ANNOUNCEMENTS

- Homework #7, due Wednesday, October 10

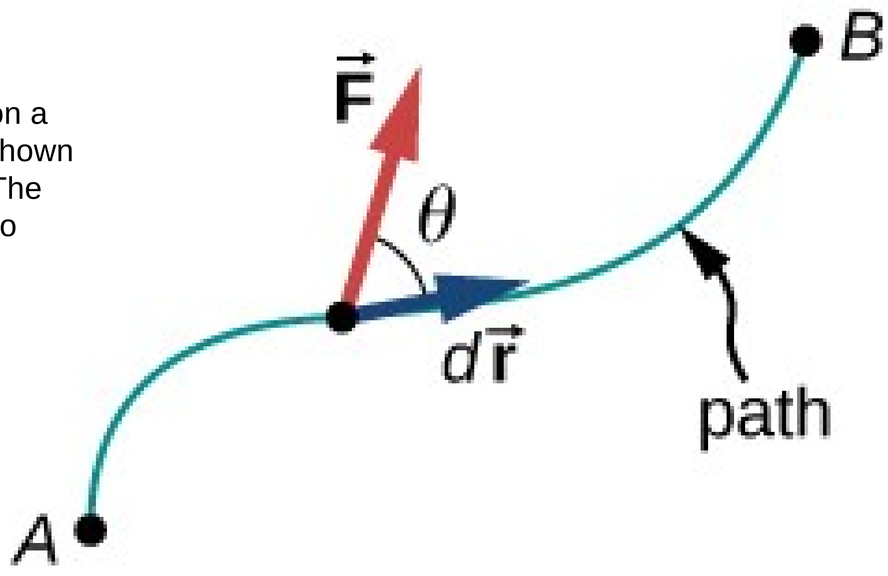
  Conceptual questions: Chapter 7, #4 and #8
  Problems: Chapter 7, #26, #28

- 5-minute quiz on Chapter 7: Friday, October 12

- Read all Chapter 7 by Wednesday.

WORK AND ENERGY

Vectors used to define work. The force acting on a particle and its infinitesimal displacement are shown at one point along the path between *A* and *B*. The infinitesimal work is the dot product of these two vectors; the total work is the integral of the dot product along the path.



$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \left|\vec{\mathbf{F}}\right| \left|d\vec{\mathbf{r}}\right| \cos\theta. \tag{7.1}$$

WORK DONE BY A FORCE

The work done by a force is the integral of the force with respect to displacement along the path of the displacement:

$$W_{AB} = \int_{\text{path } AB} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}. \tag{7.2}$$

WORK OF A CONSTANT FORCE

$$W_{AB} = \vec{\mathbf{F}} \cdot \int_A^B d\vec{\mathbf{r}} = \vec{\mathbf{F}} \cdot \left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = \left|\vec{\mathbf{F}}\right| \left|\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right| \cos\theta \quad \text{(constant force)}.$$

WORK DONE BY GRAVITY

$$W_{\text{grav},AB} = -mg\hat{\mathbf{j}} \cdot \left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = -mg\left(y_B - y_A\right).$$

WORK DONE BY A SPRING

$$W_{\text{spring},AB} = \int_A^B F_x dx = -k\int_A^B x dx = -k\frac{x^2}{2}\bigg|_A^B = -\frac{1}{2}k\left(x_B^2 - x_A^2\right).$$

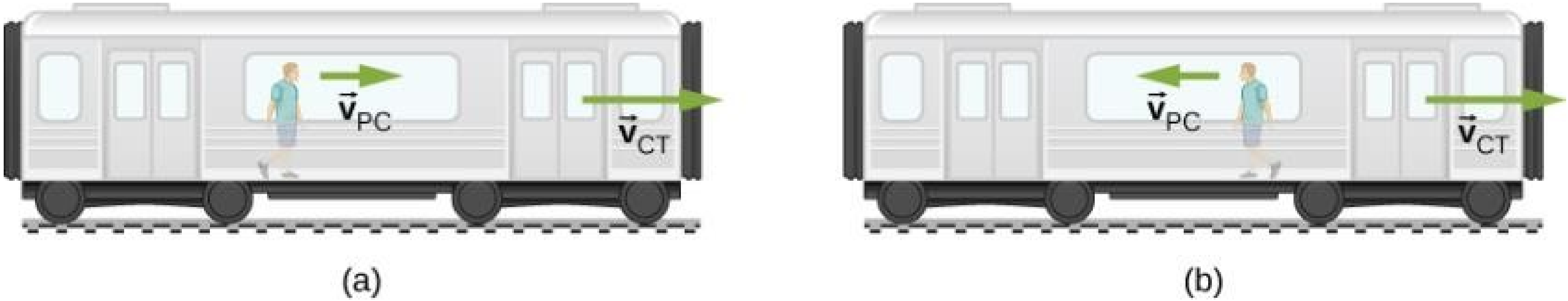
KINETIC ENERGY

The kinetic energy of a particle is one-half the product of the particle's mass *m* and the square of its speed *v*:

$$K = \frac{1}{2}mv^2.$$

Kinetic Energy Relative to Different Frames

A 75.0-kg person walks down the central aisle of a subway car at a speed of 1.50 m/s relative to the car, whereas the train is moving at 15.0 m/s relative to the tracks. (a) What is the person's kinetic energy relative to the car? (b) What is the person's kinetic energy relative to the tracks? (c) What is the person's kinetic energy relative to a frame moving with the person?



Solution

a. $K = \frac{1}{2}(75.0\ \text{kg})(1.50\ \text{m/s})^2 = 84.4\ \text{J}.$

b. $v_{PT} = (15.0 \pm 1.50)\ \text{m/s}$. Therefore, the two possible values for kinetic energy relative to the car are

$$K = \frac{1}{2}(75.0\ \text{kg})(13.5\ \text{m/s})^2 = 6.83\ \text{kJ}$$

and

$$K = \frac{1}{2}(75.0\ \text{kg})(16.5\ \text{m/s})^2 = 10.2\ \text{kJ}.$$

c. In a frame where $v_P = 0, K = 0$ as well.

WORK-ENERGY THEOREM

The net work done on a particle equals the change in the particle's kinetic energy:

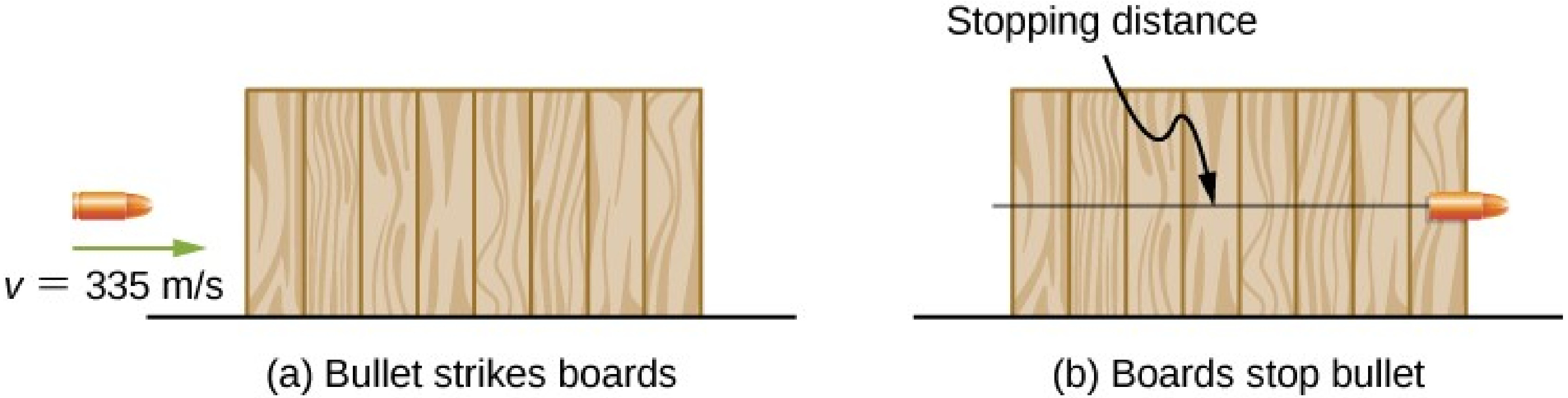
$$W_{\text{net}} = K_B - K_A\,.$$

Problem-Solving Strategy: Work-Energy Theorem

1. Draw a free-body diagram for each force on the object.
2. Determine whether or not each force does work over the displacement in the diagram. Be sure to keep any positive or negative signs in the work done.
3. Add up the total amount of work done by each force.
4. Set this total work equal to the change in kinetic energy and solve for any unknown parameter.
5. Check your answers. If the object is traveling at a constant speed or zero acceleration, the total work done should be zero and match the change in kinetic energy. If the total work is positive, the object must have sped up or increased kinetic energy. If the total work is negative, the object must have slowed down or decreased kinetic energy.

Determining a Stopping Force

A bullet from a 0.22LR-caliber cartridge has a mass of 40 grains (2.60 g) and a muzzle velocity of 1100 ft./s (335 m/s). It can penetrate eight 1-inch pine boards, each with thickness 0.75 inches. What is the average stopping force exerted by the wood, as shown in Figure 7.13?



Solution

Applying the work-energy theorem, we get

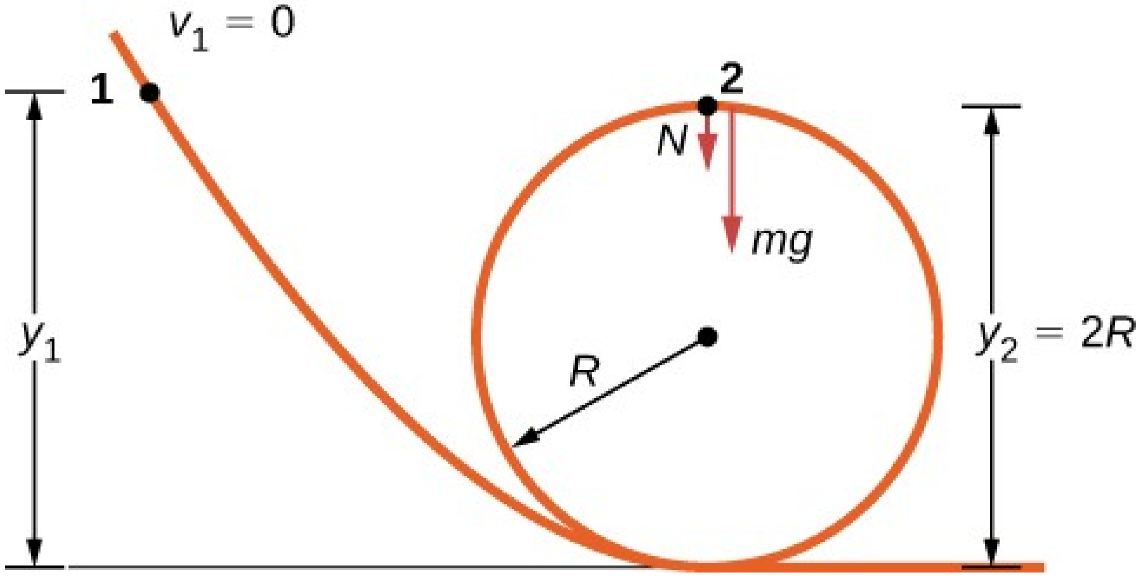
$$W_{\text{net}} = -F_{\text{ave}}\Delta s_{\text{stop}} = -K_{\text{initial}},$$

so

$$F_{\text{ave}} = \frac{\frac{1}{2}mv^2}{\Delta s_{\text{stop}}} = \frac{\frac{1}{2}(2.6 \times 10^{-3}\,\text{kg})(335\text{ m/s})^2}{0.152\text{ m}} = 960\text{ N}.$$

EXAMPLE

A frictionless track for a toy car has a loop-the-loop in it. How high must the car start so that it can go around the loop without falling off?



$$-mg(y_2 - y_1) = \frac{1}{2}m{v_2}^2,$$

$$a_{top} = \frac{F}{m} = \frac{N + mg}{m} = \frac{v_2^2}{R}.$$

The condition for maintaining contact with the track is that there must be some normal force, however slight; that is, $N > 0$. Substituting for v_2^2 and N, we can find the condition for y_1.

Solution

Implement the steps in the strategy to arrive at the desired result:

$$N = \frac{-mg + mv_2^2}{R} = \frac{-mg + 2mg(y_1 - 2R)}{R} > 0 \quad \text{or} \quad y_1 > \frac{5R}{2}.$$

POWER (RATE OF DOING WORK)

POWER

Power is defined as the rate of doing work, or the limit of the average power for time intervals approaching zero,

$$P = \frac{dW}{dt}. \quad (7.11)$$

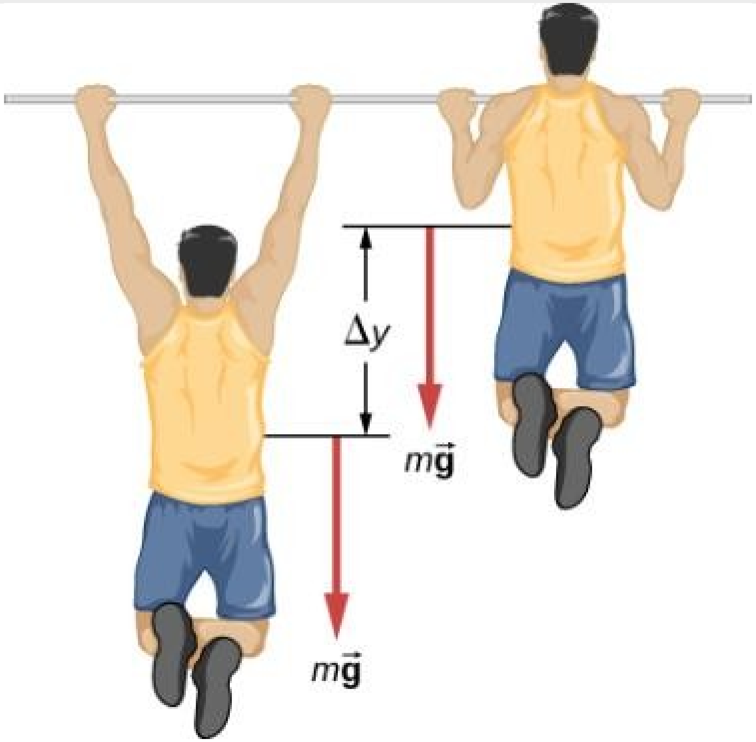
$$W = \int P dt.$$

$$P = \frac{dW}{dt} = \frac{\vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}}{dt} = \vec{\mathbf{F}} \cdot \left(\frac{d\vec{\mathbf{r}}}{dt}\right) = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}, \quad (7.12)$$

The work-energy theorem relates how work can be transformed into kinetic energy. Since there are other forms of energy as well, as we discuss in the next chapter, we can also define power as the rate of transfer of energy. Work and energy are measured in units of joules, so power is measured in units of joules per second, which has been given the SI name watts, abbreviation W: $1 \text{ J/s} = 1 \text{ W}$. Another common unit for expressing the power capability of everyday devices is horsepower: $1 \text{ hp} = 746 \text{ W}$.

Pull-Up Power

An 80-kg army trainee does 10 pull-ups in 10 s (Figure 7.14). How much average power do the trainee's muscles supply moving his body? (*Hint:* Make reasonable estimates for any quantities needed.)



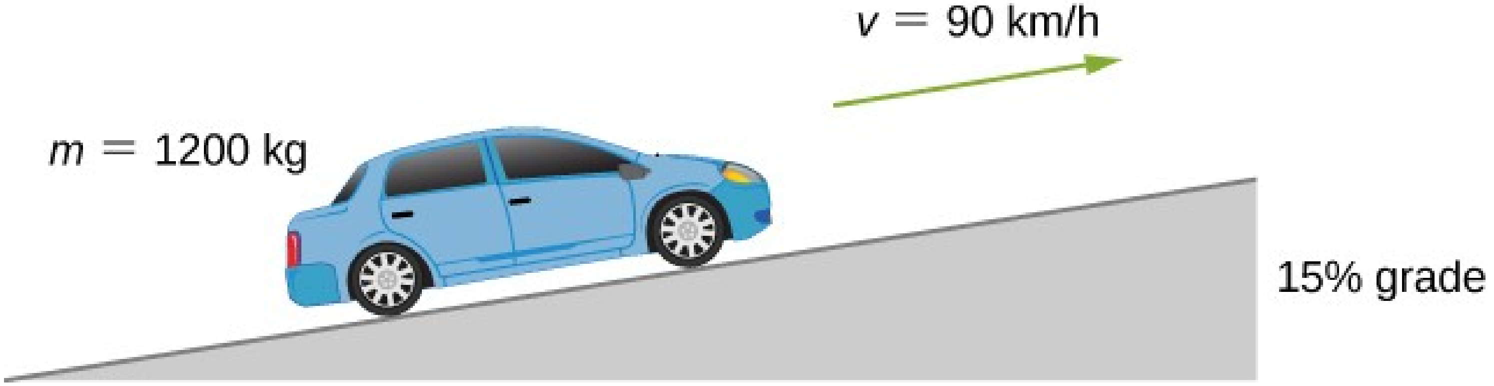
Solution

The result we get, applying our assumptions, is

$$P_{\text{ave}} = \frac{10 \times 2(0.9 \times 80\ \text{kg})(9.8\ \text{m/s}^2)(0.6\ \text{m})}{10\ \text{s}} = 850\ \text{W}.$$

Automotive Power Driving Uphill

How much power must an automobile engine expend to move a 1200-kg car up a 15% grade at 90 km/h (Figure 7.15)? Assume that 25% of this power is dissipated overcoming air resistance and friction.



Strategy

At constant velocity, there is no change in kinetic energy, so the net work done to move the car is zero. Therefore the power supplied by the engine to move the car equals the power expended against gravity and air resistance. By assumption, 75% of the power is supplied against gravity, which equals $m\vec{\mathbf{g}} \cdot \vec{\mathbf{v}} = mgv \sin \theta$, where θ is the angle of the incline. A 15% grade means $\tan \theta = 0.15$. This reasoning allows us to solve for the power required.

Solution

Carrying out the suggested steps, we find

$$0.75\, P = mgv \sin(\tan^{-1} 0.15),$$

or

$$P = \frac{(1200 \times 9.8 \text{ N})(90 \text{ m}/3.6 \text{ s})\sin(8.53°)}{0.75} = 58 \text{ kW},$$

or about 78 hp. (You should supply the steps used to convert units.)