ANNOUNCEMENTS

- Homework #9, due Wednesday, October 24

  Conceptual questions: Chapter 9, #4 and #10
  Problems: Chapter 9, #30, #54

- 5-minute quiz on Chapter 9 Friday, October 26

- Study all Chapter 9, sections 9.1-9.6 by Wednesday

- Second in-class test: Friday, November 2

KEY EQUATIONS

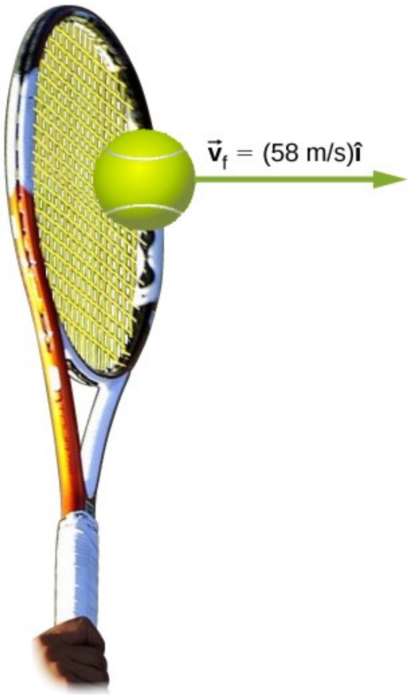
Definition of momentum	$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$
Impulse	$\vec{J} \equiv \int_{t_i}^{t_f} \vec{F}(t)dt$ or $\vec{\mathbf{J}} = \vec{\mathbf{F}}_{\text{ave}}\Delta t$
Impulse-momentum theorem	$\vec{\mathbf{J}} = \Delta\vec{\mathbf{p}}$
Average force from momentum	$\vec{\mathbf{F}} = \frac{\Delta\vec{\mathbf{p}}}{\Delta t}$
Instantaneous force from momentum (Newton's second law)	$\vec{\mathbf{F}}(t) = \frac{d\vec{\mathbf{p}}}{dt}$
Conservation of momentum	$\frac{d\vec{\mathbf{p}}_1}{dt} + \frac{d\vec{\mathbf{p}}_2}{dt} = 0$ or $\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$
Generalized conservation of momentum	$\sum_{j=1}^{N} \vec{\mathbf{p}}_j = \text{constant}$
Conservation of momentum in two dimensions	$p_{f,x} = p_{1,i,x} + p_{2,i,x}$ $p_{f,y} = p_{1,i,y} + p_{2,i,y}$
External forces	$\vec{\mathbf{F}}_{\text{ext}} = \sum_{j=1}^{N} \frac{d\vec{\mathbf{p}}_j}{dt}$

NEWTON'S SECOND LAW OF MOTION IN TERMS OF MOMENTUM

The net external force on a system is equal to the rate of change of the momentum of that system caused by the force:

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}}{dt}.$$

Calculating Force: Venus Williams' Tennis Serve
During the 2007 French Open, Venus Williams hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What is the average force exerted on the 0.057-kg tennis ball by Venus Williams' racquet? Assume that the ball's speed just after impact is 58 m/s, as shown in Figure 9.13, that the initial horizontal component of the velocity before impact is negligible, and that the ball remained in contact with the racquet for 5.0 ms.



To determine the change in momentum, insert the values for the initial and final velocities into the equation above:

$$\begin{aligned} \Delta p &= m(v_f - v_i) \\ &= (0.057 \text{ kg})(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.3 \frac{\text{kg}\cdot\text{m}}{\text{s}}. \end{aligned}$$

Now the magnitude of the net external force can be determined by using

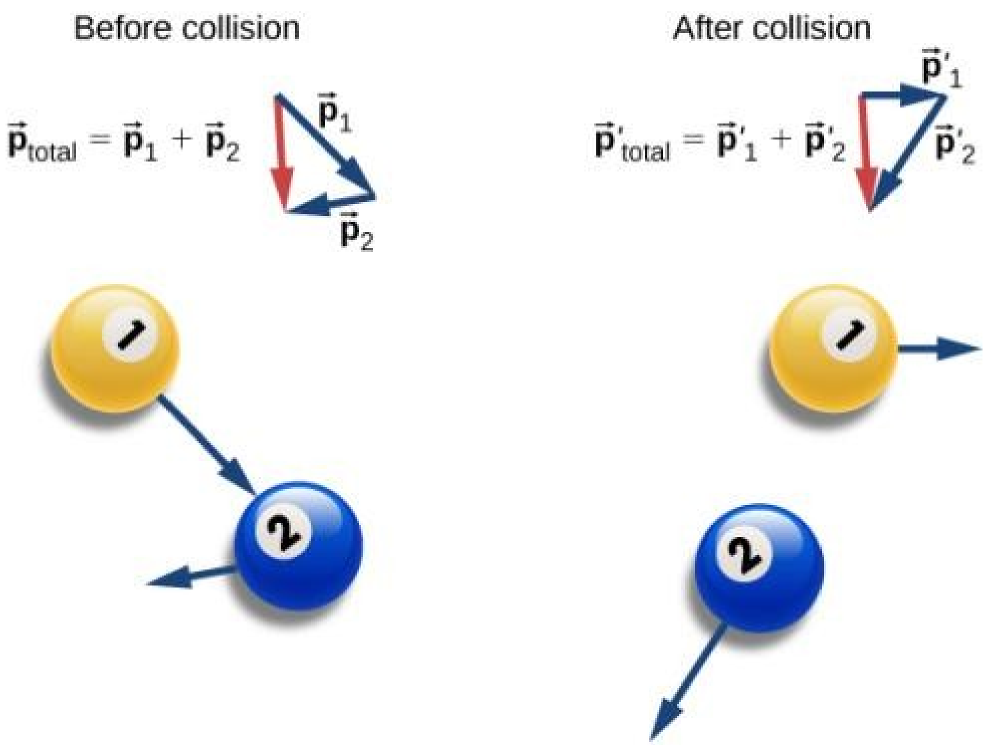
$$F = \frac{\Delta p}{\Delta t} = \frac{3.3 \frac{\text{kg}\cdot\text{m}}{\text{s}}}{5.0 \times 10^{-3} \text{ s}} = 6.6 \times 10^2 \text{ N}.$$

where we have retained only two significant figures in the final step.

LAW OF CONSERVATION OF MOMENTUM

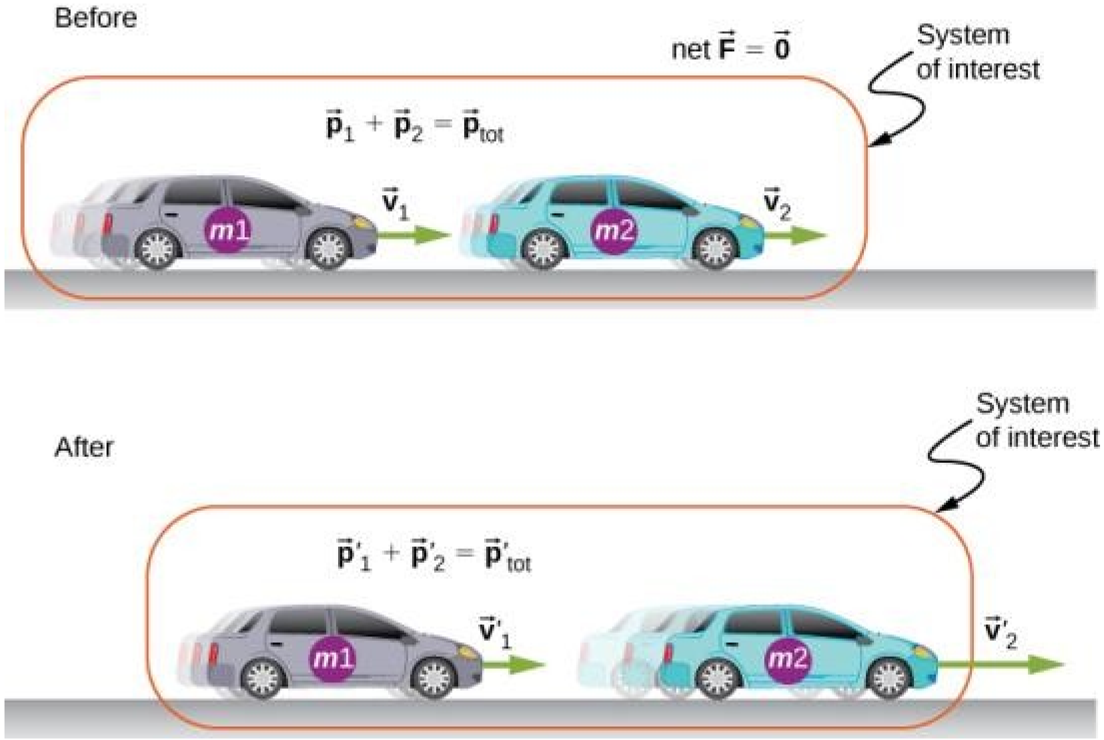
The total momentum of a closed system is conserved:

$$\sum_{j=1}^{N} \vec{\mathbf{p}}_j = \text{constant}.$$



Before the collision, the two billiard balls travel with momenta $\vec{\mathbf{p}}_1$ and $\vec{\mathbf{p}}_3$. The total momentum of the system is the sum of these, as shown by the red vector labeled $\vec{\mathbf{p}}_{\text{total}}$ on the left. After the collision, the two billiard balls travel with different momenta $\vec{\mathbf{p}}'_1$ and $\vec{\mathbf{p}}'_3$. The total momentum, however, has not changed, as shown by the red vector arrow $\vec{\mathbf{p}}'_{\text{total}}$ on the right.

EXAMPLE

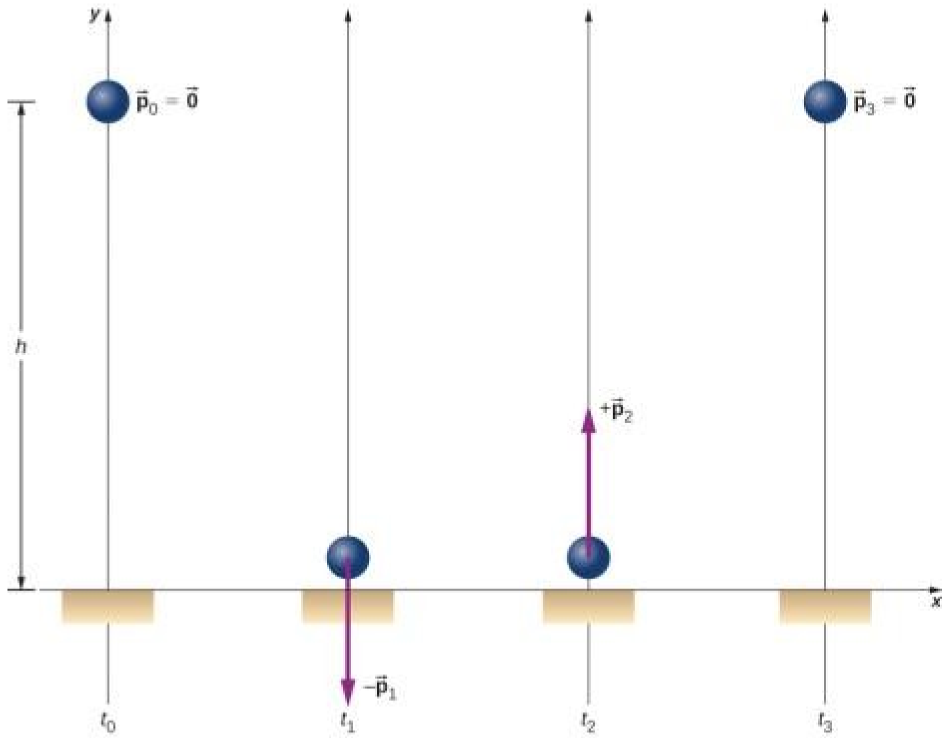


The two cars together form the system that is to be analyzed. It is important to remember that the contents (the mass) of the system do not change before, during, or after the objects in the system interact.

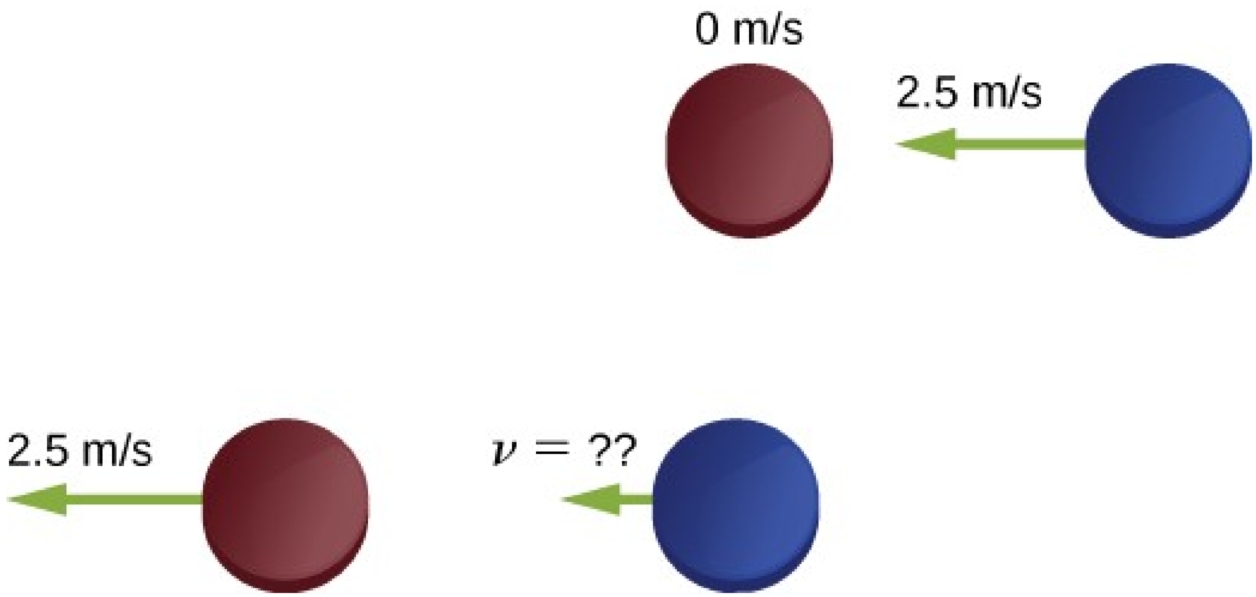
A Bouncing Superball

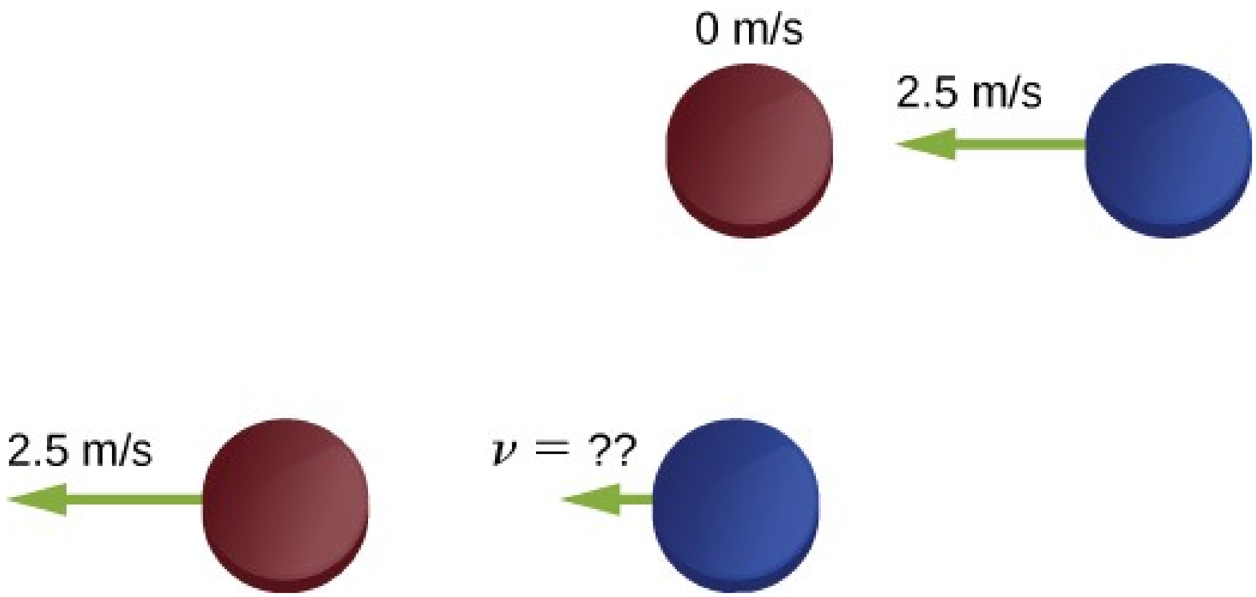
A superball of mass 0.25 kg is dropped from rest from a height of $h = 1.50\text{ m}$ above the floor. It bounces with no loss of energy and returns to its initial height (Figure 9.17).

a. What is the superball's change of momentum during its bounce on the floor?
b. What was Earth's change of momentum due to the ball colliding with the floor?
c. What was Earth's change of velocity as a result of this collision?



A superball is dropped to the floor (t_0), hits the floor (t_1), bounces (t_2), and returns to its initial height (t_3).


0 m/s
2.5 m/s
2.5 m/s
$\nu = ??$



Define the +x-direction to point to the right. Conservation of momentum then reads

$$\begin{aligned}\vec{\mathbf{p}}_\mathrm{f} &= \vec{\mathbf{p}}_\mathrm{i} \\ mv_{\mathrm{r_f}}\hat{\mathbf{i}} + mv_{\mathrm{b_f}}\hat{\mathbf{i}} &= mv_{\mathrm{r_i}}\hat{\mathbf{i}} - mv_{\mathrm{b_i}}\hat{\mathbf{i}}.\end{aligned}$$

Before the collision, the momentum of the system is entirely and only in the blue puck. Thus,

$$\begin{aligned}mv_{\mathrm{r_f}}\hat{\mathbf{i}} + mv_{\mathrm{b_f}}\hat{\mathbf{i}} &= -mv_{\mathrm{b_i}}\hat{\mathbf{i}} \\ v_{\mathrm{r_f}}\hat{\mathbf{i}} + v_{\mathrm{b_f}}\hat{\mathbf{i}} &= -v_{\mathrm{b_i}}\hat{\mathbf{i}}.\end{aligned}$$

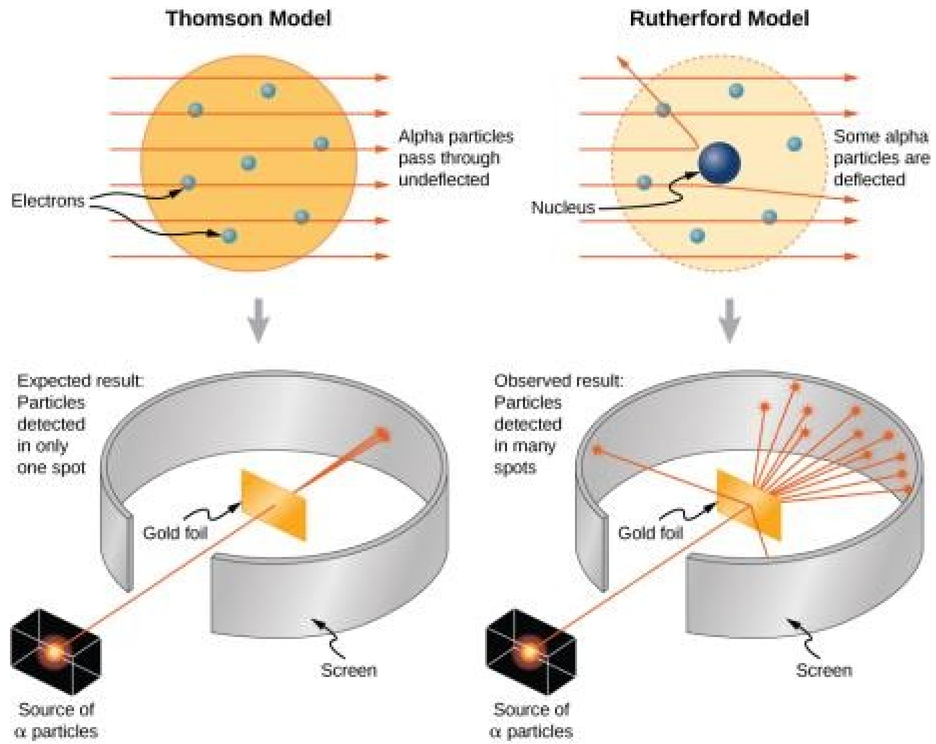
(Remember that the masses of the pucks are equal.) Substituting numbers:

$$\begin{aligned}-(2.5\ \mathrm{m/s})\hat{\mathbf{i}} + \vec{\mathbf{v}}_{\mathrm{b_f}} &= -(2.5\ \mathrm{m/s})\hat{\mathbf{i}} \\ \vec{\mathbf{v}}_{\mathrm{b_f}} &= 0.\end{aligned}$$

MOVING A COMET

An artist’s rendering of *Philae* landing on a comet. (credit: modification of work by “DLR German Aerospace Center”/Flickr)

COLLISIONS



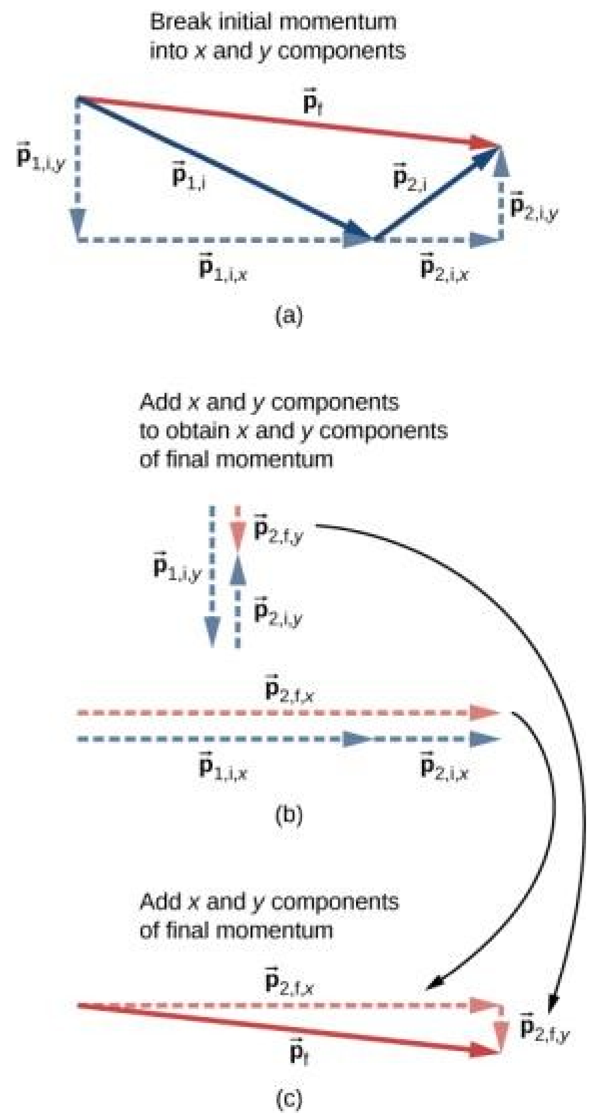
The Thomson and Rutherford models of the atom. The Thomson model predicted that nearly all of the incident alpha-particles would be scattered and at small angles. Rutherford and Geiger found that nearly none of the alpha particles were scattered, but those few that were deflected did so through very large angles. The results of Rutherford's experiments were inconsistent with the Thomson model. Rutherford used conservation of momentum and energy to develop a new, and better model of the atom—the nuclear model.

FIGURE 9.22

(a) For two-dimensional momentum problems, break the initial momentum vectors into their *x*- and *y*-components.

(b) Add the *x*- and *y*-components together separately. This gives you the *x*- and *y*-components of the final momentum, which are shown as red dashed vectors.

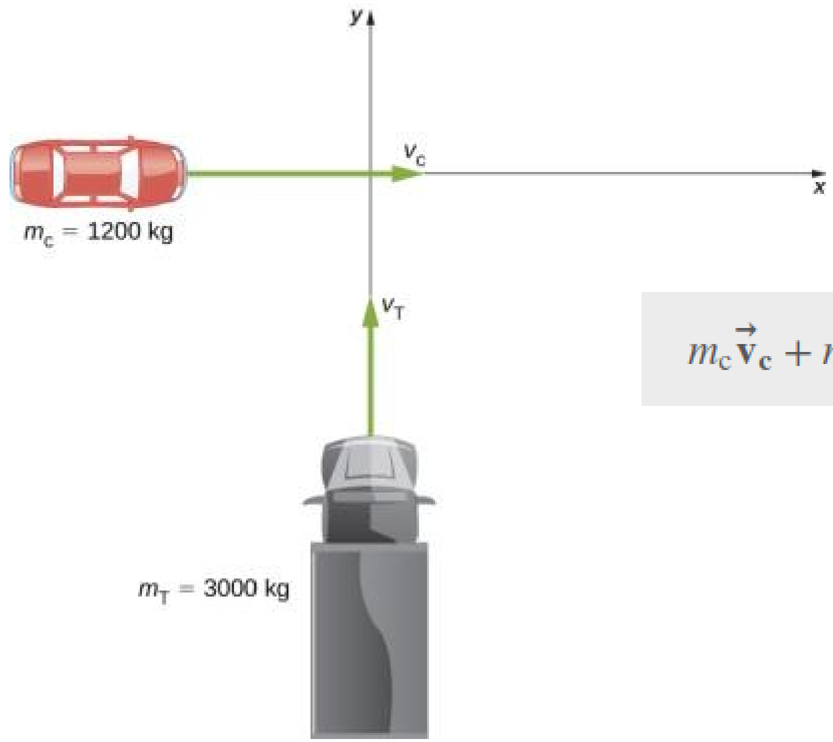
(c) Adding these components together gives the final momentum.



EXAMPLE

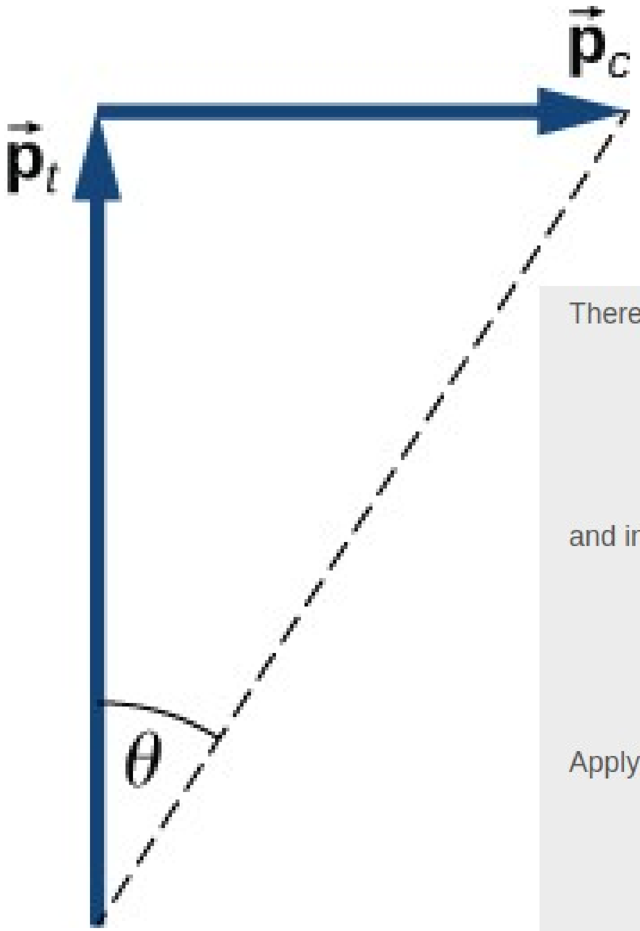
Traffic Collision

A small car of mass 1200 kg traveling east at 60 km/hr collides at an intersection with a truck of mass 3000 kg that is traveling due north at 40 km/hr (Figure 9.23). The two vehicles are locked together. What is the velocity of the combined wreckage?



$$m_c \vec{\mathbf{v}}_c + m_T \vec{\mathbf{v}}_T = (m_c + m_T) \vec{\mathbf{v}}_w.$$

A large truck moving north is about to collide with a small car moving east. The final momentum vector has both *x*- and *y*-components.



Graphical addition of momentum vectors. Notice that, although the car's velocity is larger than the truck's, its momentum is smaller.

Therefore, in the x-direction:

$$\begin{aligned} m_c v_c &= (m_c + m_T)\, v_{w,x} \\ v_{w,x} &= \left(\frac{m_c}{m_c+m_T}\right) v_c \end{aligned}$$

and in the y-direction:

$$\begin{aligned} m_T v_T &= (m_c + m_T)\, v_{w,y} \\ v_{w,y} &= \left(\frac{m_T}{m_c+m_T}\right) v_T. \end{aligned}$$

Applying the Pythagorean theorem gives

$$\begin{aligned} \left|\vec{\mathbf{v}}_w\right| &= \sqrt{\left[\left(\frac{m_c}{m_c+m_t}\right) v_c\right]^2 + \left[\left(\frac{m_t}{m_c+m_t}\right) v_t\right]^2} \\ &= \sqrt{\left[\left(\frac{1200\text{ kg}}{4200\text{ kg}}\right)\left(16.67\ \frac{\text{m}}{\text{s}}\right)\right]^2 + \left[\left(\frac{3000\text{ kg}}{4200\text{ kg}}\right)\left(11.1\ \frac{\text{m}}{\text{s}}\right)\right]^2} \\ &= \sqrt{\left(4.76\ \frac{\text{m}}{\text{s}}\right)^2 + \left(7.93\ \frac{\text{m}}{\text{s}}\right)^2} \\ &= 9.25\ \frac{\text{m}}{\text{s}} \approx 33.3\ \frac{\text{km}}{\text{hr}}. \end{aligned}$$

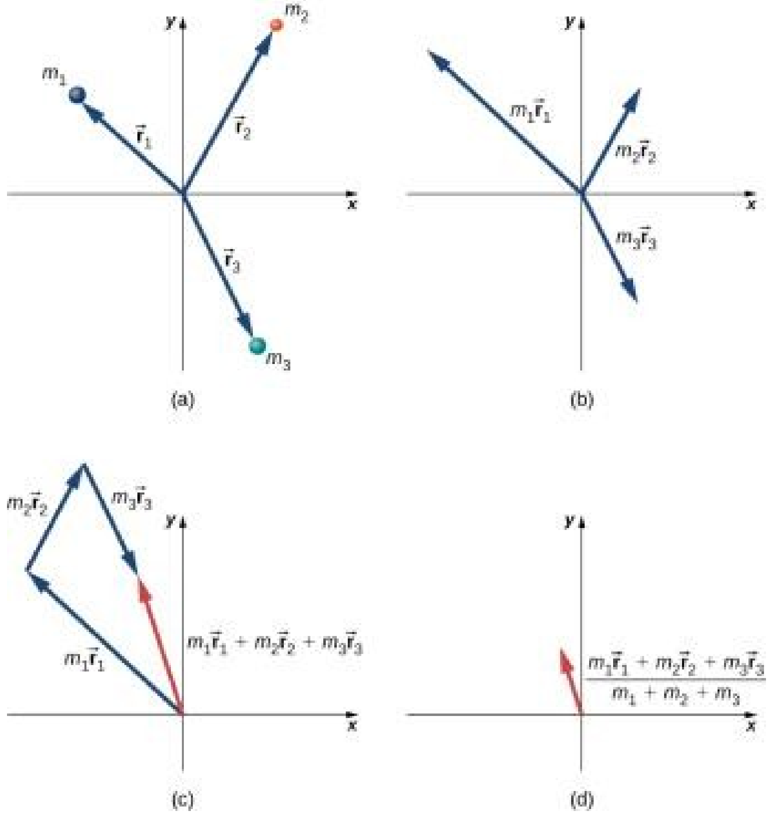
As for its direction, using the angle shown in the figure,

$$\theta = \tan^{-1}\left(\frac{v_{w,x}}{v_{w,y}}\right) = \tan^{-1}\left(\frac{7.93\text{ m/s}}{4.76\text{ m/s}}\right) = 59°.$$

CENTER OF MASS

$$M = \sum_{j=1}^{N} m_j.$$

$$\vec{\mathbf{r}}_{CM} \equiv \frac{1}{M}\sum_{j=1}^{N} m_j \vec{\mathbf{r}}_j.$$



$$r_{CM,x} = \frac{1}{M}\sum_{j=1}^{N} m_j x_j$$

$$r_{CM,y} = \frac{1}{M}\sum_{j=1}^{N} m_j y_j$$

$$r_{CM,z} = \frac{1}{M}\sum_{j=1}^{N} m_j z_j$$

$$\vec{\mathbf{r}}_{CM} = r_{CM,x}\hat{\mathbf{i}} + r_{CM,y}\hat{\mathbf{j}} + r_{CM,z}\hat{\mathbf{k}}$$

$$r_{CM} = \left|\vec{\mathbf{r}}_{CM}\right| = \left(r_{CM,x}^2 + r_{CM,y}^2 + r_{CM,z}^2\right)^{1/2}.$$

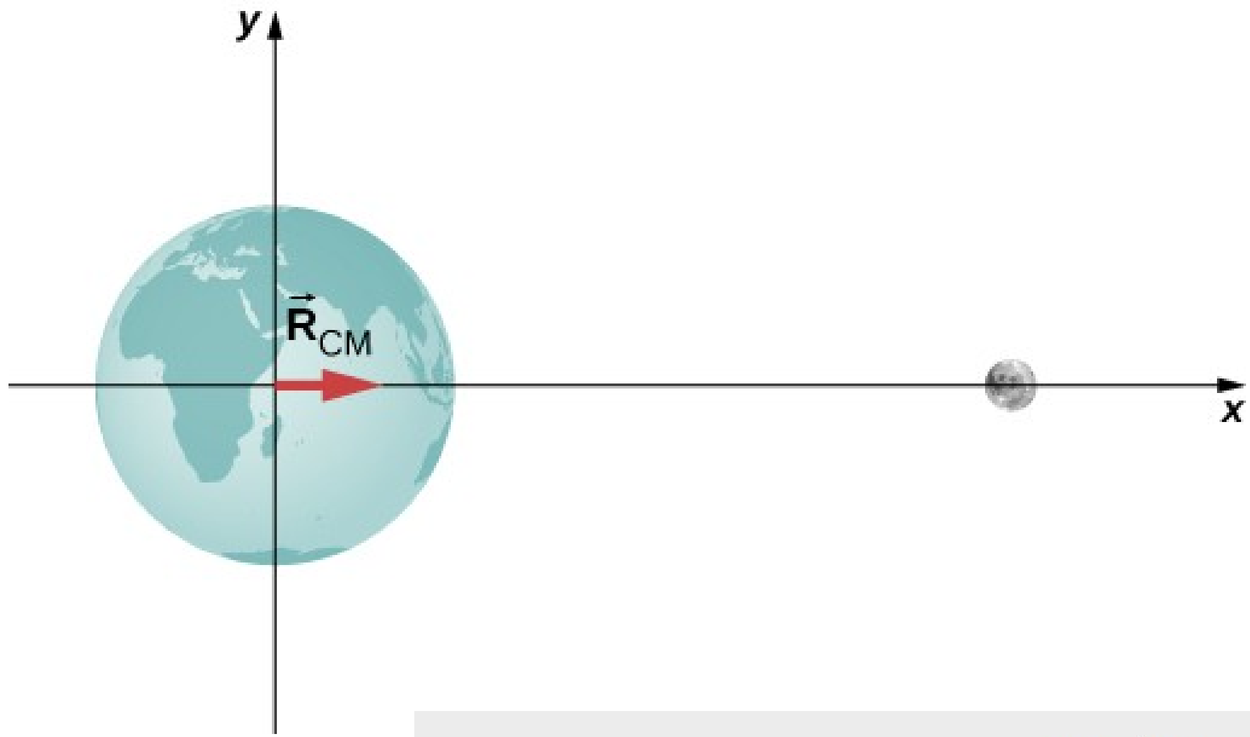
Finding the center of mass of a system of three different particles.

(a) Position vectors are created for each object.

(b) The position vectors are multiplied by the mass of the corresponding object.

(c) The scaled vectors from part (b) are added together.

(d) The final vector is divided by the total mass. This vector points to the center of mass of the system. Note that no mass is actually present at the center of mass of this system.



$$R = \frac{m_e r_e + m_m r_m}{m_e + m_m}.$$

From Appendix D,

$$m_e = 5.97 \times 10^{24}\ \text{kg}$$

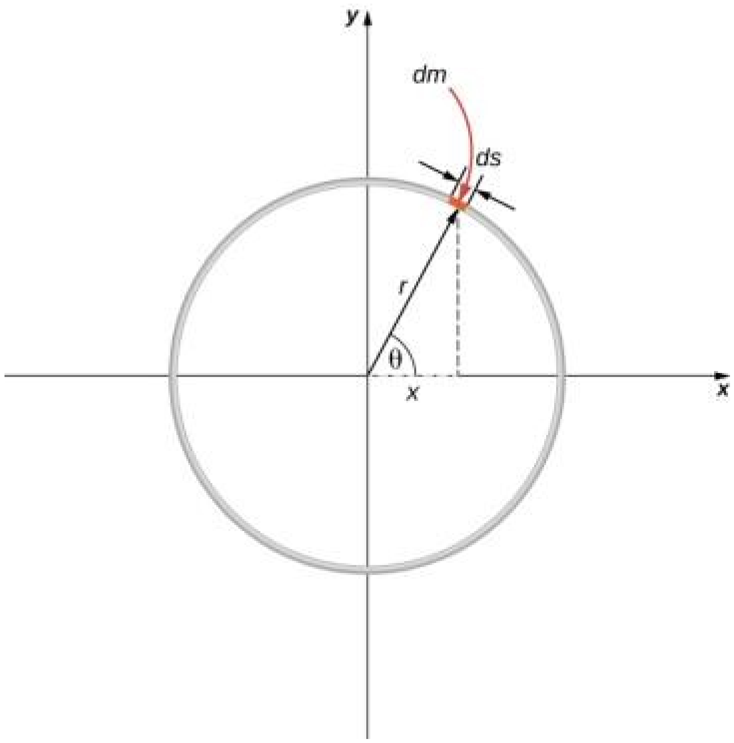
$$m_m = 7.36 \times 10^{22}\ \text{kg}$$

$$r_m = 3.82 \times 10^{8}\ \text{m}.$$

We defined the center of Earth as the origin, so $r_e = 0\ \text{m}$. Inserting these into the equation for *R* gives

$$\begin{aligned} R &= \frac{(5.97 \times 10^{24}\ \text{kg})(0\ \text{m}) + (7.36 \times 10^{22}\ \text{kg})(3.82 \times 10^{8}\ \text{m})}{5.97 \times 10^{24}\ \text{kg} + 7.36 \times 10^{22}\ \text{kg}} \\ &= 4.64 \times 10^{6}\ \text{m}. \end{aligned}$$

CENTER OF MASS OF EXTENDED OBJECTS



Finding the center of mass of a uniform hoop. We express the coordinates of a differential piece of the hoop, and then integrate around the hoop.

$$\vec{\mathbf{v}}_{\mathrm{CM,f}} = \vec{\mathbf{v}}_{\mathrm{CM,i}}. \tag{9.37}$$

That is to say, *in the absence of an external force*, *the velocity of the center of mass never changes*.

$$\vec{\mathbf{F}} = \frac{d\vec{\mathbf{p}}_{\mathrm{CM}}}{dt}.$$

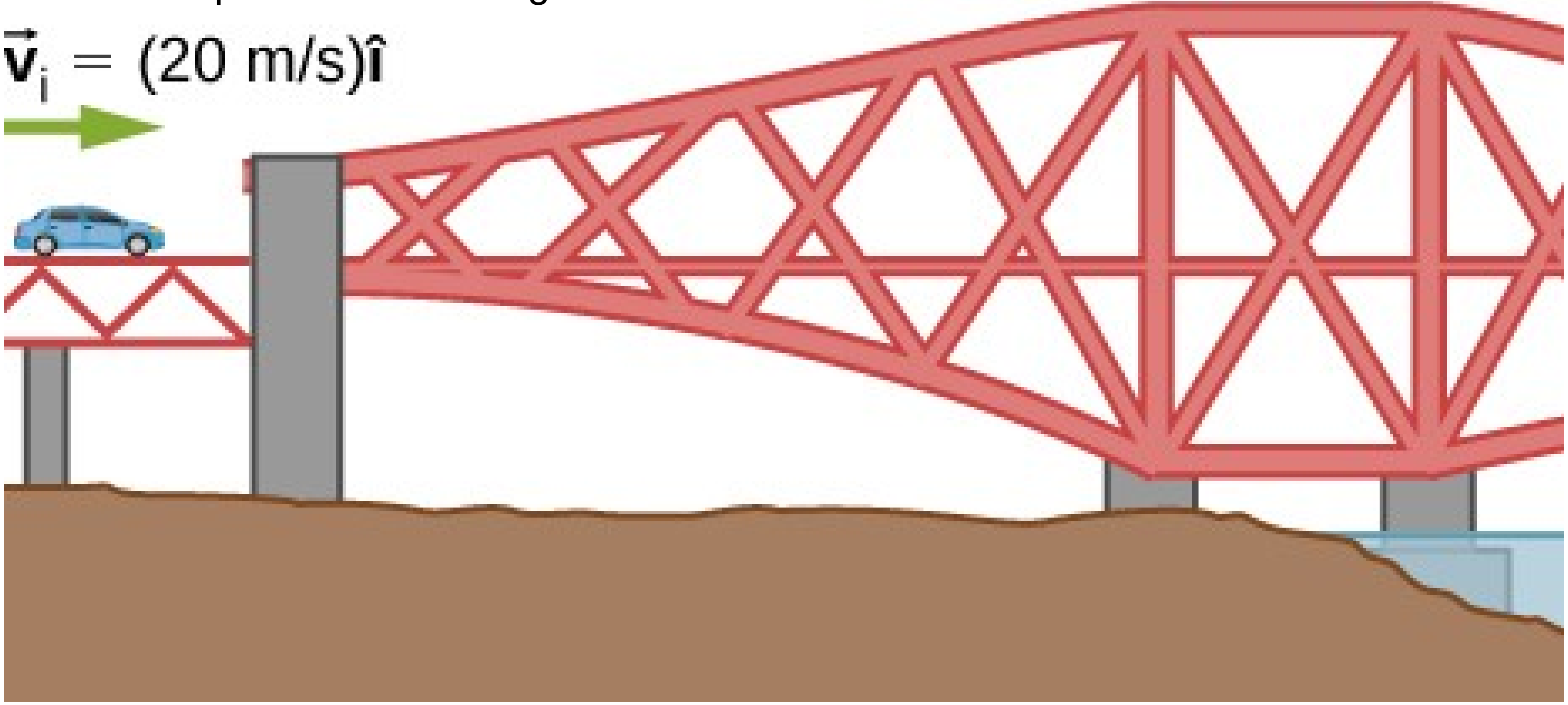
These exploding fireworks are a vivid example of conservation of momentum and the motion of the center of mass.

EXERCISE 25

A 75.0-kg person is riding in a car moving at 20.0 m/s when the car runs into a bridge abutment

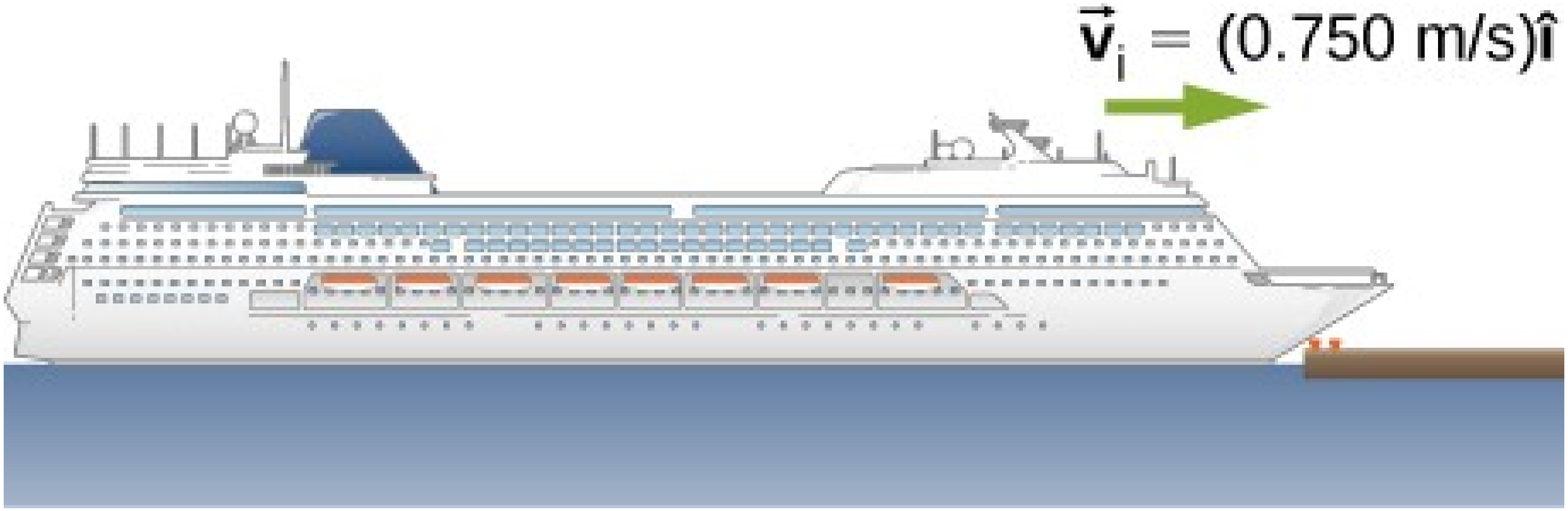
a) Calculate the average force on the person if he is stopped by a padded dashboard that compresses an average of 1.00 cm.

b) Calculate the average force on the person if he is stopped by an air bag that compresses an average of 15.0 cm.



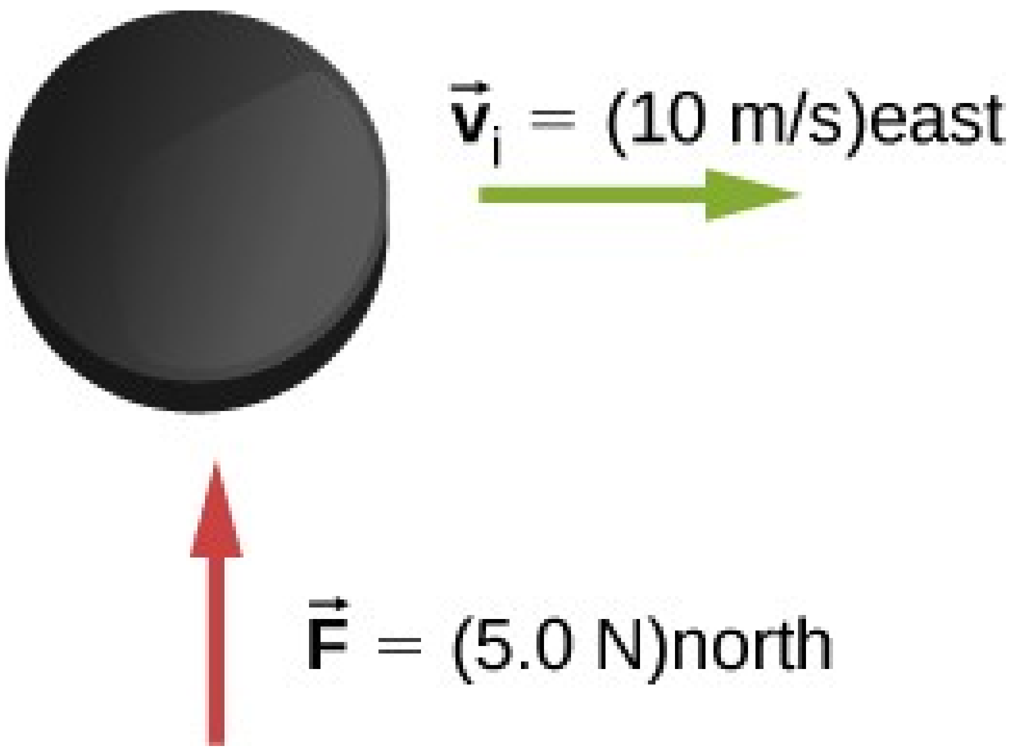
EXERCISE 27

A cruise ship with a mass of 1.00×10^7 kg strikes a pier at a speed of 0.750 m/s. It comes to rest after traveling 6.00 m, damaging the ship, the pier, and the tugboat captain's finances. Calculate the average force exerted on the pier using the concept of impulse. (*Hint*: First calculate the time it took to bring the ship to rest, assuming a constant force.)

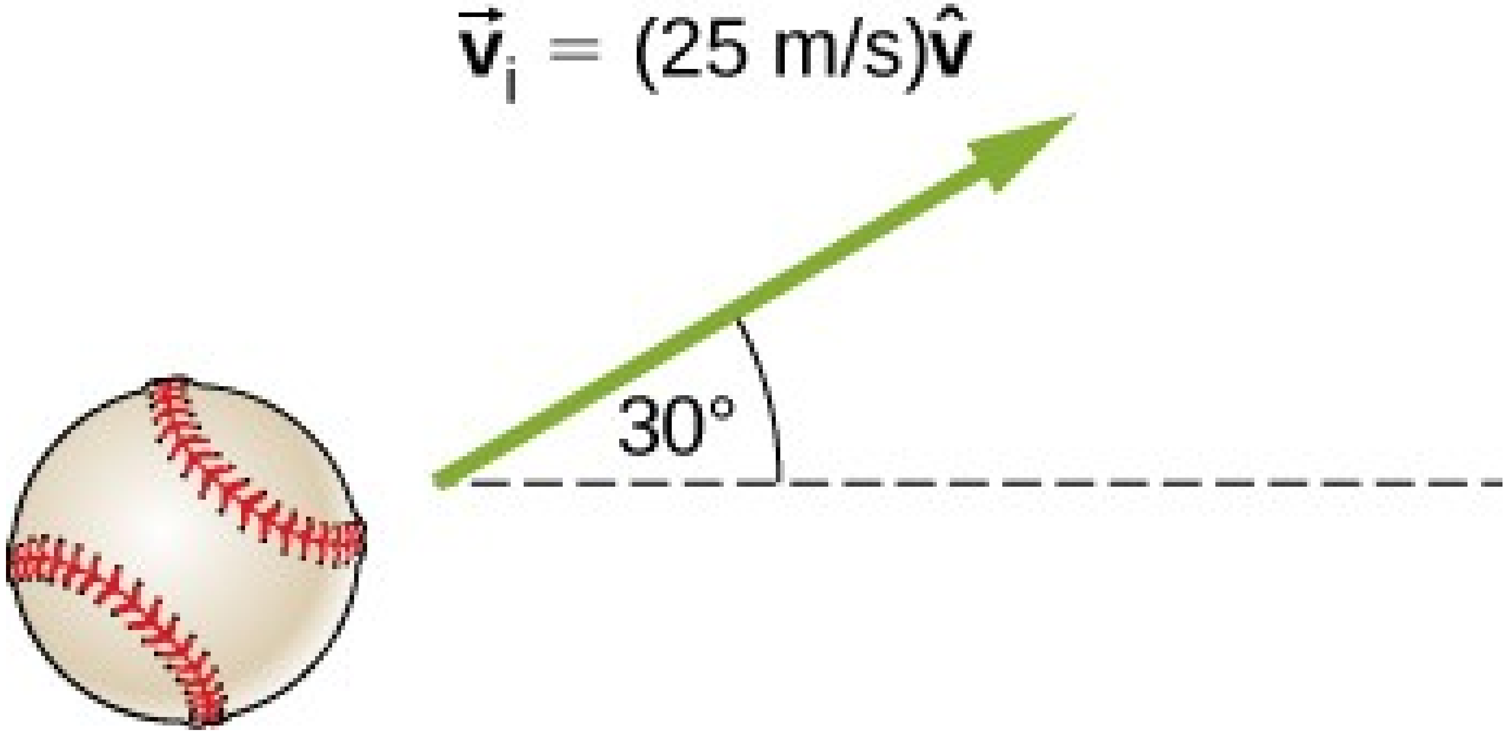


EXERCISE 33

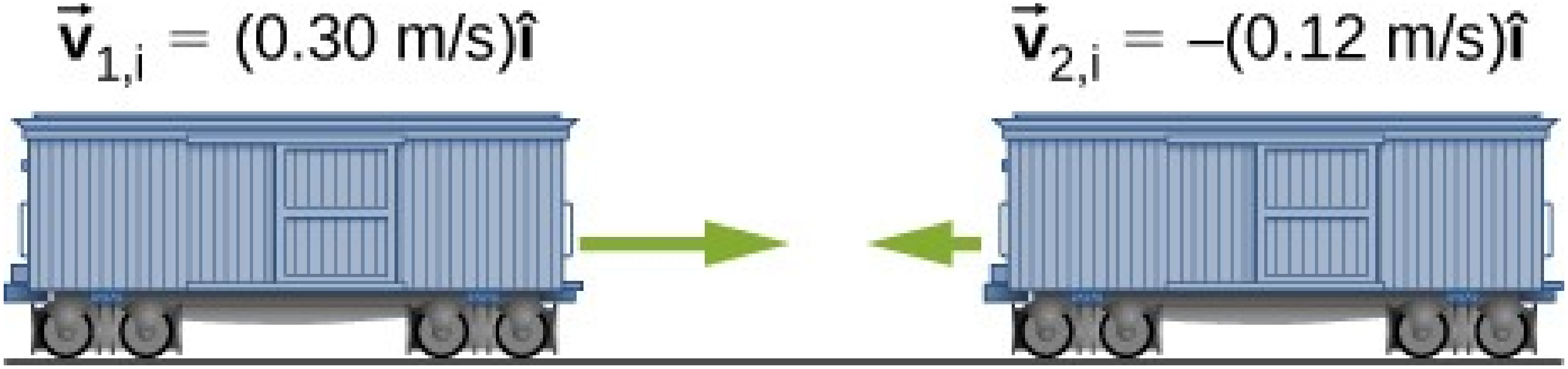
A hockey puck of mass 150 g is sliding due east on a frictionless table with a speed of 10 m/s. Suddenly, a constant force of magnitude 5 N and direction due north is applied to the puck for 1.5 s. Find the north and east components of the momentum at the end of the 1.5-s interval.



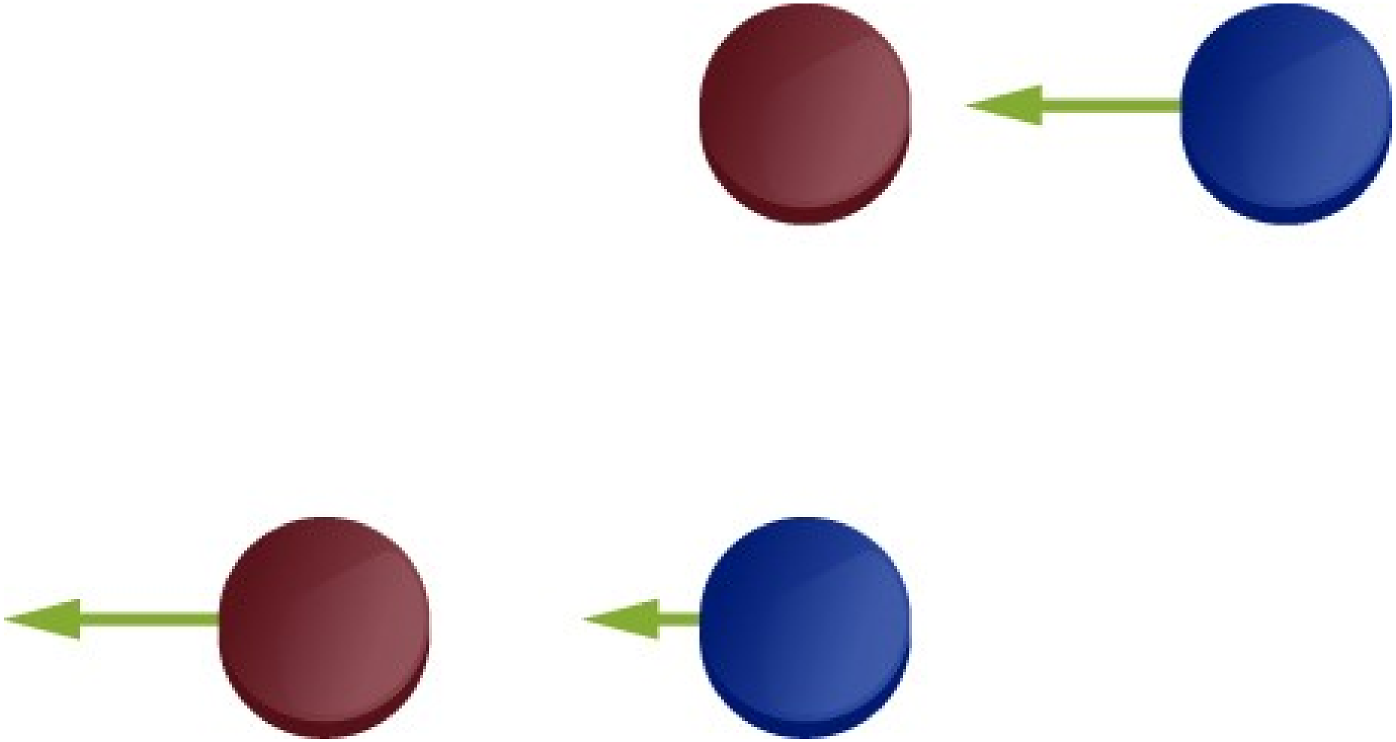
EXERCISE 34



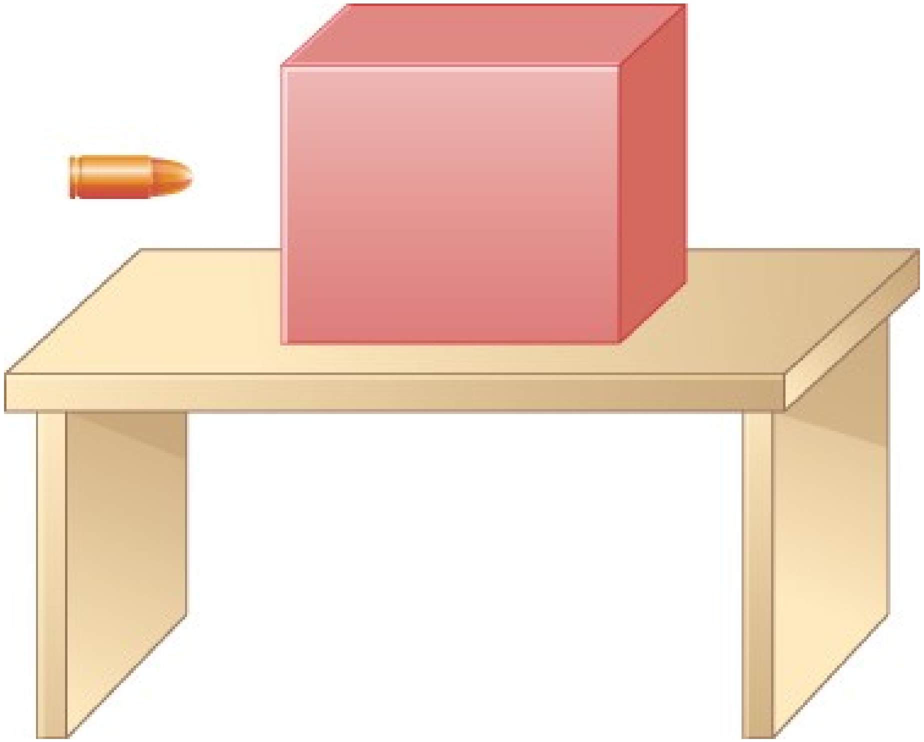
EXERCISE 35



EXERCISE 36

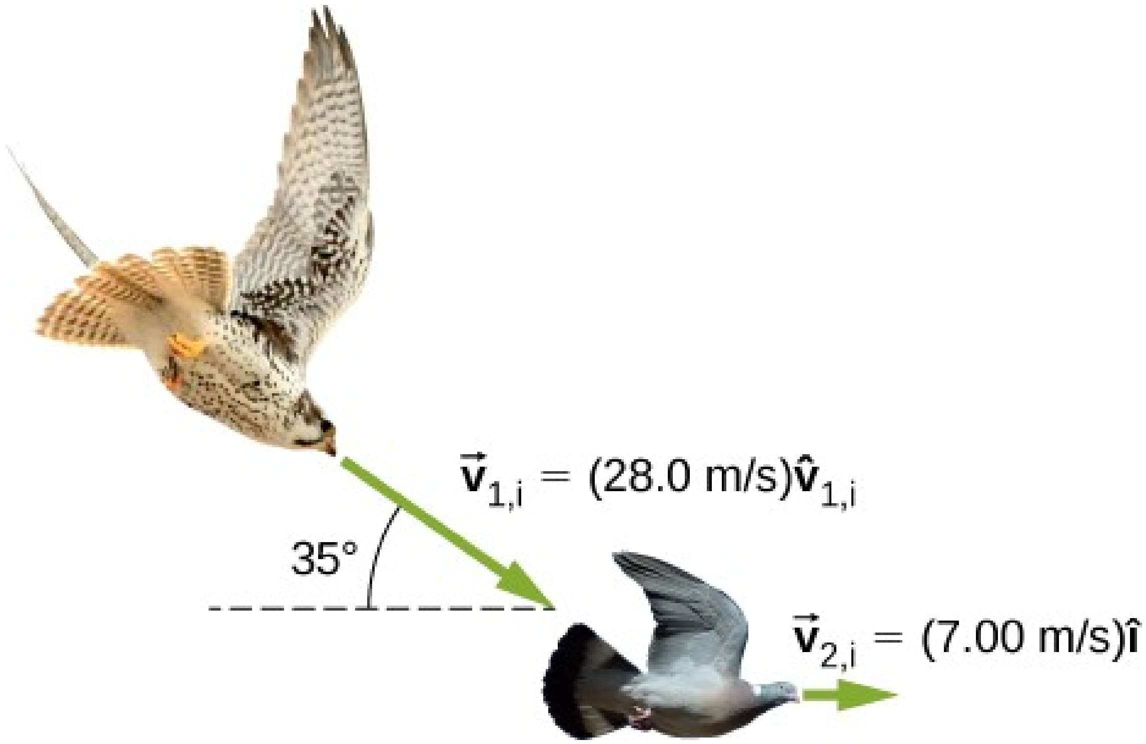


EXERCISE 37



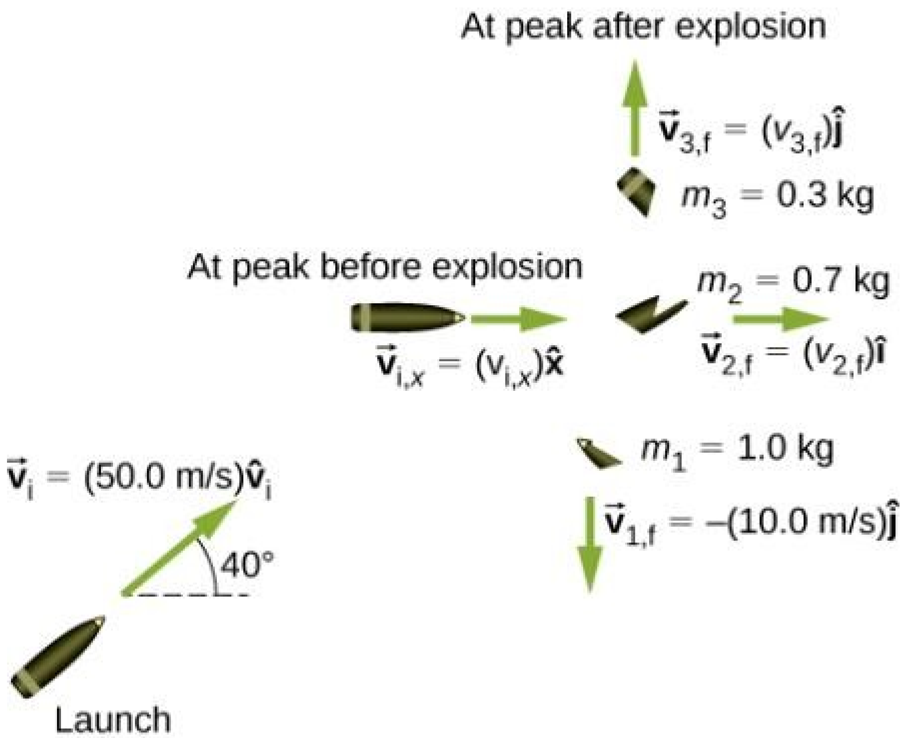
EXERCISE 44

FIGURE 9.34



(credit “hawk”: modification of work by “USFWS Mountain-Prairie”/Flickr; credit “dove”: modification of work by Jacob Spinks)

EXERCISE 55



At peak after explosion
$\vec{v}_{3,f} = (v_{3,f})\hat{j}$
$m_3 = 0.3$ kg
At peak before explosion
$m_2 = 0.7$ kg
$\vec{v}_{i,x} = (v_{i,x})\hat{x}$
$\vec{v}_{2,f} = (v_{2,f})\hat{i}$
$m_1 = 1.0$ kg
$\vec{v}_i = (50.0 \text{ m/s})\hat{v}_i$
$\vec{v}_{1,f} = -(10.0 \text{ m/s})\hat{j}$
40°
Launch

EXERCISE 63

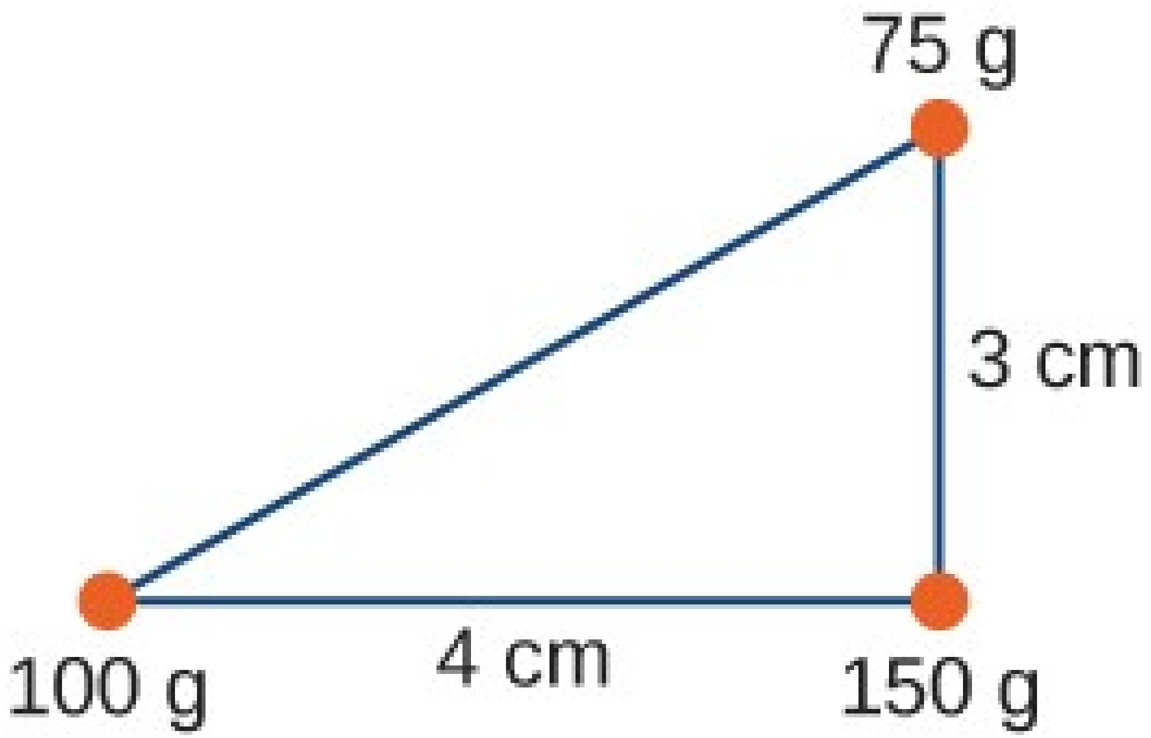
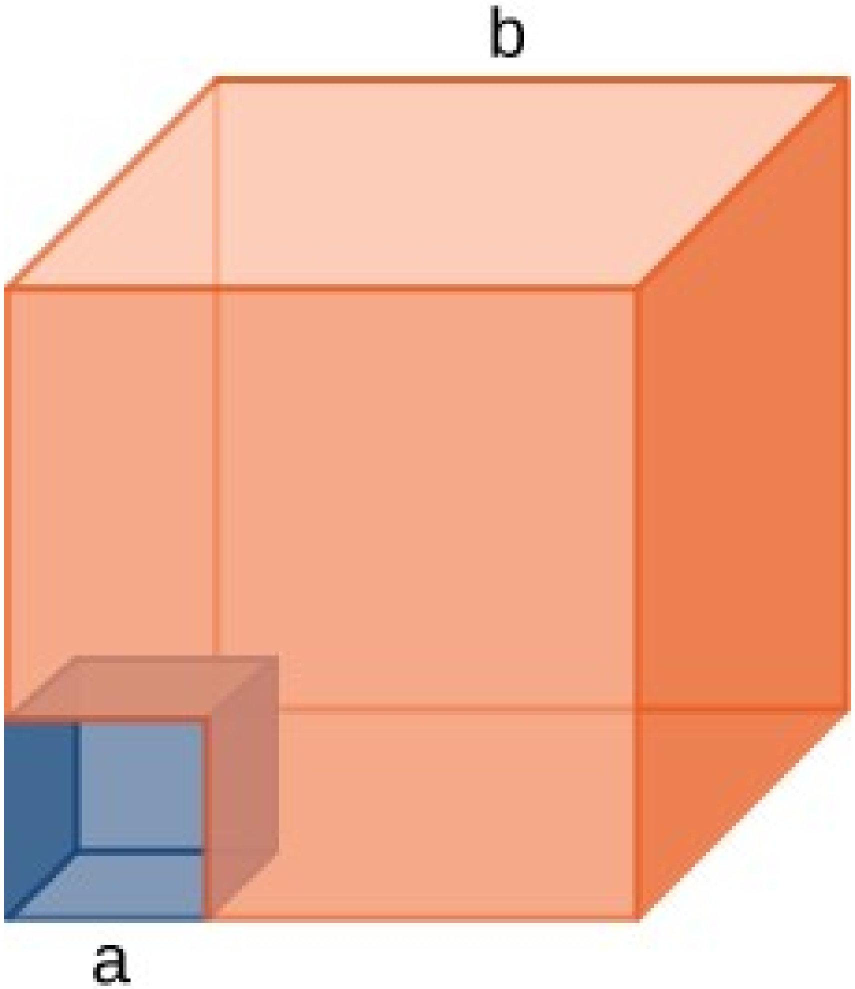
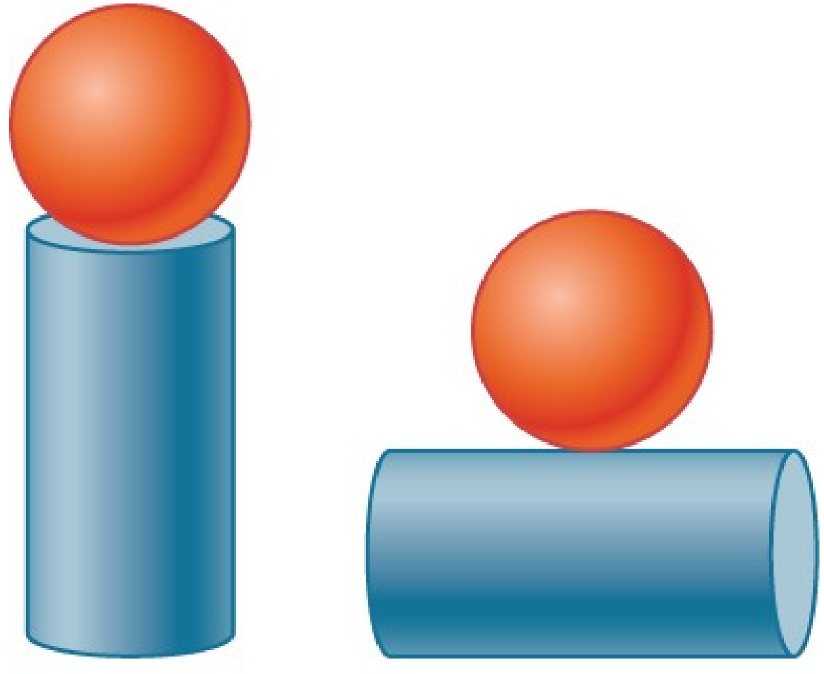


FIGURE 72

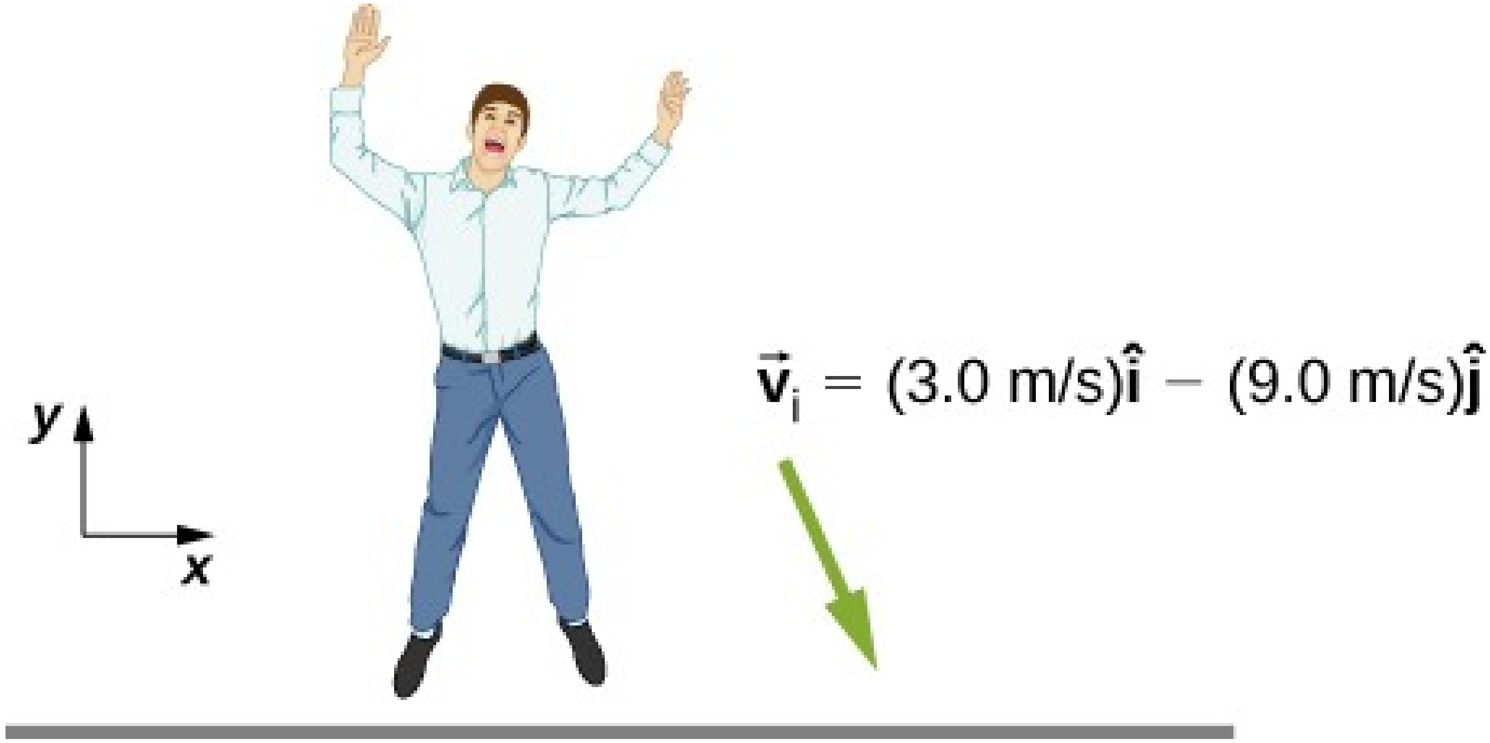


EXERCISE 76

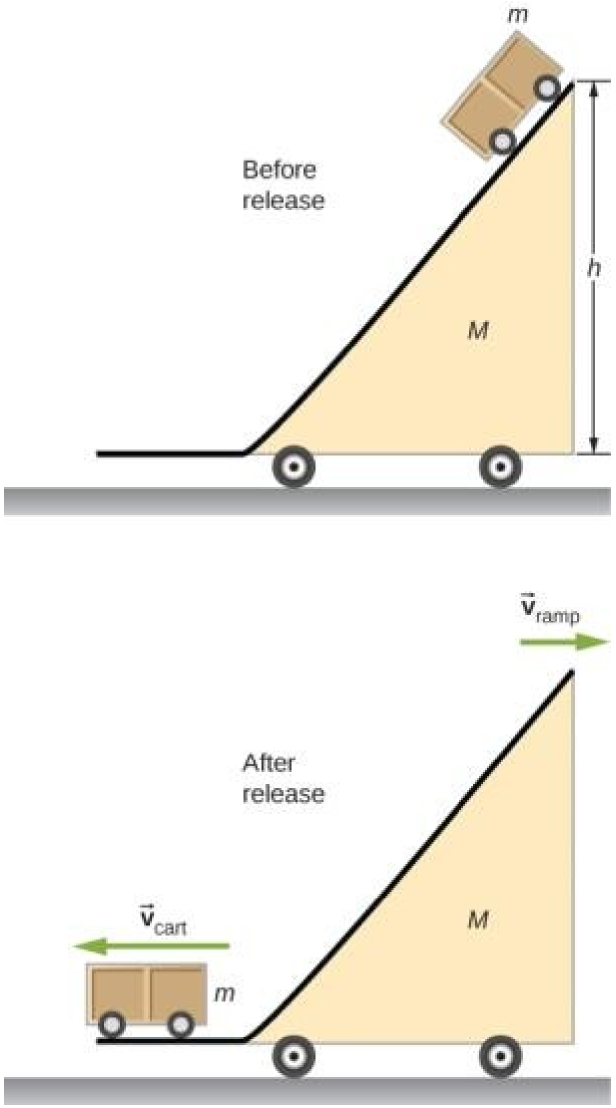


(a) (b)

EXERCISE 114



EXERCISE 117



EXERCISE 118