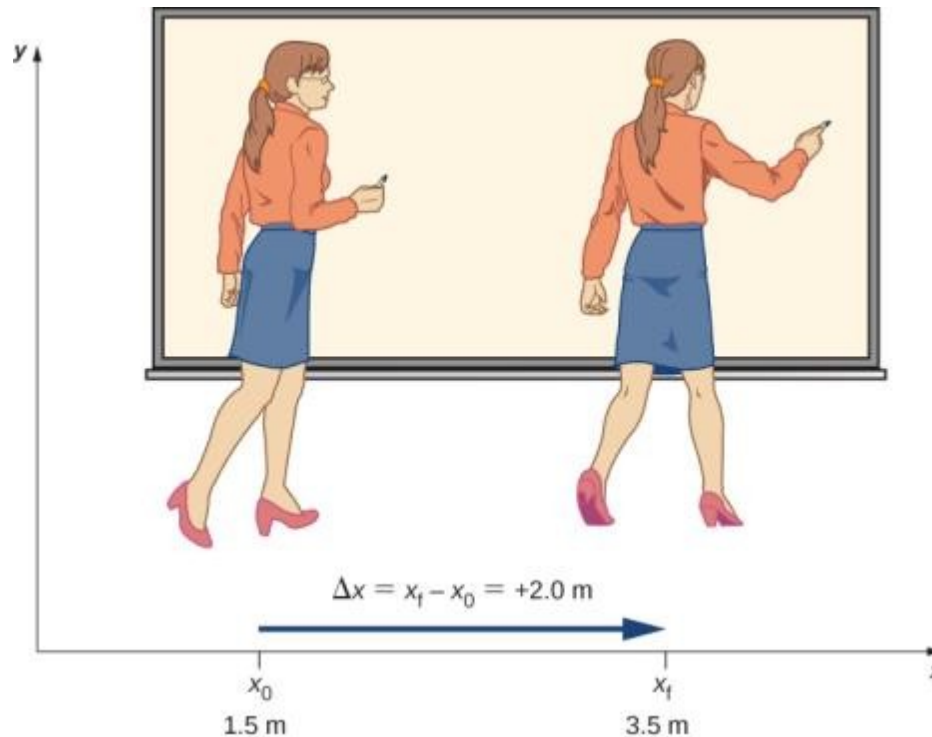


ANNOUNCEMENTS

- Homework #3, due Monday, Sep. 10 before class:
Answer conceptual questions: Chapter 3, #6 and #12
Solve problems: Chapter 3, #30 and #48.
- Read Sections 3.1-3.4 before next class (Friday)
- 5-minute quiz on Chapter 3:
Wednesday, September 12 at beginning of class
- First in-class test will be Friday, September 21
- Grades for homework #2 are online.

DISPLACEMENT: CHANGE IN POSITION



A professor paces left and right while lecturing. Her position relative to Earth is given by x . The +2.0-m displacement of the professor relative to Earth is represented by an arrow pointing to the right.

DISPLACEMENT

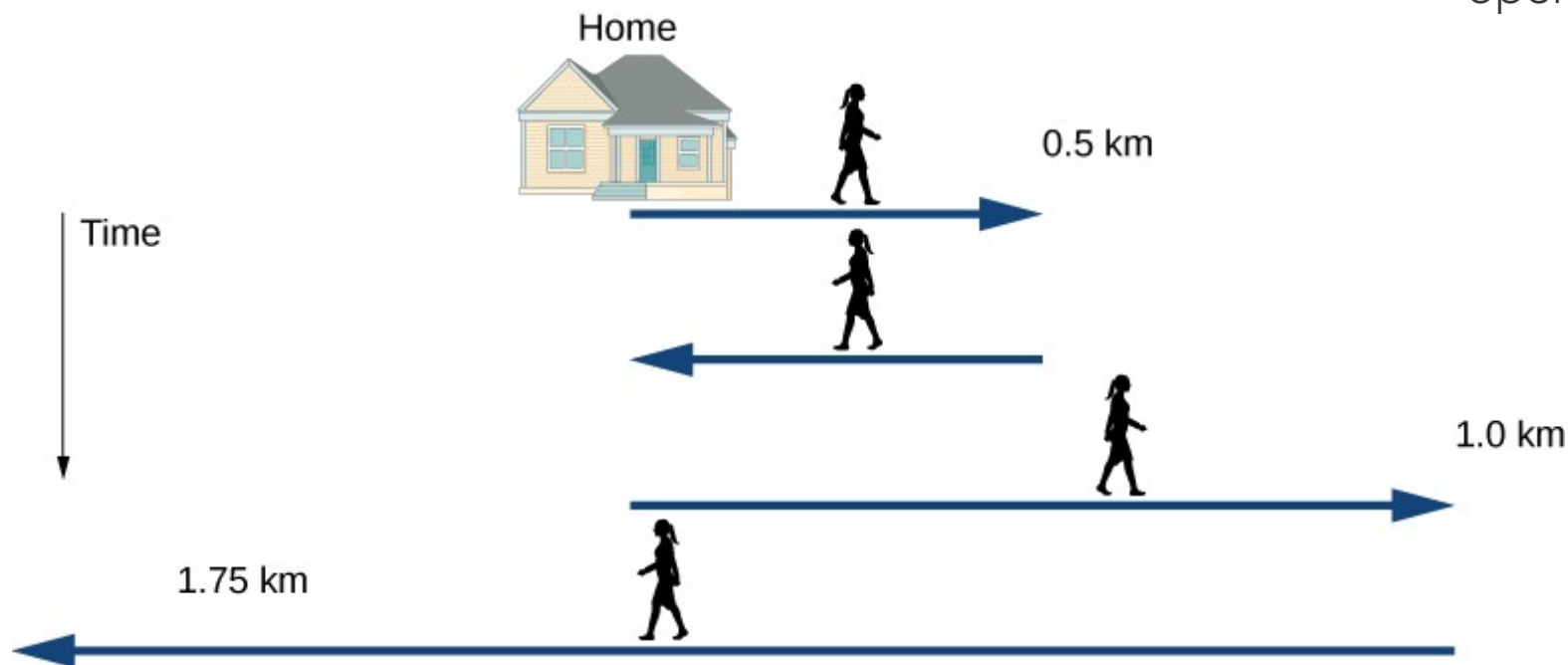
Displacement Δx is the change in position of an object:

$$\Delta x = x_f - x_0,$$

(3.1)

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

TOTAL DISPLACEMENT



Timeline of Jill's movements.

$$\Delta x_{\text{Total}} = \sum \Delta x_i,$$

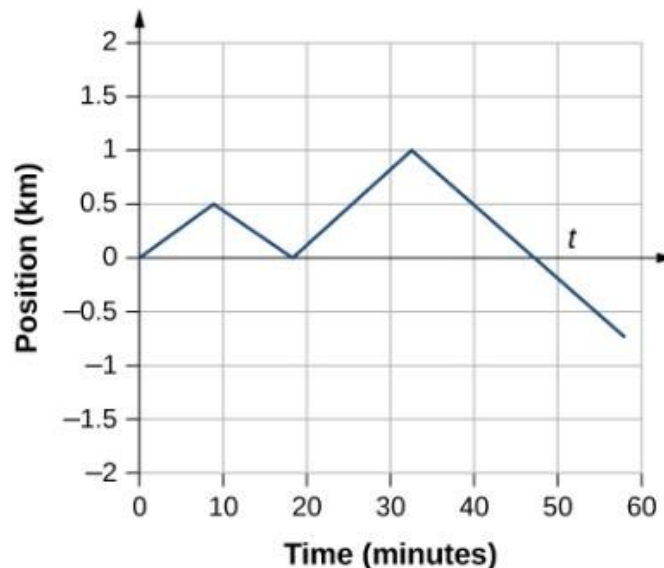
(3.2)

The magnitude of the total displacement should not be confused with the distance traveled. Distance traveled x_{Total} , is the total length of the path traveled between two positions. In the previous problem, the **distance traveled** is the sum of the magnitudes of the individual displacements:

$$x_{\text{Total}} = |\Delta x_1| + |\Delta x_2| = 2 + 4 = 6 \text{ m.}$$

POSITION VS TIME AND AVERAGE VELOCITY

Position vs. Time



This graph depicts Jill's position versus time. The average velocity is the slope of a line connecting the initial and final points.

AVERAGE VELOCITY

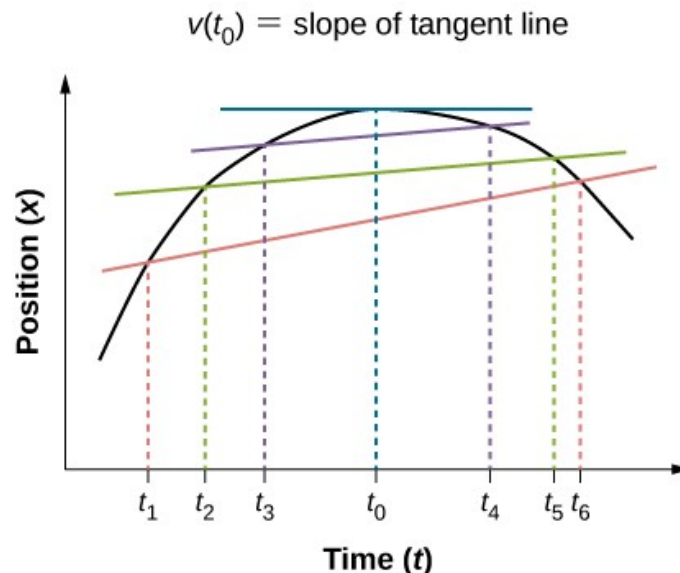
If x_1 and x_2 are the positions of an object at times t_1 and t_2 , respectively, then

$$\text{Average velocity} = \bar{v} = \frac{\text{Displacement between two points}}{\text{Elapsed time between two points}}$$

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_2 - x_1}{t_2 - t_1}.$$

(3.3)

INSTANTANEOUS VELOCITY



In a graph of position versus time, the instantaneous velocity is the slope of the tangent line at a given point. The average velocities $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$ between times $\Delta t = t_6 - t_1$, $\Delta t = t_5 - t_2$, and $\Delta t = t_4 - t_3$ are shown. When $\Delta t \rightarrow 0$, the average velocity approaches the instantaneous velocity at $t = t_0$.

INSTANTANEOUS VELOCITY

The instantaneous velocity of an object is the limit of the average velocity as the elapsed time approaches zero, or the derivative of x with respect to t :

$$v(t) = \frac{d}{dt}x(t).$$

(3.4)

ANNOUNCEMENTS

- Homework #3, due Monday, Sep. 10 before class:
Answer conceptual questions: Chapter 3, #6 and #12
Solve problems: Chapter 3, #30 and #48.
- Read Sections 3.5 before next class (Monday)
- 5-minute quiz on Chapter 3:
Wednesday, September 12 at beginning of class
- First in-class test will be Friday, September 21
- Grades for quiz #2 and current averages are online.

SUMMARY

DISPLACEMENT

Displacement Δx is the change in position of an object:

$$\Delta x = x_f - x_0, \quad (3.1)$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

$$\Delta x_{\text{Total}} = \sum \Delta x_i, \quad (3.2)$$

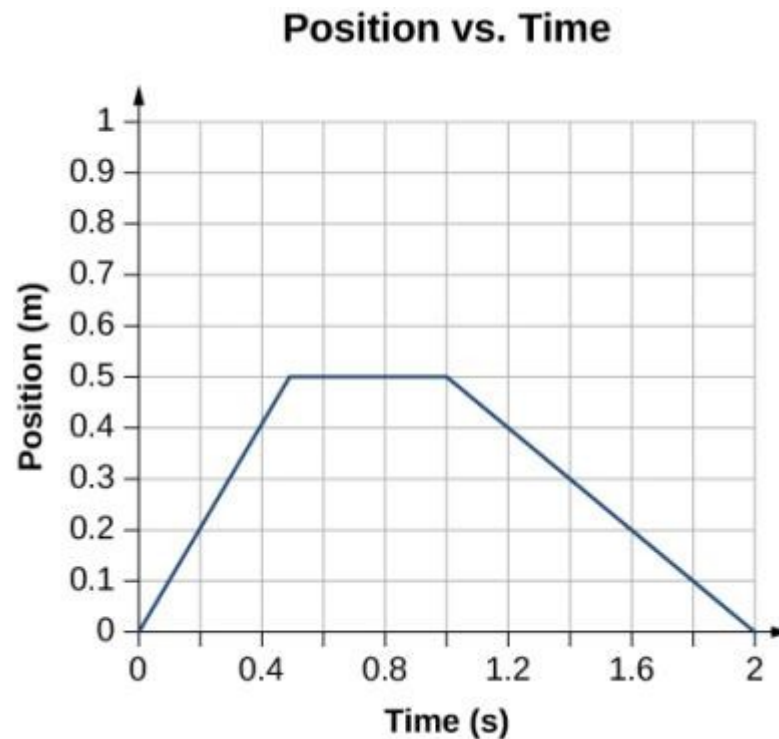
INSTANTANEOUS VELOCITY

The instantaneous velocity of an object is the limit of the average velocity as the elapsed time approaches zero, or the derivative of x with respect to t :

$$v(t) = \frac{d}{dt}x(t). \quad (3.4)$$

$$a(t) = \frac{d}{dt}v(t). \quad (3.9)$$

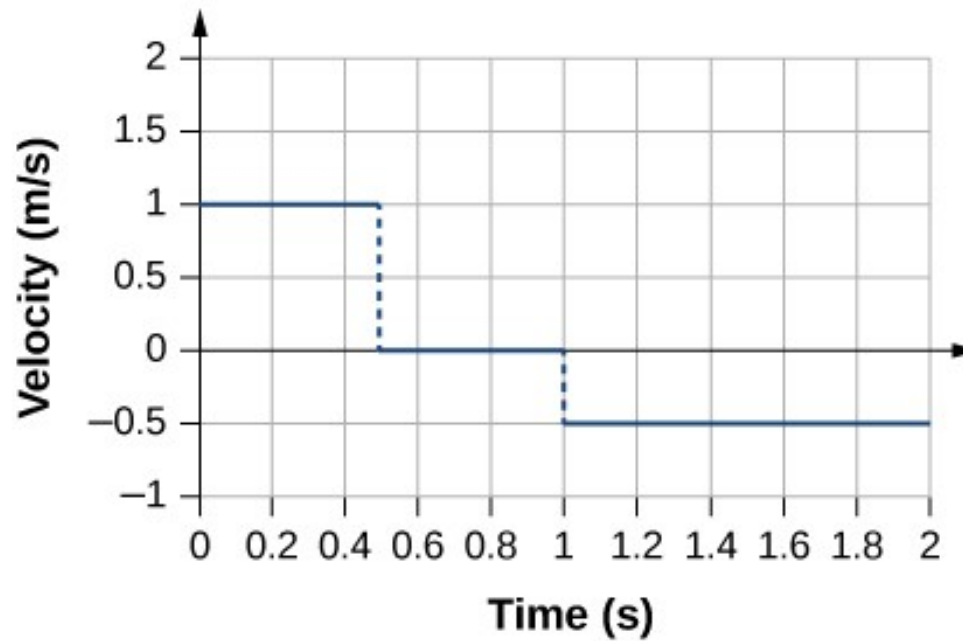
EXAMPLE OF POSITION VS. TIME



The object starts out in the positive direction, stops for a short time, and then reverses direction, heading back toward the origin. Notice that the object comes to rest instantaneously, which would require an infinite force. Thus, the graph is an approximation of motion in the real world. (The concept of force is discussed in [Newton's Laws of Motion](#).)

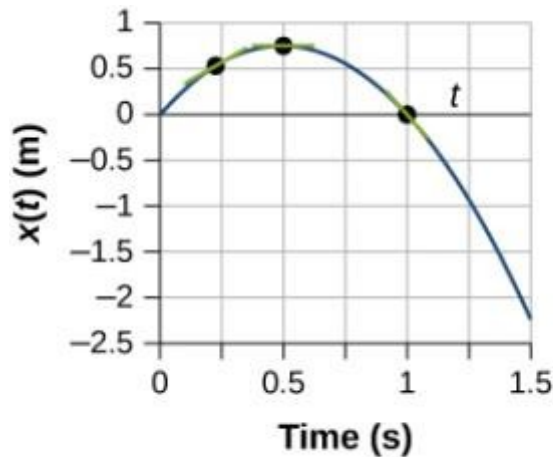
VELOCITY VS. TIME

Velocity vs. Time

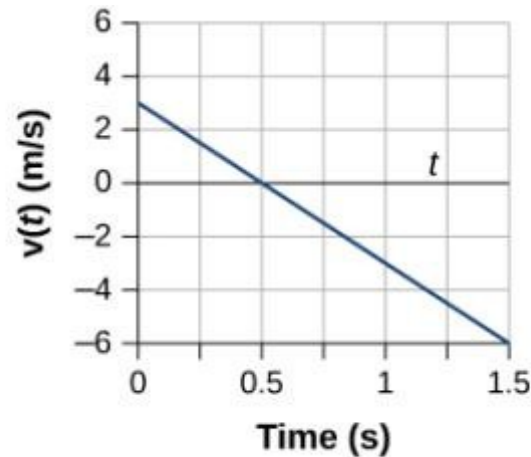


The velocity is positive for the first part of the trip, zero when the object is stopped, and negative when the object reverses direction.

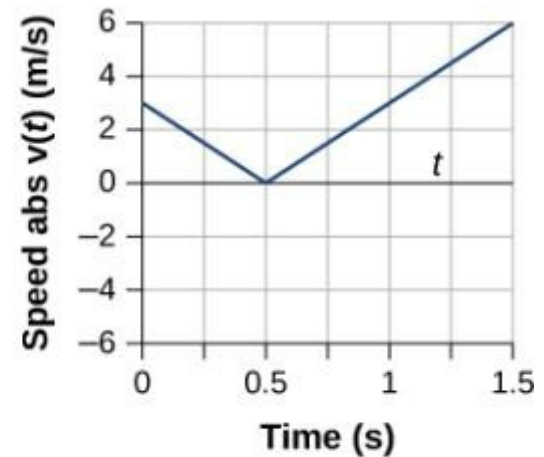
POSITION, VELOCITY AND SPEED



(a) Position



(b) Velocity



(c) Speed

- (a) Position: $x(t)$ versus time.
- (b) Velocity: $v(t)$ versus time. The slope of the position graph is the velocity. A rough comparison of the slopes of the tangent lines in (a) at 0.25 s, 0.5 s, and 1.0 s with the values for velocity at the corresponding times indicates they are the same values.
- (c) Speed: $|v(t)|$ versus time. Speed is always a positive number.

AVERAGE AND INSTANTANEOUS SPEED

$$\text{Average speed} = \bar{s} = \frac{\text{Total distance}}{\text{Elapsed time}} . \quad (3.5)$$

Average speed is not necessarily the same as the magnitude of the average velocity, which is found by dividing the magnitude of the total displacement by the elapsed time. For example, if a trip starts and ends at the same location, the total displacement is zero, and therefore the average velocity is zero. The average speed, however, is not zero, because the total distance traveled is greater than zero. If we take a road trip of 300 km and need to be at our destination at a certain time, then we would be interested in our average speed.

However, we can calculate the **instantaneous speed** from the magnitude of the instantaneous velocity:

$$\text{Instantaneous speed} = |v(t)| . \quad (3.6)$$

Speed	m/s	mi/h
Continental drift	10^{-7}	2×10^{-7}
Brisk walk	1.7	3.9
Cyclist	4.4	10
Sprint runner	12.2	27
Rural speed limit	24.6	56
Official land speed record	341.1	763
Speed of sound at sea level	343	768
Space shuttle on reentry	7800	17,500
Escape velocity of Earth*	11,200	25,000
Orbital speed of Earth around the Sun	29,783	66,623
Speed of light in a vacuum	299,792,458	670,616,629

Table 3.1 Speeds of Various Objects

AVERAGE ACCELERATION

AVERAGE ACCELERATION

Average acceleration is the rate at which velocity changes:

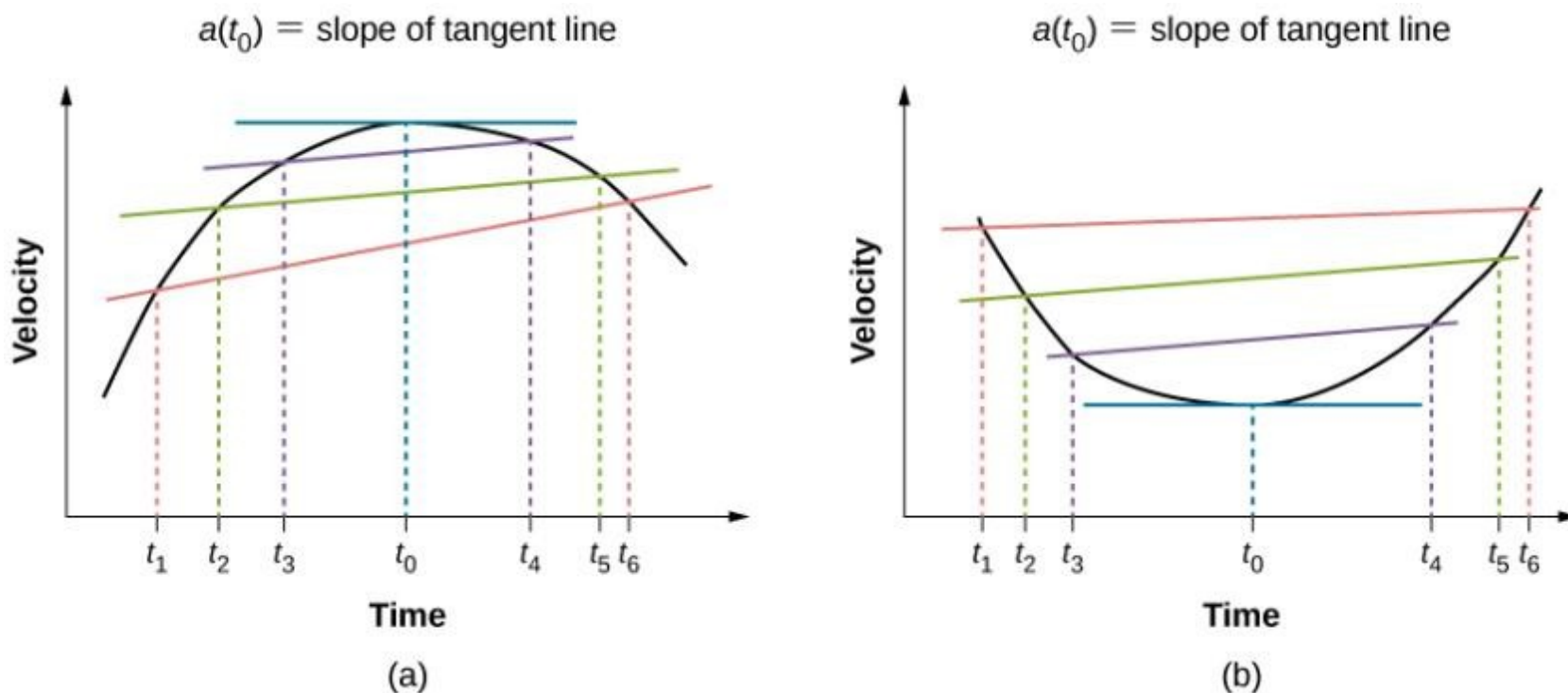
$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}, \quad (3.8)$$

where \bar{a} is **average acceleration**, v is velocity, and t is time. (The bar over the a means *average* acceleration.)

INSTANTANEOUS ACCELERATION

$$a(t) = \frac{d}{dt}v(t). \quad (3.9)$$

FIGURE 3.14



In a graph of velocity versus time, instantaneous acceleration is the slope of the tangent line.

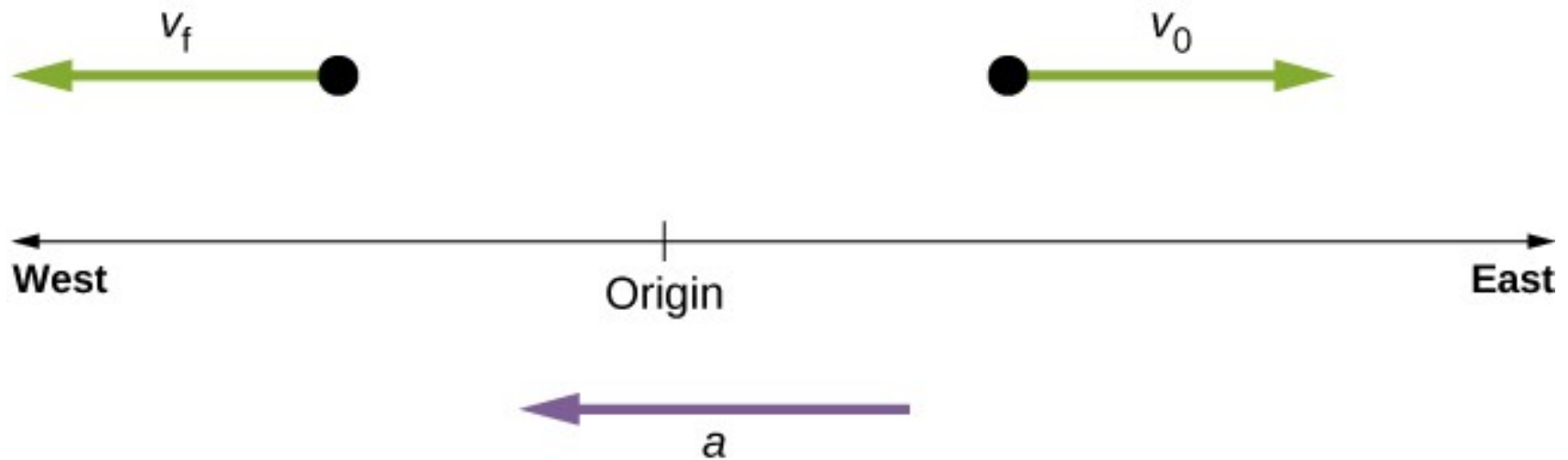
- (a) Shown is average acceleration $\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$ between times $\Delta t = t_6 - t_1$, $\Delta t = t_5 - t_2$, and $\Delta t = t_4 - t_3$. When $\Delta t \rightarrow 0$, the average acceleration approaches instantaneous acceleration at time t_0 . In view (a), instantaneous acceleration is shown for the point on the velocity curve at maximum velocity. At this point, instantaneous acceleration is the slope of the tangent line, which is zero. At any other time, the slope of the tangent line—and thus instantaneous acceleration—would not be zero.
- (b) Same as (a) but shown for instantaneous acceleration at minimum velocity.

ACCELERATION



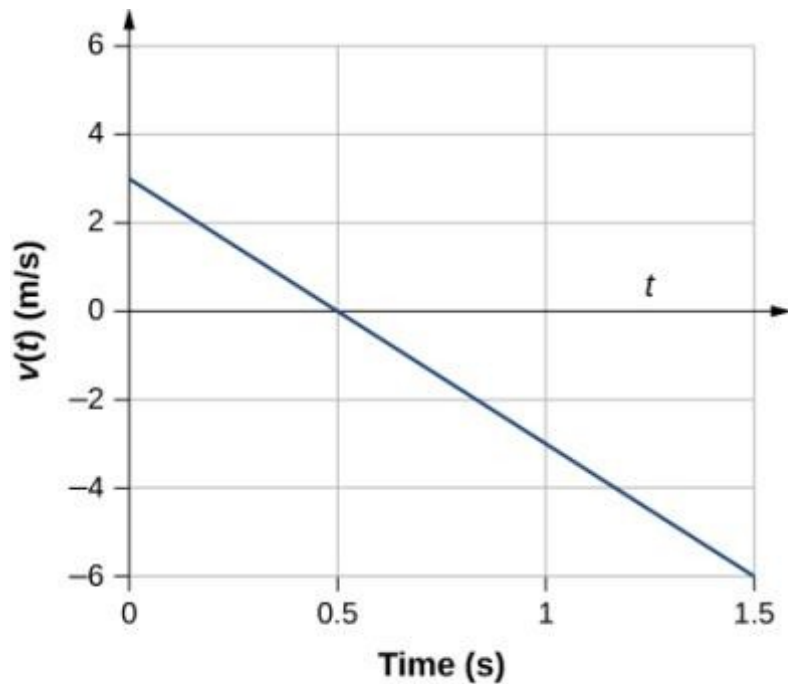
A subway train in Sao Paulo, Brazil, decelerates as it comes into a station. It is accelerating in a direction opposite to its direction of motion. (credit: Yusuke Kawasaki)

FIGURE 3.11

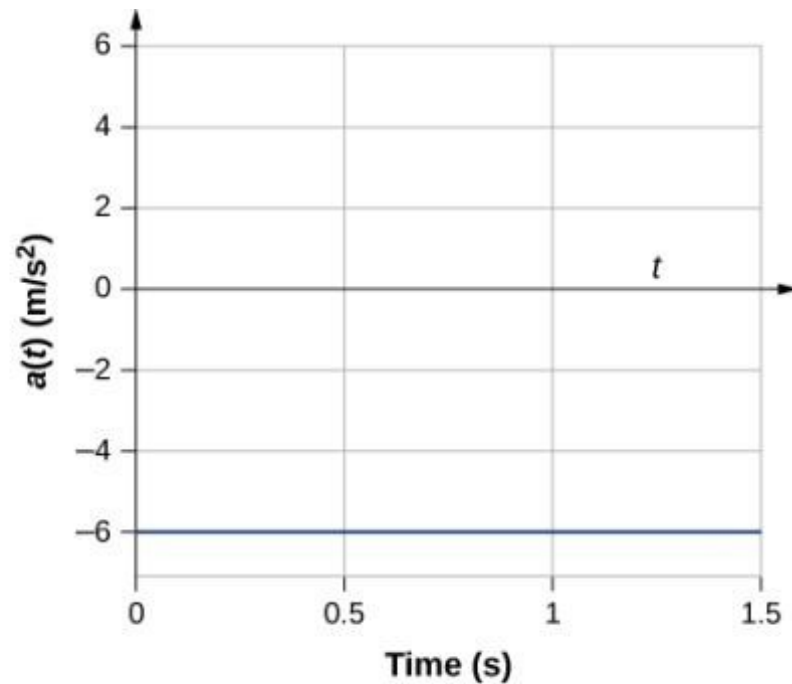


An object in motion with a velocity vector toward the east under negative acceleration comes to a rest and reverses direction. It passes the origin going in the opposite direction after a long enough time.

CONSTANT ACCELERATION



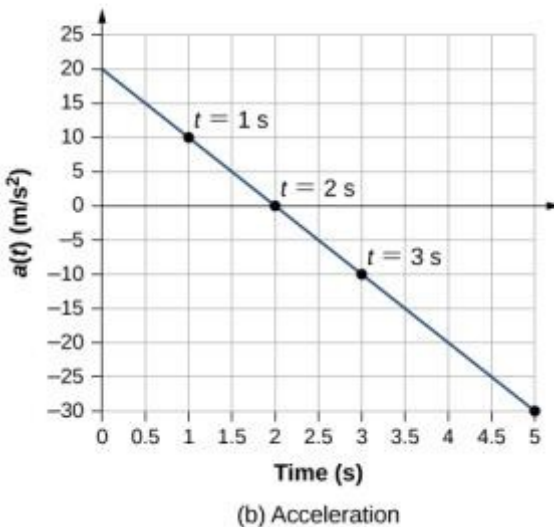
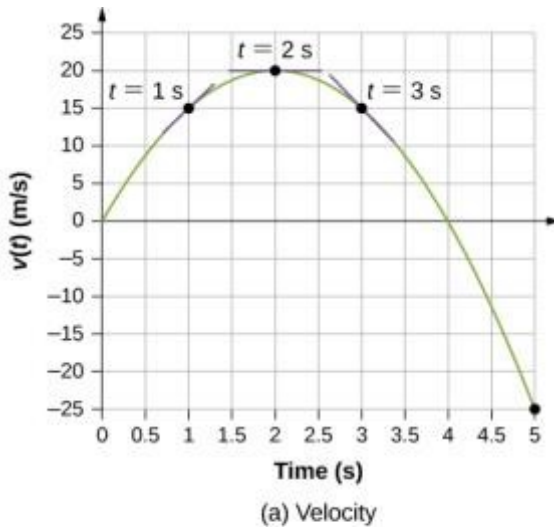
(a) Velocity



(b) Acceleration

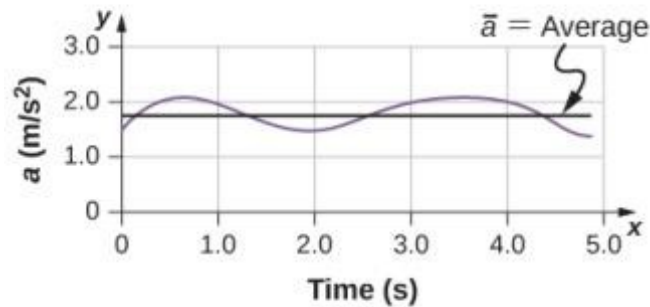
(a, b) The velocity-versus-time graph is linear and has a negative constant slope (a) that is equal to acceleration, shown in (b).

LINEAR ACCELERATION

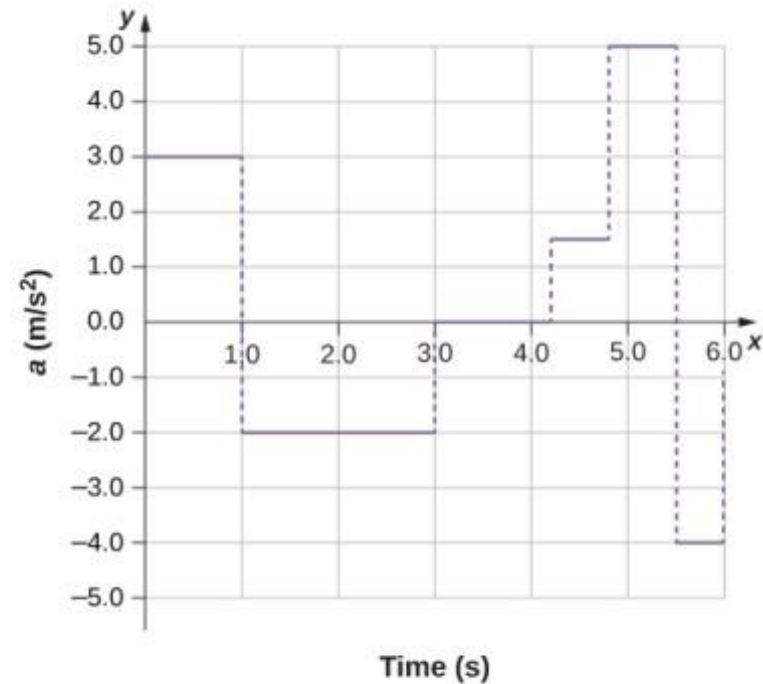


- a) Velocity versus time. Tangent lines are indicated at times 1, 2, and 3 s. The slopes of the tangent lines are the accelerations. At $t = 3$ s, velocity is positive. At $t = 5$ s, velocity is negative, indicating the particle has reversed direction.
- b) Acceleration versus time. Comparing the values of accelerations given by the black dots with the corresponding slopes of the tangent lines (slopes of lines through black dots) in (a), we see they are identical.

EXAMPLES OF ACCELERATION



(a)



(b)

Graphs of instantaneous acceleration versus time for two different one-dimensional motions.

- a) Acceleration varies only slightly and is always in the same direction, since it is positive. The average over the interval is nearly the same as the acceleration at any given time.
- b) Acceleration varies greatly, perhaps representing a package on a post office conveyor belt that is accelerated forward and backward as it bumps along. It is necessary to consider small time intervals (such as from 0–1.0 s) with constant or nearly constant acceleration in such a situation.

Acceleration	Value (m/s ²)
High-speed train	0.25
Elevator	2
Cheetah	5
Object in a free fall without air resistance near the surface of Earth	9.8
Space shuttle maximum during launch	29
Parachutist peak during normal opening of parachute	59
F16 aircraft pulling out of a dive	79
Explosive seat ejection from aircraft	147
Sprint missile	982
Fastest rocket sled peak acceleration	1540
Jumping flea	3200
Baseball struck by a bat	30,000
Closing jaws of a trap-jaw ant	1,000,000
Proton in the large Hadron collider	1.9×10^9

Table 3.2 Typical Values of Acceleration (credit: Wikipedia: Orders of Magnitude (acceleration))

CONSTANT ACCELERATION

SUMMARY OF KINEMATIC EQUATIONS (CONSTANT A)

$$x = x_0 + \bar{v}t$$

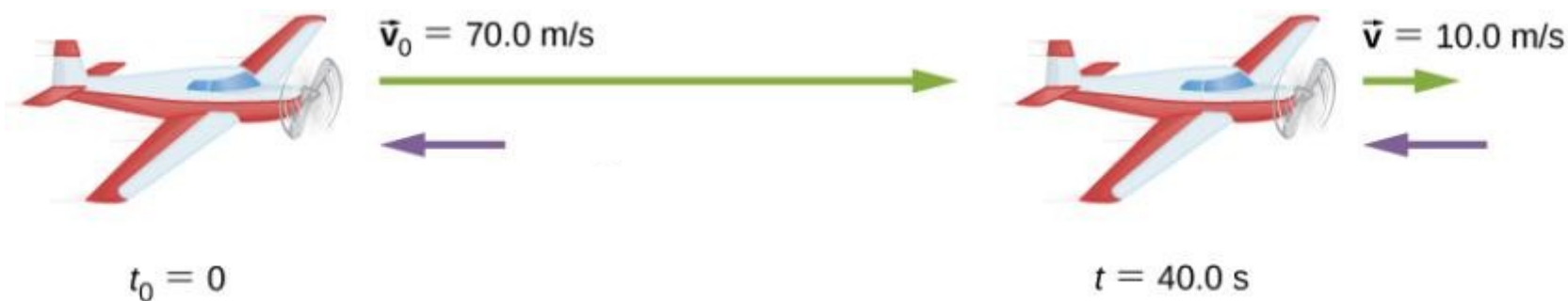
$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

EXAMPLE



The airplane lands with an initial velocity of 70.0 m/s and slows to a final velocity of 10.0 m/s before heading for the terminal. Note the acceleration is negative because its direction is opposite to its velocity, which is positive.

SUMMARY OF KINEMATIC EQUATIONS (CONSTANT A)

$$x = x_0 + \bar{v}t$$

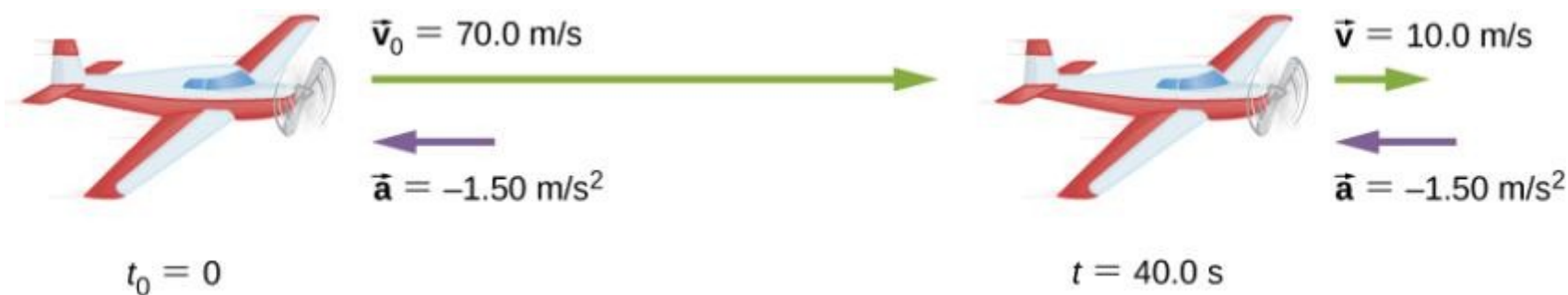
$$\bar{v} = \frac{v_0 + v}{2}$$

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SUMMARY OF KINEMATIC EQUATIONS (CONSTANT A)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

ANNOUNCEMENTS

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- Read Sections 4.1-4.3 before next class (Wednesday)
- 5-minute quiz on Chapter 3:
Wednesday, September 12 at beginning of class
- First in-class test will be Friday, September 21

SUMMARY

DISPLACEMENT

Displacement Δx is the change in position of an object:

$$\Delta x = x_f - x_0, \quad (3.1)$$

where Δx is displacement, x_f is the final position, and x_0 is the initial position.

$$\Delta x_{\text{Total}} = \sum \Delta x_i, \quad (3.2)$$

INSTANTANEOUS VELOCITY

The instantaneous velocity of an object is the limit of the average velocity as the elapsed time approaches zero, or the derivative of x with respect to t :

$$v(t) = \frac{d}{dt}x(t). \quad (3.4)$$

$$a(t) = \frac{d}{dt}v(t). \quad (3.9)$$

SPECIAL CASE: CONSTANT ACCELERATION

SUMMARY OF KINEMATIC EQUATIONS (CONSTANT A)

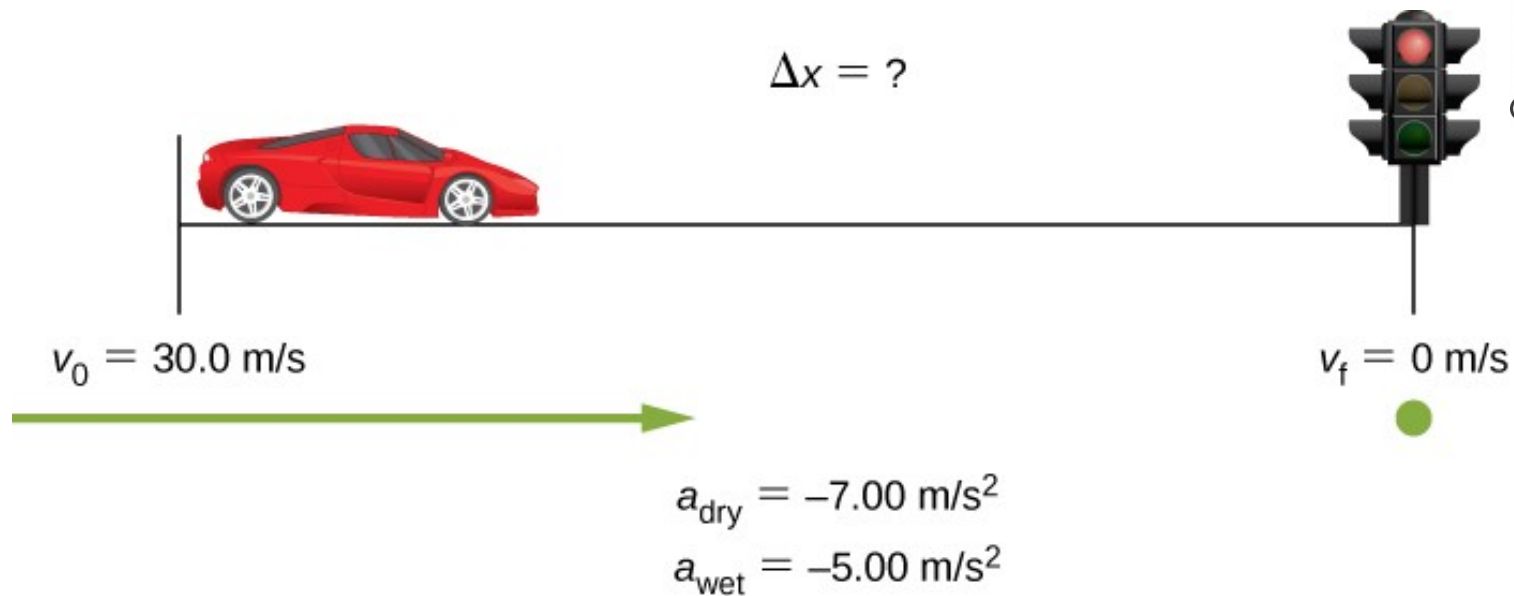
$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$



SUMMARY OF KINEMATIC EQUATIONS (CONSTANT A)

$$x = x_0 + \bar{v}t$$

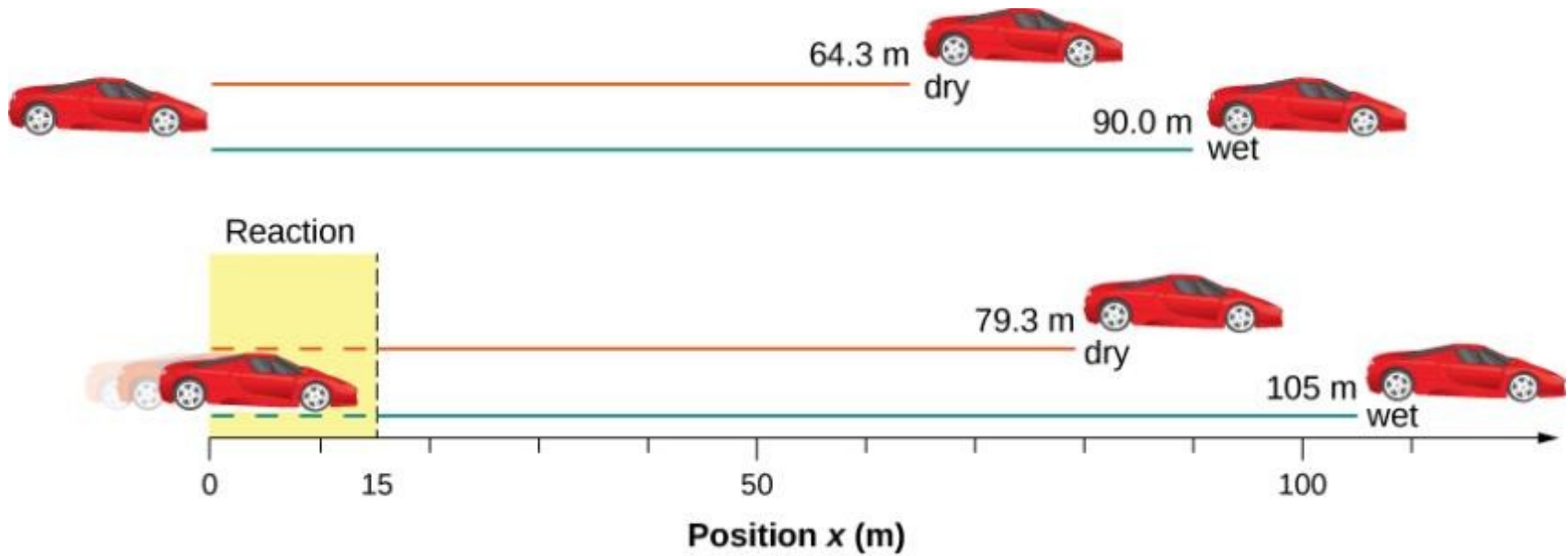
$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

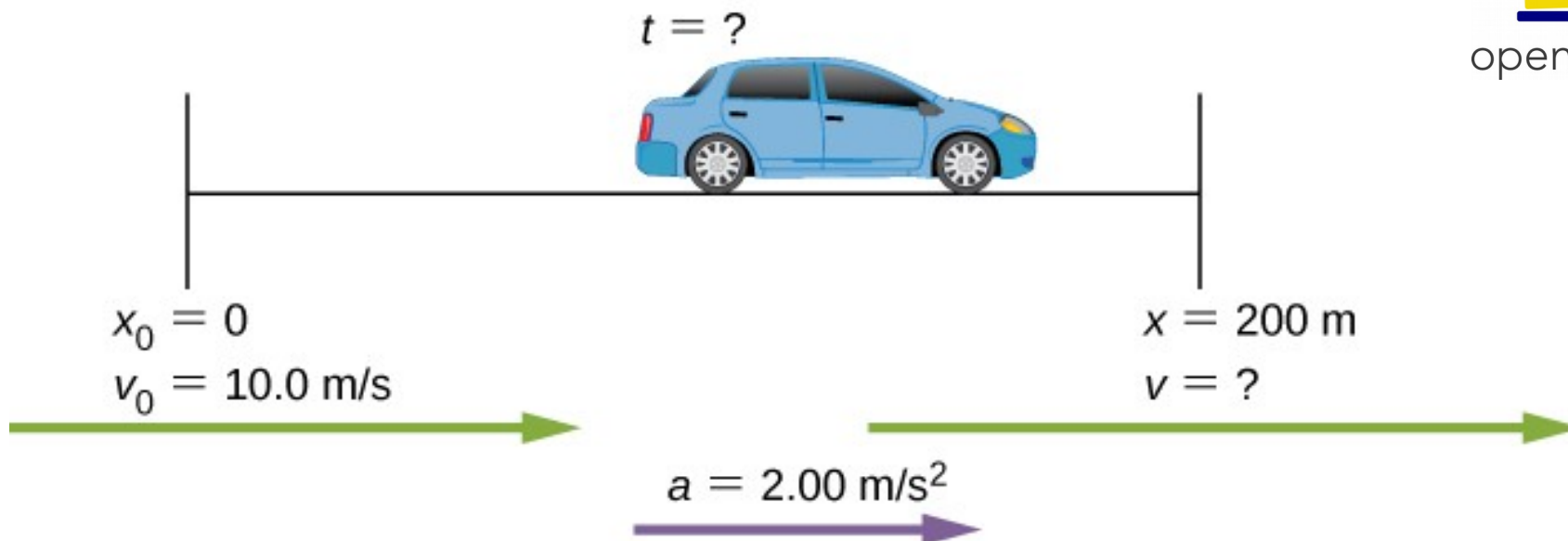
$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

EXAMPLE



The distance necessary to stop a car varies greatly, depending on road conditions and driver reaction time. Shown here are the braking distances for dry and wet pavement, as calculated in this example, for a car traveling initially at 30.0 m/s . Also shown are the total distances traveled from the point when the driver first sees a light turn red, assuming a 0.500 s reaction time.



SUMMARY OF KINEMATIC EQUATIONS (CONSTANT A)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

FIGURE 3.25

$v_1 \neq \text{constant}$

$v_2 = \text{constant}$



Car 1 accelerates toward Car 2



At a later time Car 1 catches Car 2

A two-body pursuit scenario where car 2 has a constant velocity and car 1 is behind with a constant acceleration. Car 1 catches up with car 2 at a later time.

SUMMARY OF KINEMATIC EQUATIONS (CONSTANT A)

$$x = x_0 + \bar{v}t$$

$$\bar{v} = \frac{v_0 + v}{2}$$

$$v = v_0 + at$$

$$x = x_0 + v_0t + \frac{1}{2}at^2$$

$$v^2 = v_0^2 + 2a(x - x_0)$$

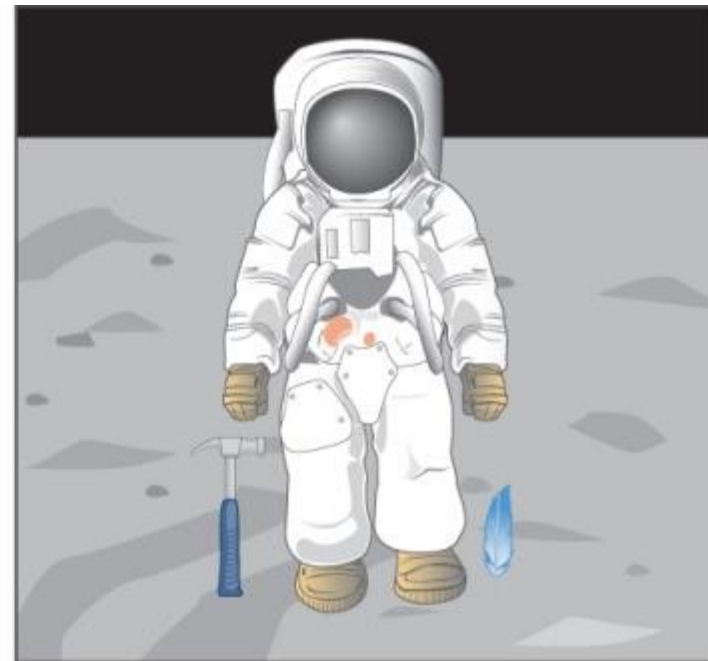
GRAVITY ACCELERATION (FREE FALL)



In air



In a vacuum



In a vacuum (the hard way)

A hammer and a feather fall with the same constant acceleration if air resistance is negligible. This is a general characteristic of gravity not unique to Earth, as astronaut David R. Scott demonstrated in 1971 on the Moon, where the acceleration from gravity is only 1.67 m/s^2 and there is no atmosphere.

FREE FALL

The positions and velocities at 1-s intervals of a ball thrown downward from a tall building at 4.9 m/s.

KINEMATIC EQUATIONS FOR OBJECTS IN FREE FALL

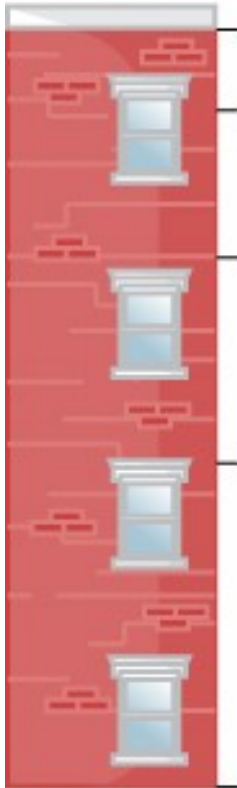
We assume here that acceleration equals $-g$ (with the positive direction upward).

$$v = v_0 - gt$$

$$y = y_0 + v_0t - \frac{1}{2}gt^2$$

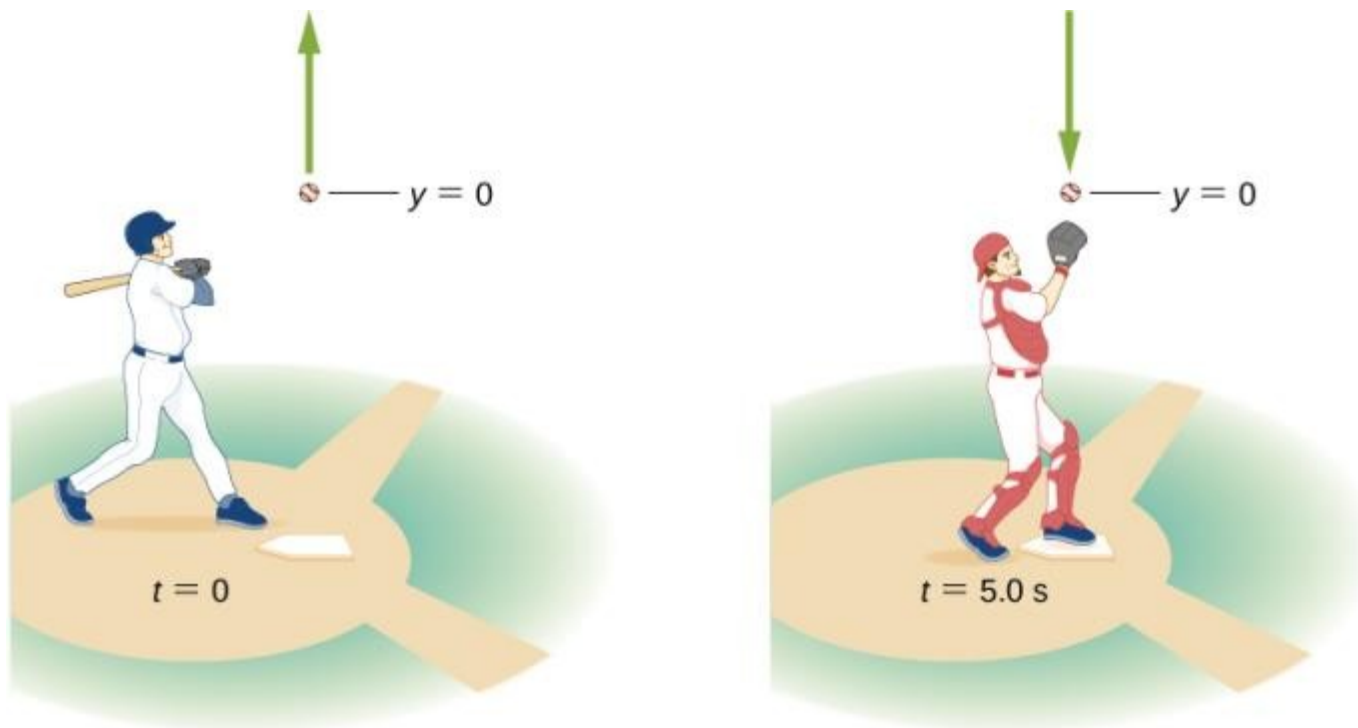
$$v^2 = v_0^2 - 2g(y - y_0)$$

$$g = 9.81\text{m/s}^2 \text{ (or } 32.2\text{ft/s}^2\text{)}$$



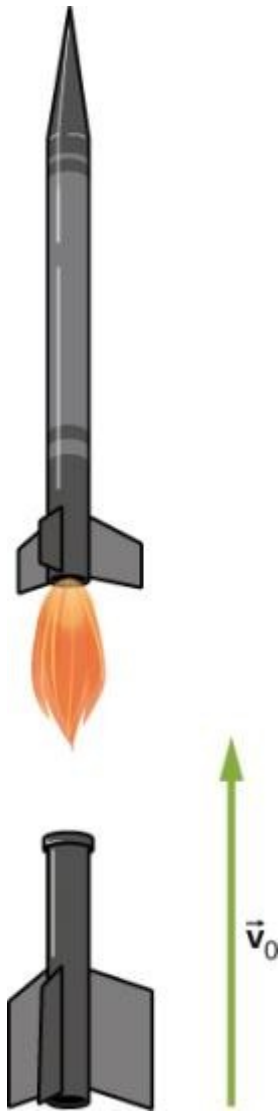
t (s)	x (m)	v (m/s)
0	0	-4.9
1	-9.8	-14.7
2	-29.4	-24.5
3	-58.8	-34.3
4	-98.0	-44.1

FIGURE 3.28



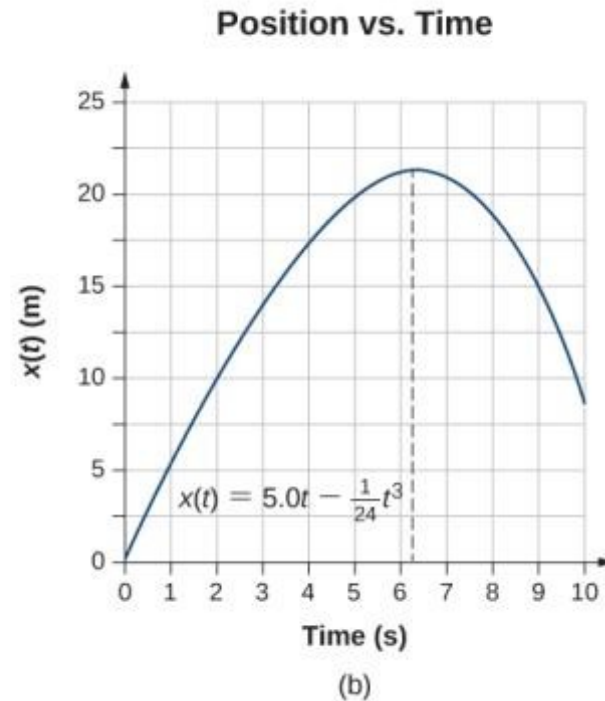
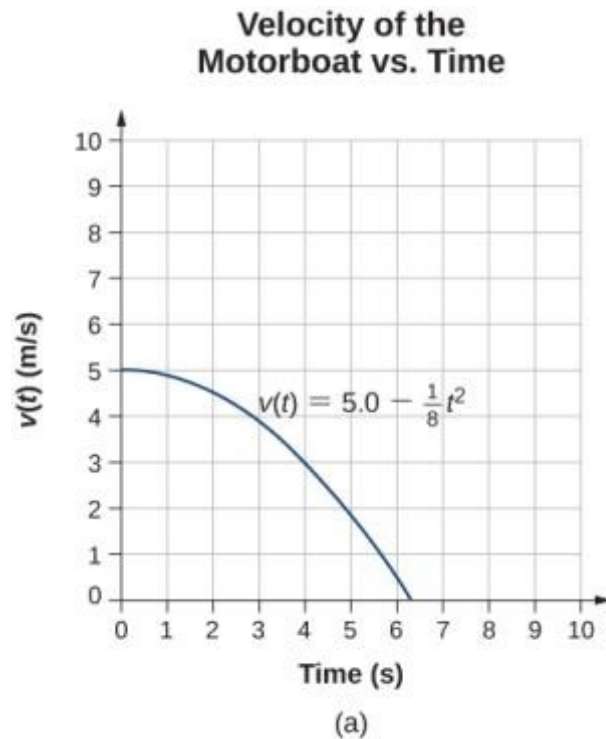
A baseball hit straight up is caught by the catcher 5.0 s later.

EXAMPLE



A rocket releases its booster at a given height and velocity. How high and how fast does the booster go?

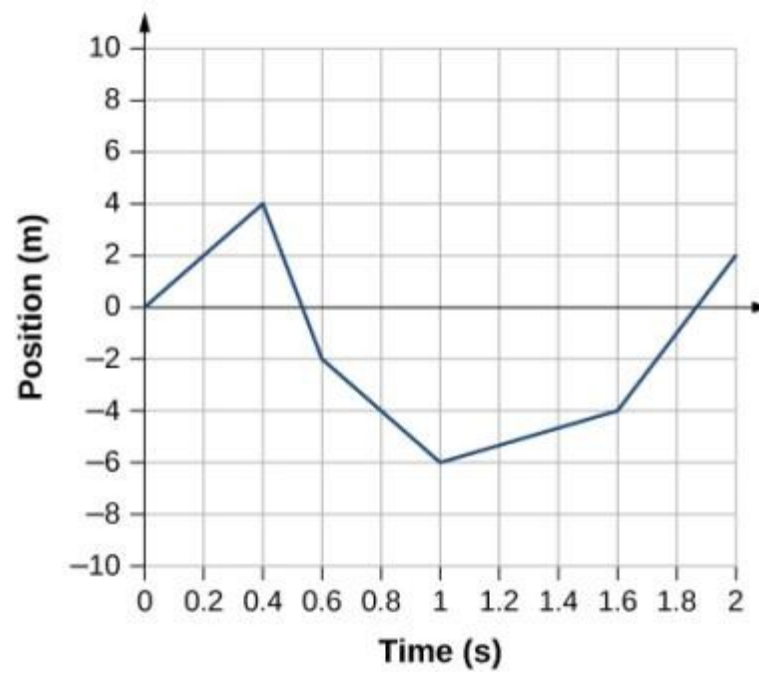
VELOCITY AND POSITION



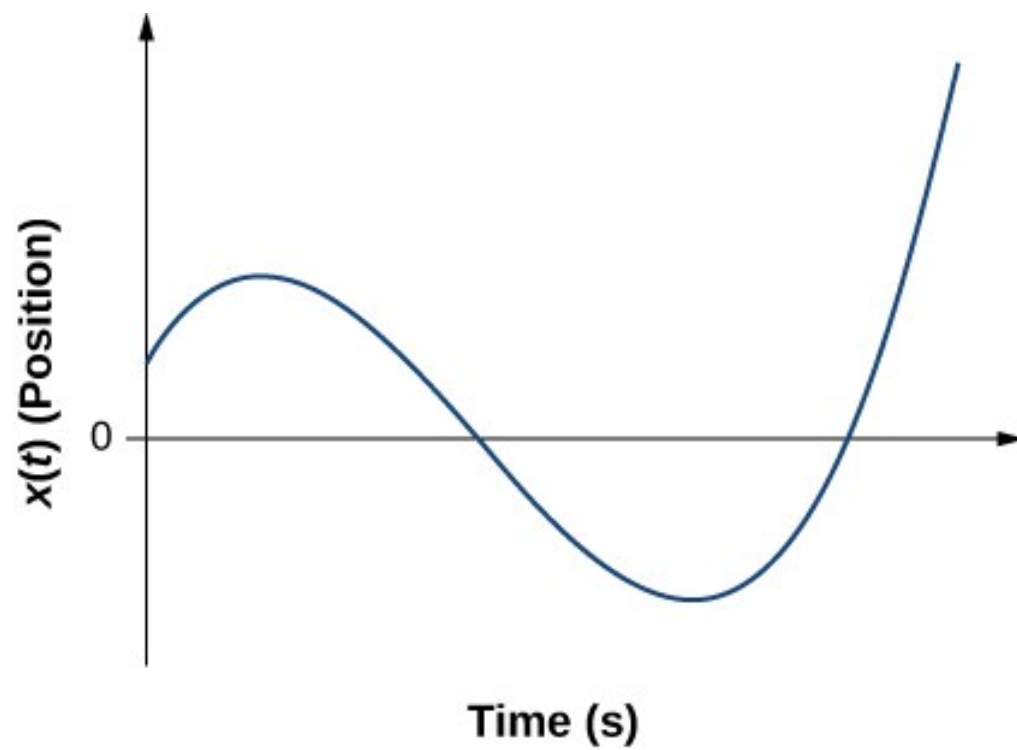
- a) Velocity of the motorboat as a function of time. The motorboat decreases its velocity to zero in 6.3 s. At times greater than this, velocity becomes negative—meaning, the boat is reversing direction.
- b) Position of the motorboat as a function of time. At $t = 6.3$ s, the velocity is zero and the boat has stopped. At times greater than this, the velocity becomes negative—meaning, if the boat continues to move with the same acceleration, it reverses direction and heads back toward where it originated.

EXERCISE 31

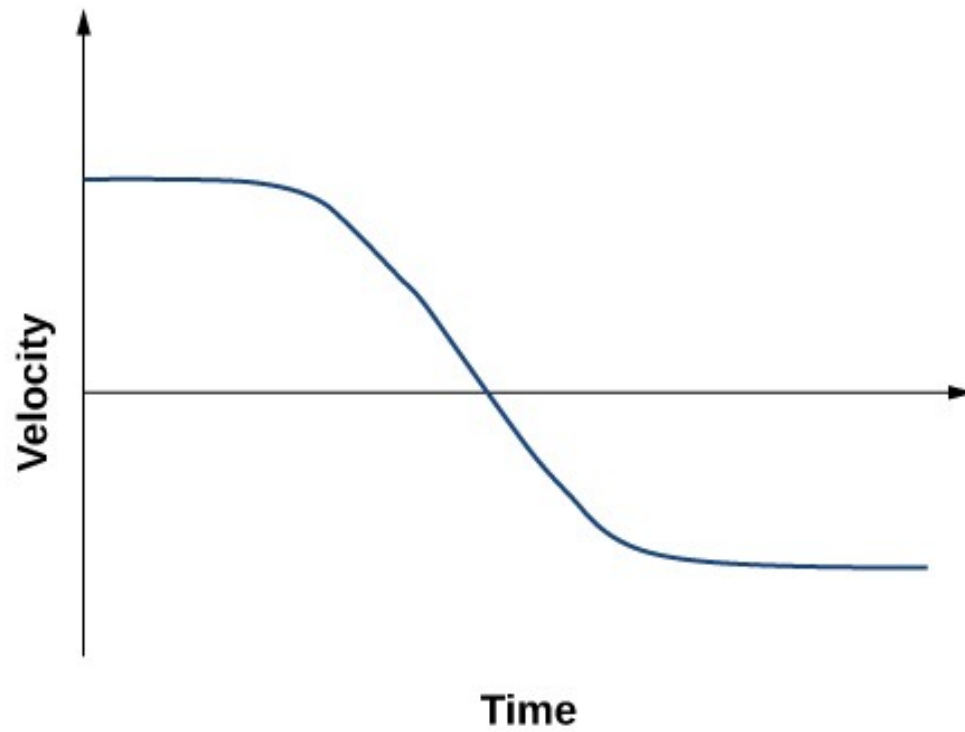
Position vs. Time



EXERCISE 32

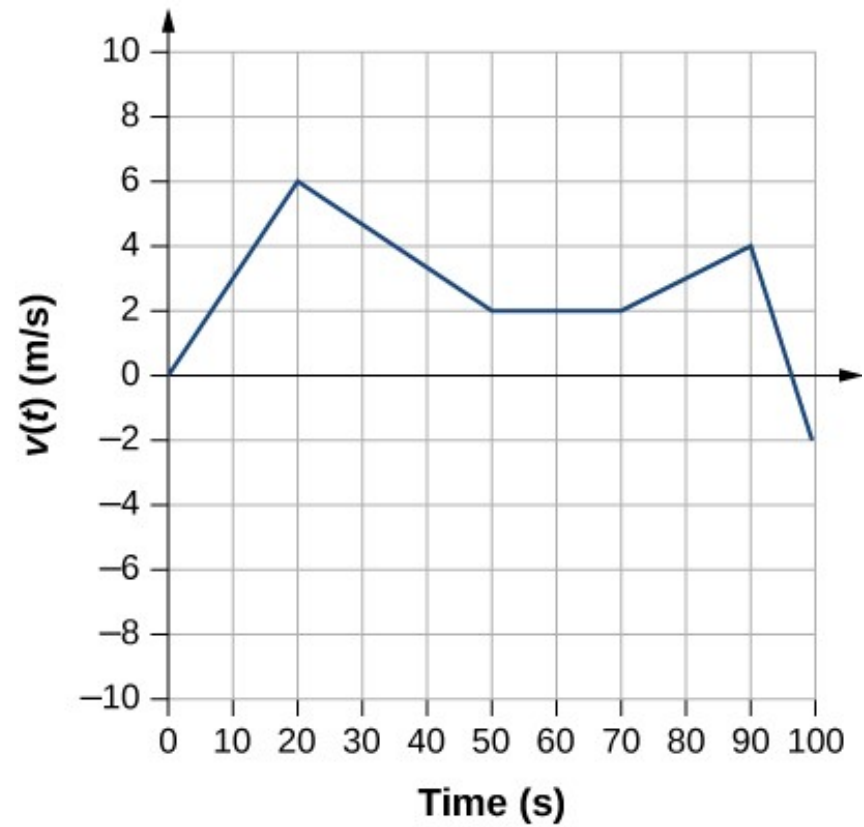


EXERCISE 33

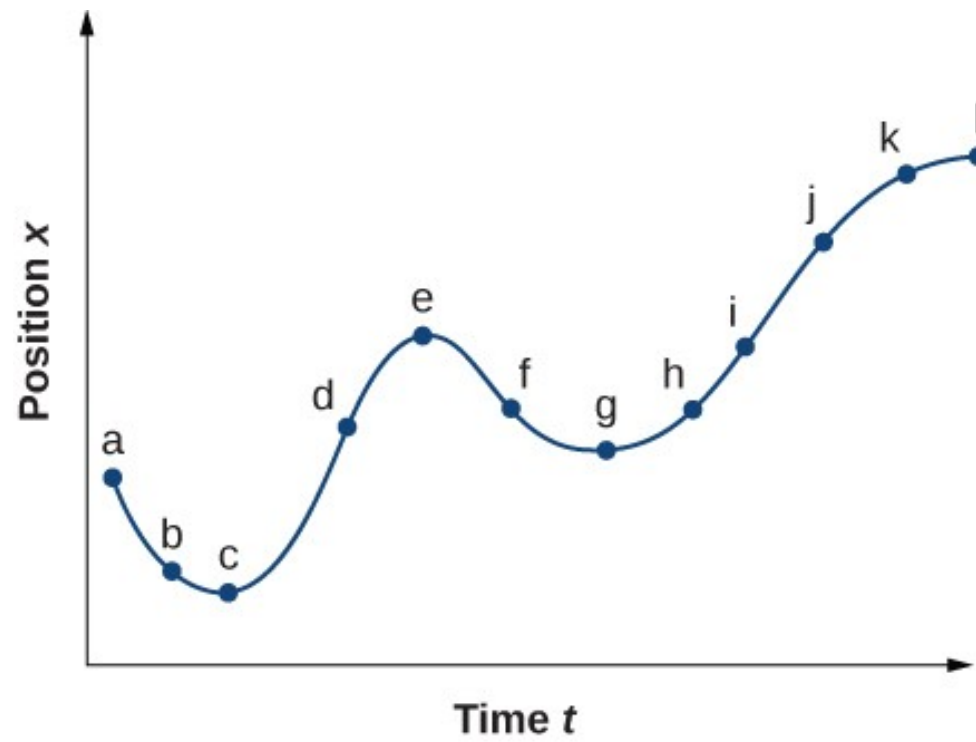


EXERCISE 39

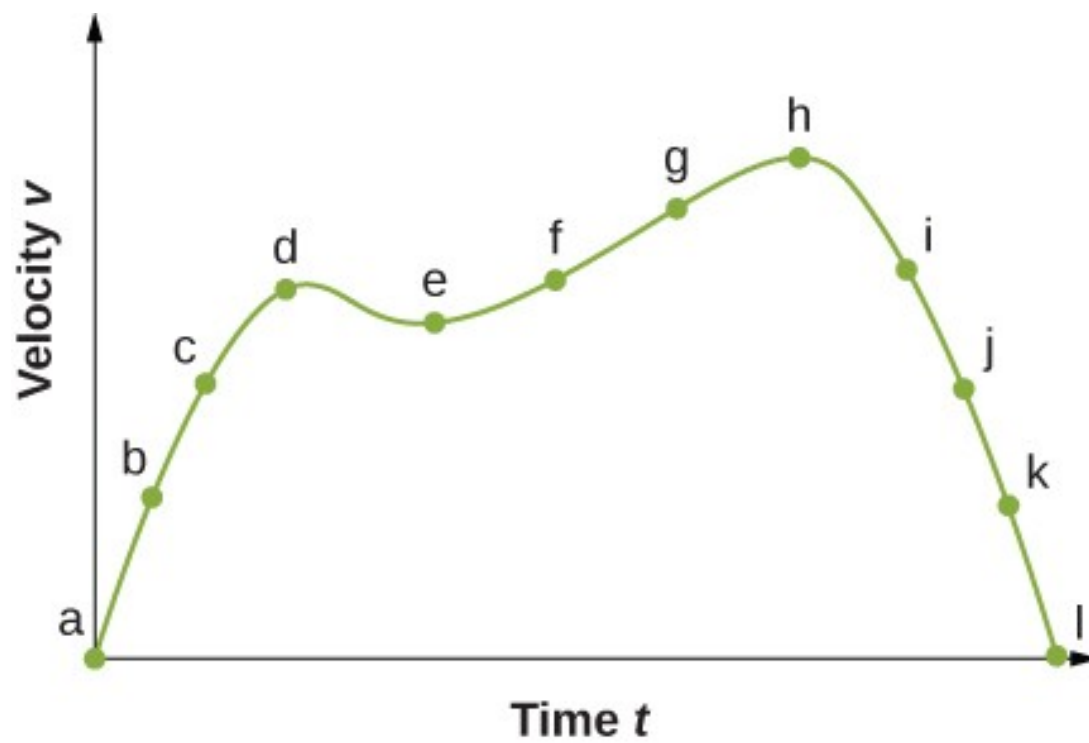
Velocity vs. Time



EXERCISE 46



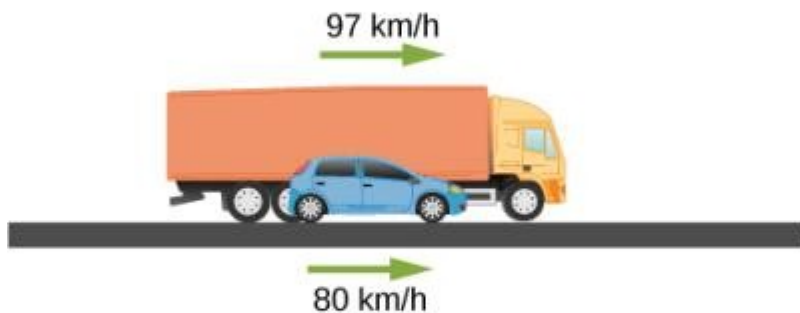
EXERCISE 47



EXERCISE 94



Before



After