ANNOUNCEMENTS

- Homework #7, due Wednesday, October 10

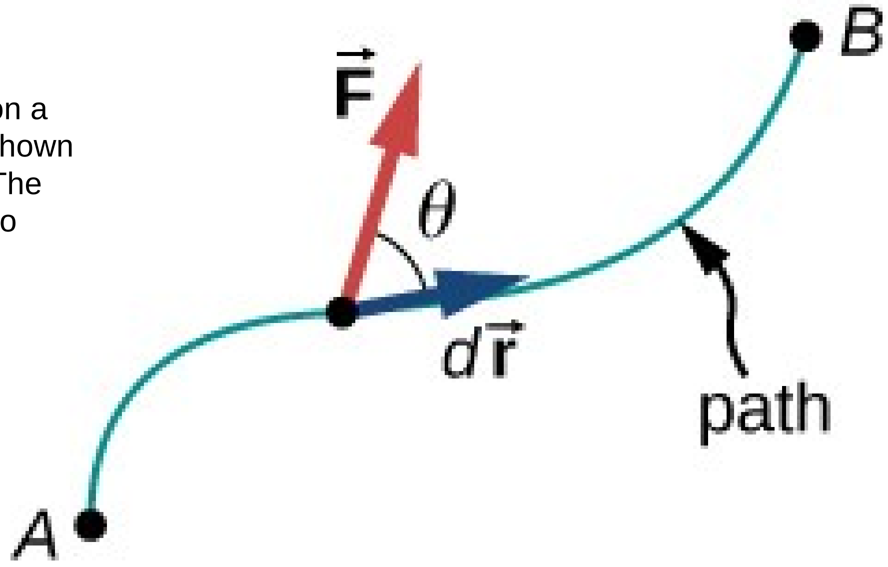
  Conceptual questions: Chapter 7, #4 and #8
  Problems: Chapter 7, #26, #28

- 5-minute quiz on Chapter 6: Friday, October 5

- Read all Chapter 7 by Wednesday.

WORK AND ENERGY

Vectors used to define work. The force acting on a particle and its infinitesimal displacement are shown at one point along the path between *A* and *B*. The infinitesimal work is the dot product of these two vectors; the total work is the integral of the dot product along the path.



$$dW = \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \left|\vec{\mathbf{F}}\right| \left|d\vec{\mathbf{r}}\right| \cos\theta. \tag{7.1}$$

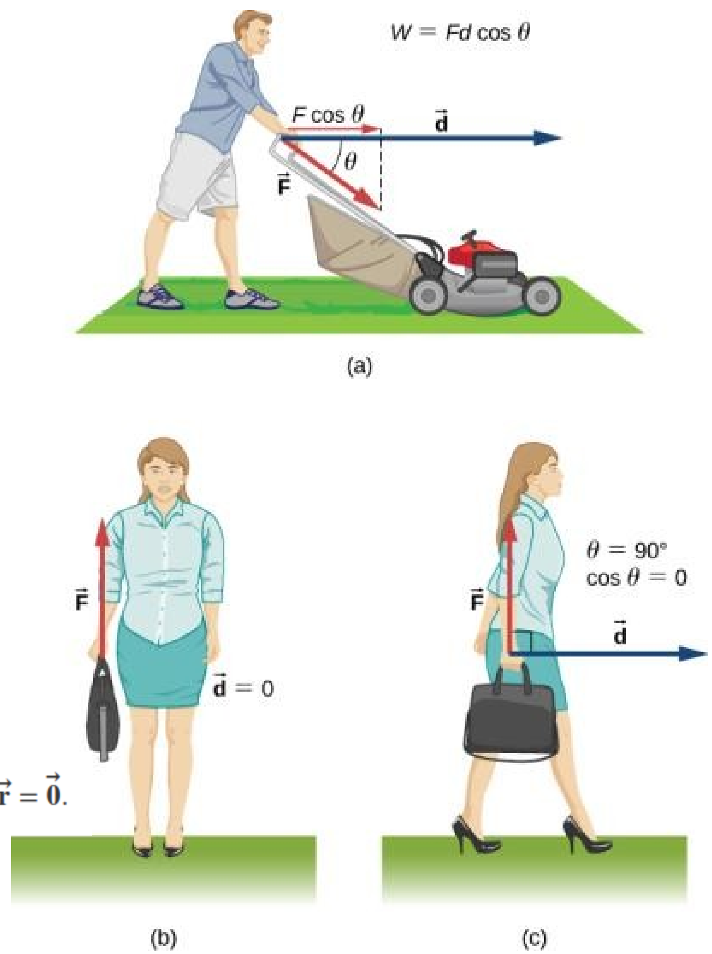
WORK DONE BY A FORCE

The work done by a force is the integral of the force with respect to displacement along the path of the displacement:

$$W_{AB} = \int_{\text{path } AB} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}. \tag{7.2}$$

WORK OF A CONSTANT FORCE

$$W_{AB} = \vec{\mathbf{F}} \cdot \int_A^B d\vec{\mathbf{r}} = \vec{\mathbf{F}} \cdot \left(\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right) = \left|\vec{\mathbf{F}}\right| \left|\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A\right| \cos\theta \quad \text{(constant force).}$$



$$dW_{\mathrm{N}} = \vec{\mathbf{N}} \cdot d\vec{\mathbf{r}} = \vec{\mathbf{0}}.$$

Work done by a constant force.

(a) A person pushes a lawn mower with a constant force. The component of the force parallel to the displacement is the work done, as shown in the equation in the figure.

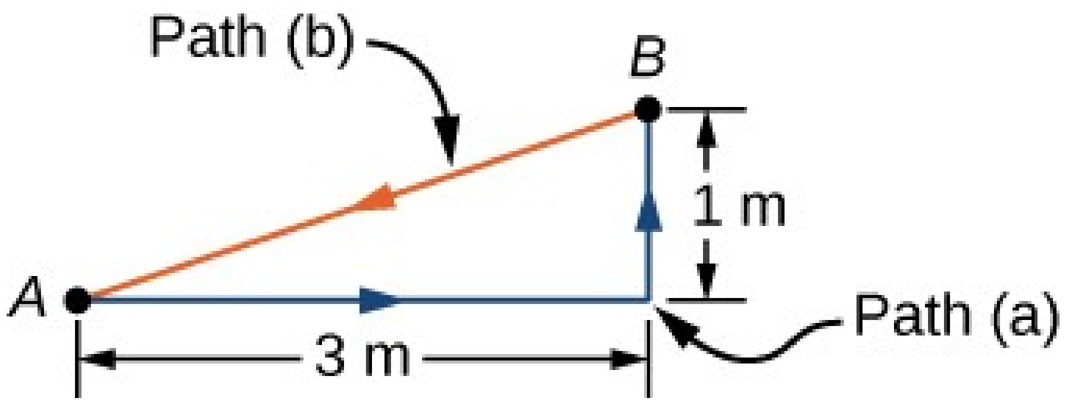
(b) A person holds a briefcase. No work is done because the displacement is zero.

(c) The person in (b) walks horizontally while holding the briefcase. No work is done because cos is zero.

WORK BY FRICTION

Moving a Couch

You decide to move your couch to a new position on your horizontal living room floor. The normal force on the couch is 1 kN and the coefficient of friction is 0.6. (a) You first push the couch 3 m parallel to a wall and then 1 m perpendicular to the wall (*A* to *B* in Figure 7.4). How much work is done by the frictional force? (b) You don't like the new position, so you move the couch straight back to its original position (*B* to *A* in Figure 7.4). What was the total work done against friction moving the couch away from its original position and back again?



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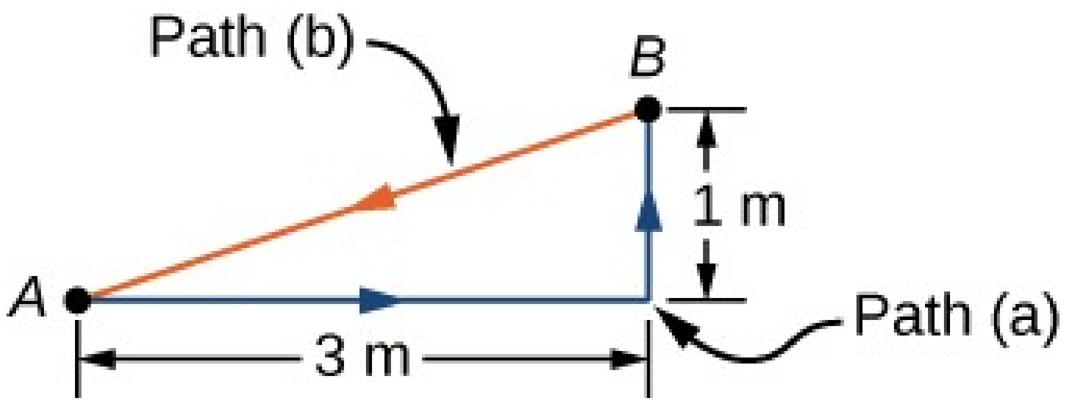


Solution

a. The work done by friction is

$$W = -(0.6)(1\ \text{kN})(3\ \text{m} + 1\ \text{m}) = -2.4\ \text{kJ}.$$

b. The length of the path along the hypotenuse is $\sqrt{10}$ m, so the total work done against friction is

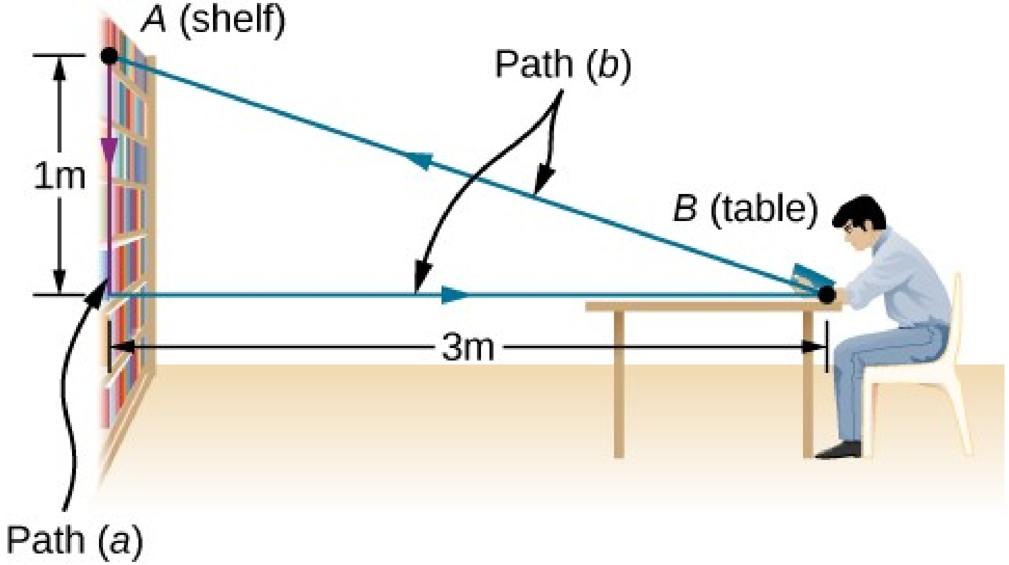
$$W = (0.6)(1\ \text{kN})(3\ \text{m} + 1\ \text{m} + \sqrt{10}\ \text{m}) = 4.3\ \text{kJ}.$$

WORK DONE BY GRAVITY

$$W_{\text{grav},AB} = -mg\hat{\mathbf{j}} \cdot (\vec{\mathbf{r}}_B - \vec{\mathbf{r}}_A) = -mg\,(y_B - y_A)\,.$$

Shelving a Book

You lift an oversized library book, weighing 20 N, 1 m vertically down from a shelf, and carry it 3 m horizontally to a table (Figure 7.5). How much work does gravity do on the book? (b) When you're finished, you move the book in a straight line back to its original place on the shelf. What was the total work done against gravity, moving the book away from its original position on the shelf and back again?



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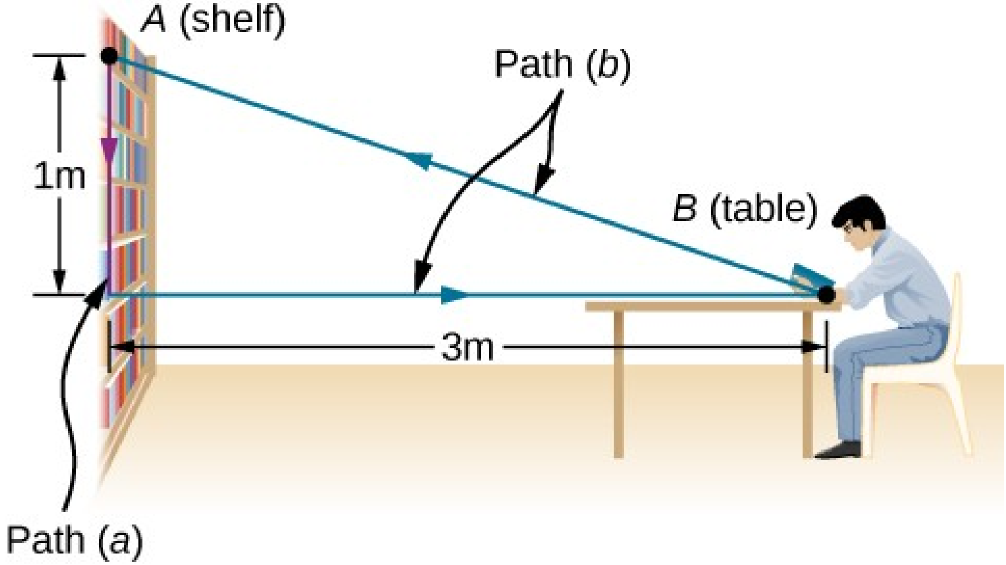


Solution

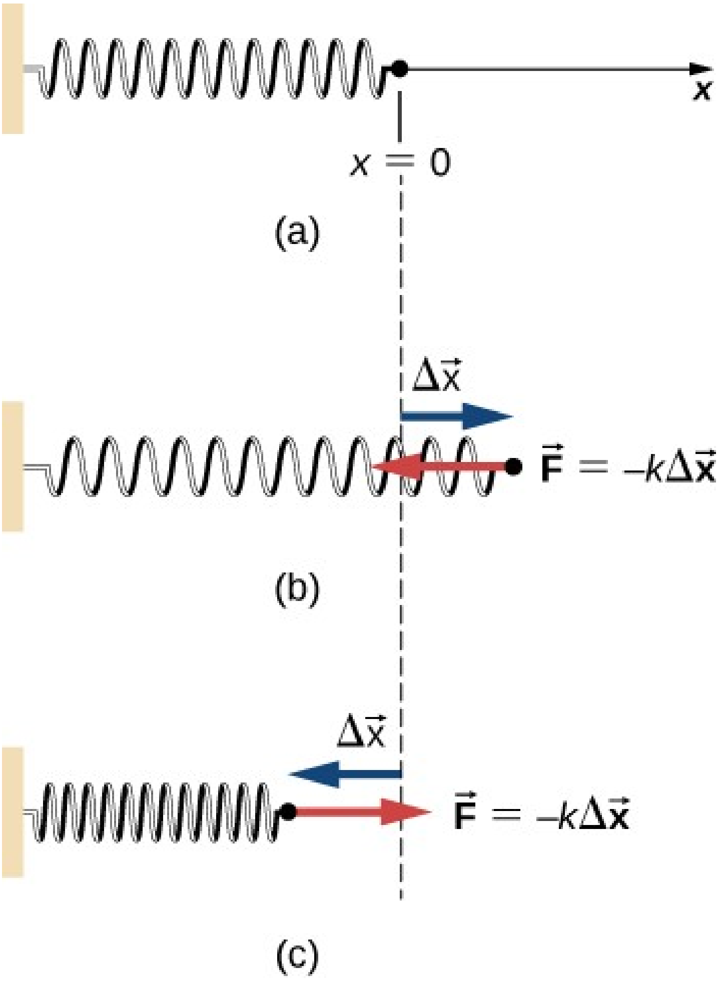
a. Since the book starts on the shelf and is lifted down $y_B - y_A = -1\text{ m}$, we have

$$W = -(20\text{ N})(-1\text{ m}) = 20\text{ J}.$$

b. There is zero difference in height for any path that begins and ends at the same place on the shelf, so $W = 0$.

WORK DONE BY A SPRING

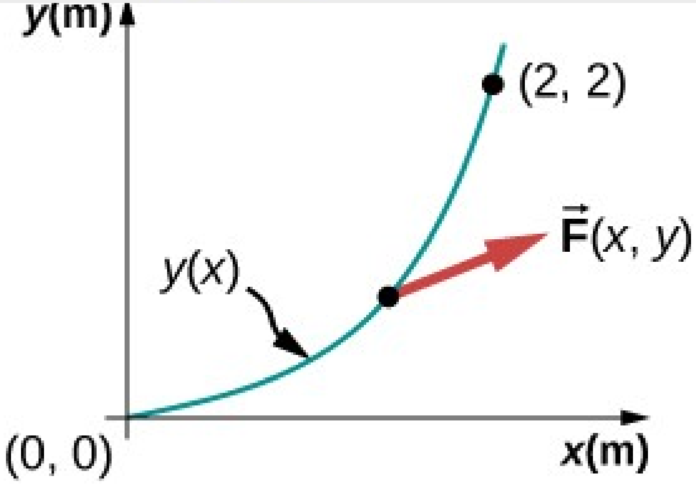
$$dW = F_x dx + F_y dy + F_z dz.$$



(a) The spring exerts no force at its equilibrium position. The spring exerts a force in the opposite direction to (b) an extension or stretch, and (c) a compression.

$$W_{\text{spring},AB} = \int_A^B F_x dx = -k\int_A^B x dx = -k\frac{x^2}{2}\bigg|_A^B = -\frac{1}{2}k\left(x_B^2 - x_A^2\right).$$

Work Done by a Variable Force over a Curved Path
An object moves along a parabolic path $y = (0.5\text{ m}^{-1})x^2$ from the origin $A = (0, 0)$ to the point $B = (2\text{ m}, 2\text{ m})$ under the action of a force $\vec{\mathbf{F}} = (5\text{ N/m})y\hat{\mathbf{i}} + (10\text{ N/m})x\hat{\mathbf{j}}$ (Figure 7.6). Calculate the work done.



Strategy
The components of the force are given functions of *x* and *y*. We can use the equation of the path to express *y* and *dy* in terms of *x* and *dx*; namely,

$$y = (0.5\text{ m}^{-1})x^2 \text{ and } dy = 2(0.5\text{ m}^{-1})x dx.$$

Then, the integral for the work is just a definite integral of a function of *x*.

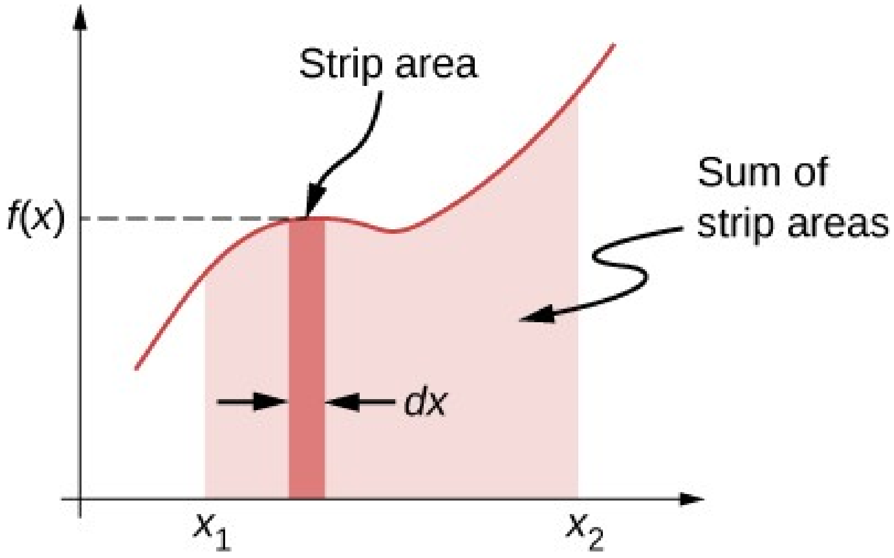
Solution
The infinitesimal element of work is

$$\begin{aligned} dW &= F_x dx + F_y dy = (5\text{ N/m})y dx + (10\text{ N/m})x dy \\ &= (5\text{ N/m})(0.5\text{ m}^{-1})x^2 dx + (10\text{ N/m})2(0.5\text{ m}^{-1})x^2 dx = (12.5\text{ N/m}^2)x^2 dx. \end{aligned}$$

The integral of x^2 is $x^3/3$, so

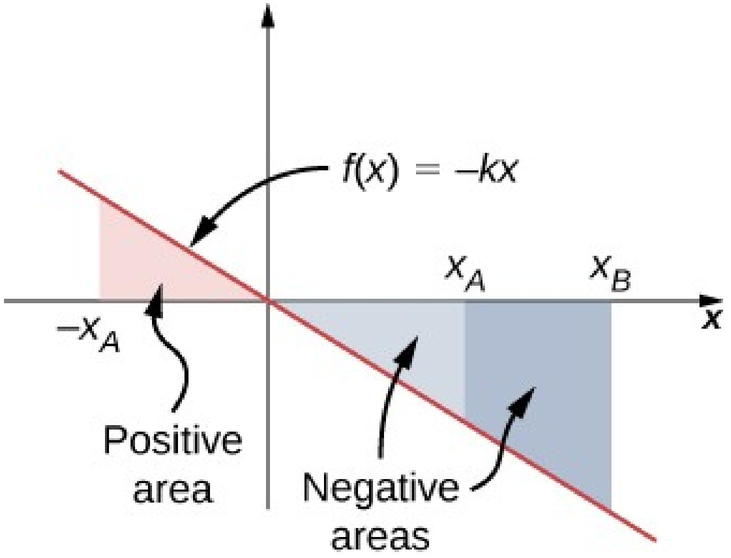
$$W = \int_0^{2\text{ m}} (12.5\text{ N/m}^2)x^2 dx = (12.5\text{ N/m}^2)\frac{x^3}{3}\Bigg|_0^{2\text{ m}} = (12.5\text{ N/m}^2)\left(\frac{8}{3}\right) = 33.3\text{ J}.$$

POSITION-DEPENDENT FORCES



A curve of $f(x)$ versus x showing the area of an infinitesimal strip, $f(x)dx$, and the sum of such areas, which is the integral of $f(x)$ from x_1 to x_2.

Curve of the spring force $f(x) = -kx$ versus x, showing areas under the line, between x_A and x_B , for both positive and negative values of x_A . When x_A is negative, the total area under the curve for the integral in **Equation 7.5** is the sum of positive and negative triangular areas. When x_A is positive, the total area under the curve is the difference between two negative triangles.



Work Done by a Spring Force

A perfectly elastic spring requires 0.54 J of work to stretch 6 cm from its equilibrium position, as in Figure 7.7(b). (a) What is its spring constant *k*? (b) How much work is required to stretch it an additional 6 cm?

Strategy

Work "required" means work done against the spring force, which is the negative of the work in Equation 7.5, that is

$$W = \frac{1}{2}k(x_B^2 - x_A^2).$$

For part (a), $x_A = 0$ and $x_B = 6\text{cm}$; for part (b), $x_B = 6\text{cm}$ and $x_B = 12\text{cm}$. In part (a), the work is given and you can solve for the spring constant; in part (b), you can use the value of *k*, from part (a), to solve for the work.

Solution

a. $W = 0.54\text{ J} = \frac{1}{2}k[(6\text{ cm})^2 - 0]$, so $k = 3\text{ N/cm}$.
b. $W = \frac{1}{2}(3\text{ N/cm})[(12\text{ cm})^2 - (6\text{ cm})^2] = 1.62\text{ J}$.

KINETIC ENERGY

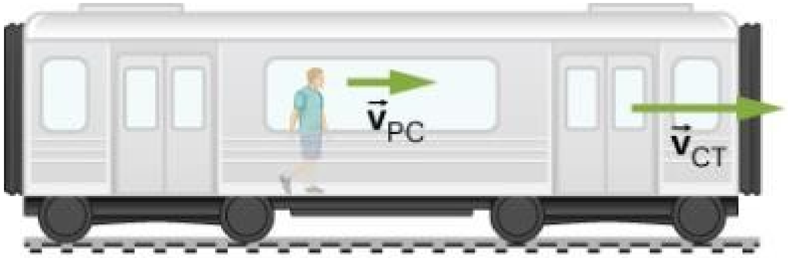
The kinetic energy of a particle is one-half the product of the particle's mass *m* and the square of its speed *v*:

$$K = \frac{1}{2}mv^2.$$

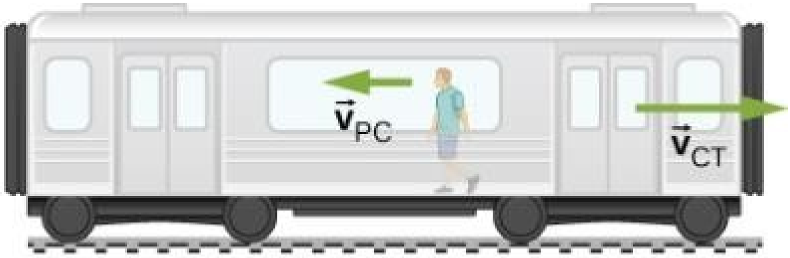
$$K = \frac{1}{2}m\left(\frac{p}{m}\right)^2 = \frac{p^2}{2m}$$

Kinetic Energy Relative to Different Frames

A 75.0-kg person walks down the central aisle of a subway car at a speed of 1.50 m/s relative to the car, whereas the train is moving at 15.0 m/s relative to the tracks. (a) What is the person's kinetic energy relative to the car? (b) What is the person's kinetic energy relative to the tracks? (c) What is the person's kinetic energy relative to a frame moving with the person?



(a)



(b)

Solution

a. $K = \frac{1}{2}(75.0\text{ kg})(1.50\text{ m/s})^2 = 84.4\text{ J}.$

b. $v_{PT} = (15.0 \pm 1.50)\text{ m/s}$. Therefore, the two possible values for kinetic energy relative to the car are

$$K = \frac{1}{2}(75.0\text{ kg})(13.5\text{ m/s})^2 = 6.83\text{ kJ}$$

and

$$K = \frac{1}{2}(75.0\text{ kg})(16.5\text{ m/s})^2 = 10.2\text{ kJ}.$$

c. In a frame where $v_P = 0, K = 0$ as well.

WORK-ENERGY THEOREM

The net work done on a particle equals the change in the particle's kinetic energy:

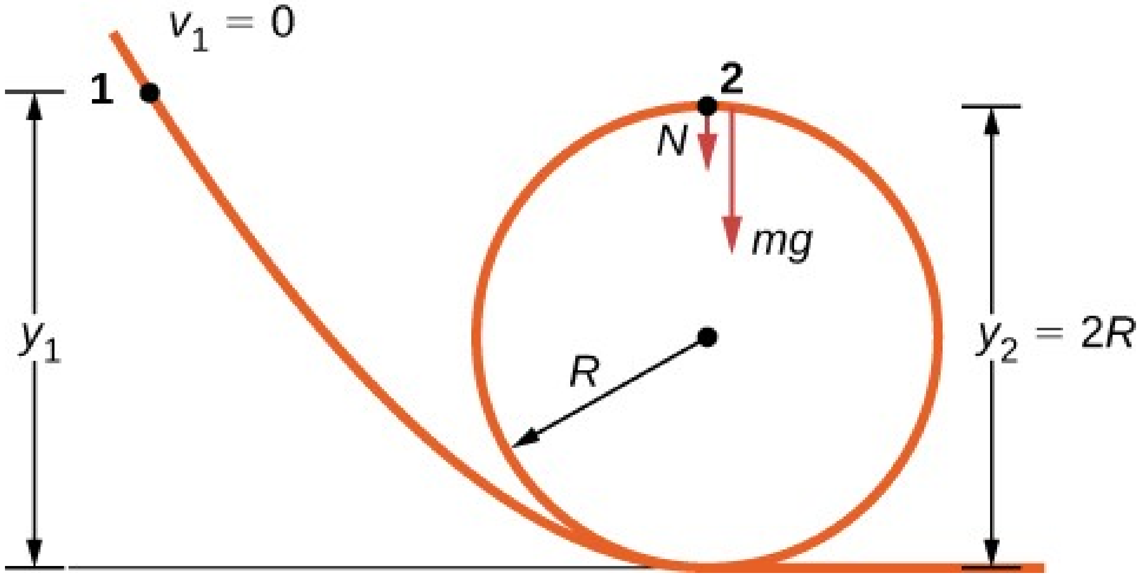
$$W_{\text{net}} = K_B - K_A.$$

Problem-Solving Strategy: Work-Energy Theorem

1. Draw a free-body diagram for each force on the object.
2. Determine whether or not each force does work over the displacement in the diagram. Be sure to keep any positive or negative signs in the work done.
3. Add up the total amount of work done by each force.
4. Set this total work equal to the change in kinetic energy and solve for any unknown parameter.
5. Check your answers. If the object is traveling at a constant speed or zero acceleration, the total work done should be zero and match the change in kinetic energy. If the total work is positive, the object must have sped up or increased kinetic energy. If the total work is negative, the object must have slowed down or decreased kinetic energy.

EXAMPLE

A frictionless track for a toy car has a loop-the-loop in it. How high must the car start so that it can go around the loop without falling off?



$$-mg(y_2 - y_1) = \frac{1}{2} m {v_2}^2,$$

$$a_{top} = \frac{F}{m} = \frac{N + mg}{m} = \frac{v_2^2}{R}.$$

The condition for maintaining contact with the track is that there must be some normal force, however slight; that is, $N > 0$. Substituting for v_2^2 and *N*, we can find the condition for y_1.

Solution

Implement the steps in the strategy to arrive at the desired result:

$$N = \frac{-mg + mv_2^2}{R} = \frac{-mg + 2mg(y_1 - 2R)}{R} > 0 \quad \text{or} \quad y_1 > \frac{5R}{2}.$$