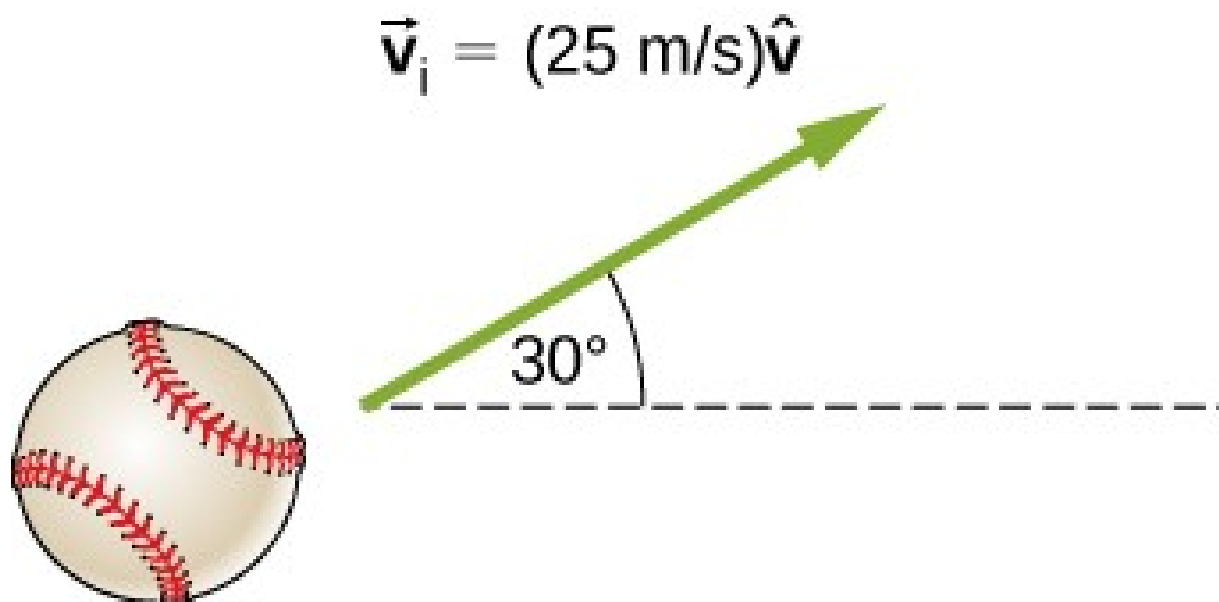
(Newton's second law)

Conservation of momentum

$$\frac{d\vec{p}_1}{dt} + \frac{d\vec{p}_2}{dt} = 0 \quad \text{or} \quad \vec{p}_1 + \vec{p}_2 = \text{constant}$$

EXERCISE

A ball of mass 250 g is thrown with an initial velocity of 25 m/s at an angle of 30° with the horizontal direction. Ignore air resistance. What is the momentum of the ball after 0.2 s? (Do this problem by finding the components of the momentum first, and then constructing the magnitude and direction of the momentum vector from the components.)



EXERCISE

Train cars are coupled together by being bumped into one another. Suppose two loaded train cars are moving toward one another, the first having a mass of $1.50 \times 10^5 \text{ kg}$ and a velocity of $(0.30 \text{ m/s})\hat{i}$, and the second having a mass of $1.10 \times 10^5 \text{ kg}$ and a velocity of $-(0.12 \text{ m/s})\hat{i}$. What is their final velocity?

$$\vec{v}_{1,i} = (0.30 \text{ m/s})\hat{i}$$

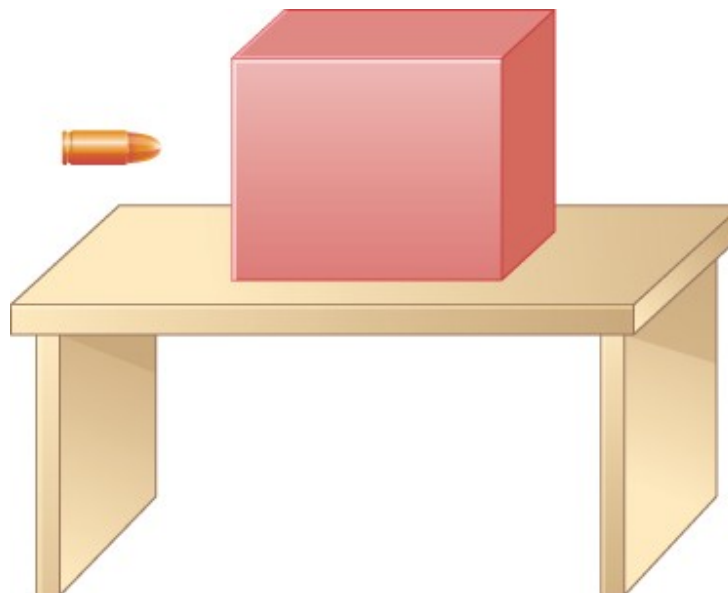


$$\vec{v}_{2,i} = -(0.12 \text{ m/s})\hat{i}$$



EXERCISE

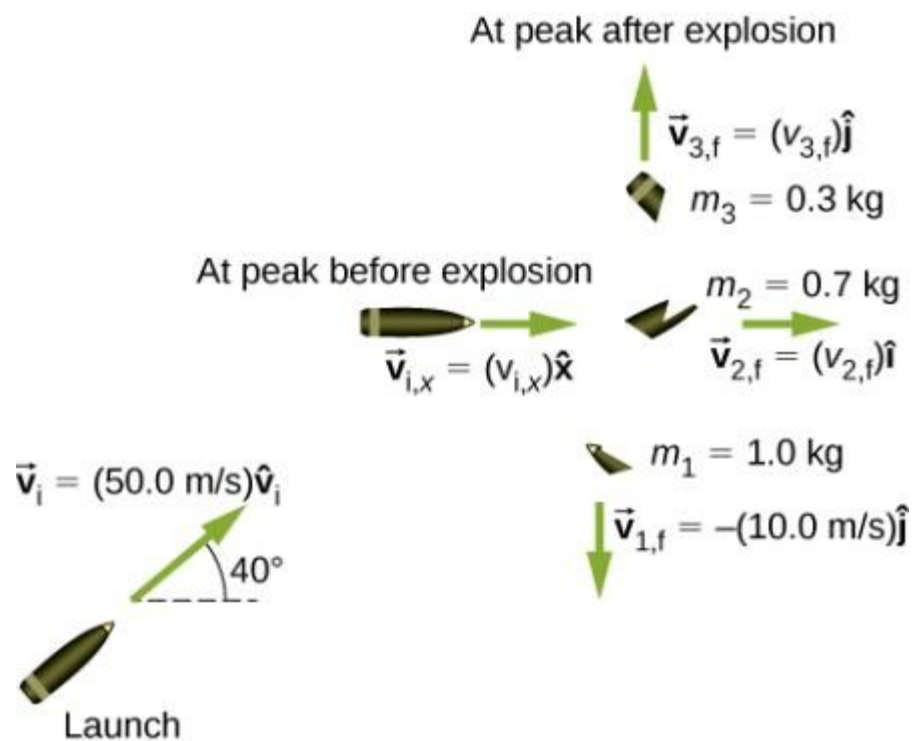
The figure below shows a bullet of mass 200 g traveling horizontally towards the east with speed 400 m/s, which strikes a block of mass 1.5 kg that is initially at rest on a frictionless table.



After striking the block, the bullet is embedded in the block and the block and the bullet move together as one unit.

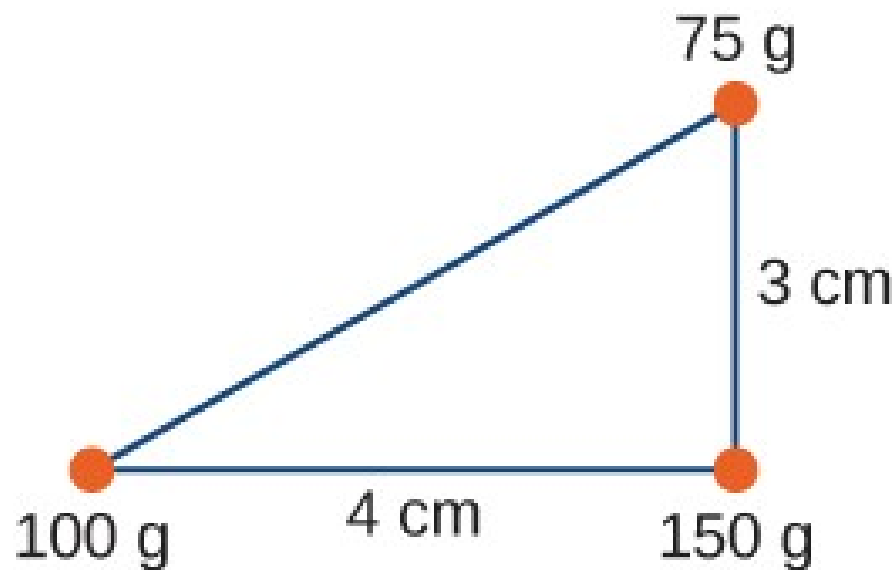
- What is the magnitude and direction of the velocity of the block/bullet combination immediately after the impact?
- What is the magnitude and direction of the impulse by the block on the bullet?
- What is the magnitude and direction of the impulse from the bullet on the block?
- If it took 3 ms for the bullet to change the speed from 400 m/s to the final speed after impact, what is the average force between the block and the bullet during this time?

EXERCISE 55



EXERCISE

Find the center of mass of the three-mass system.

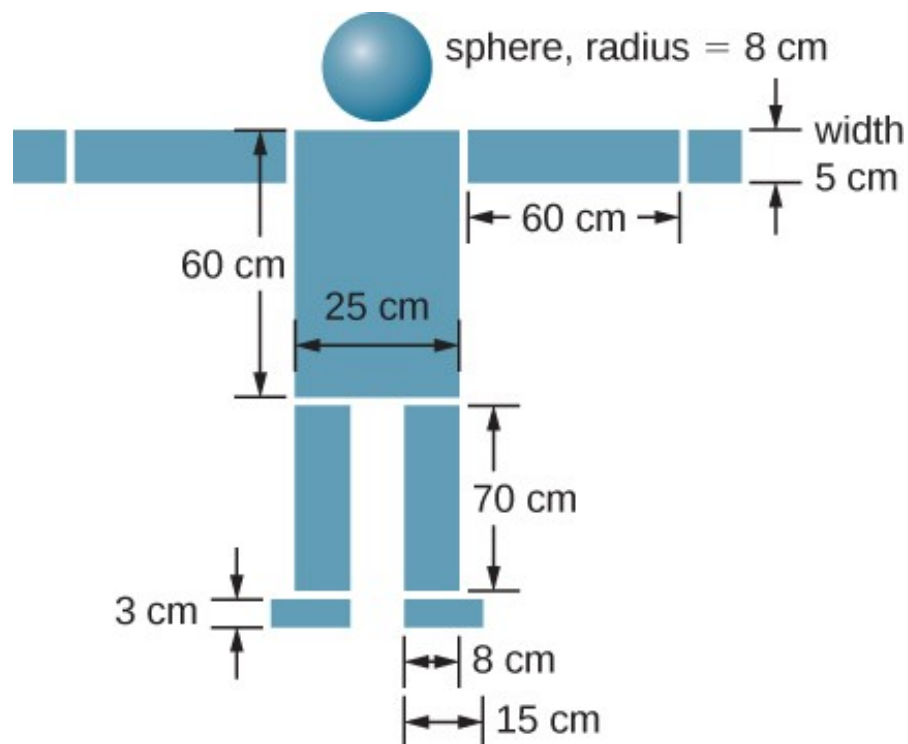


Position of the center of mass for a system
of particles

$$\vec{r}_{\text{CM}} \equiv \frac{1}{M} \sum_{j=1}^N m_j \vec{r}_j$$

EXERCISE

Find the center of mass of the structure given in the figure below. Assume a uniform thickness of 20 cm, and a uniform density of 1 g/cm^3 .



Position of the center of mass for a system
of particles

$$\vec{r}_{\text{CM}} \equiv \frac{1}{M} \sum_{j=1}^N m_j \vec{r}_j$$