ANNOUNCEMENTS

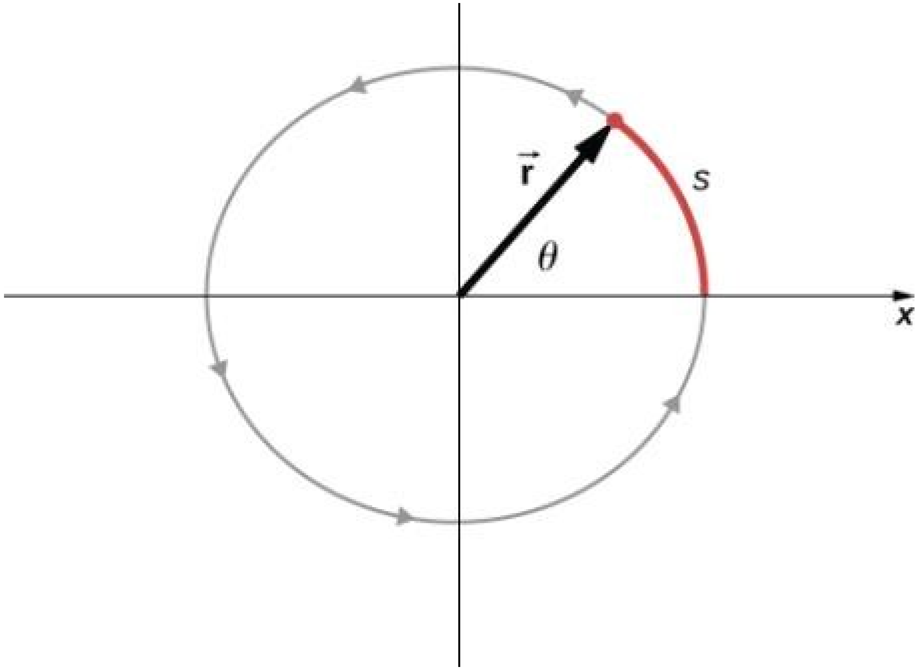
- Homework #9, due today!

  Conceptual questions: Chapter 9, #4 and #10
  Problems: Chapter 9, #30, #54

- 5-minute quiz on Chapter 9 Friday, October 26

- Second in-class test: Friday, November 2

In Figure 10.2, we show a particle moving in a circle. The coordinate system is fixed and serves as a frame of reference to define the particle's position. Its position vector from the origin of the circle to the particle sweeps out the angle θ, which increases in the counterclockwise direction as the particle moves along its circular path. The angle θ is called the **angular position** of the particle. As the particle moves in its circular path, it also traces an arc length *s*.



$$\theta = \frac{s}{r}.$$

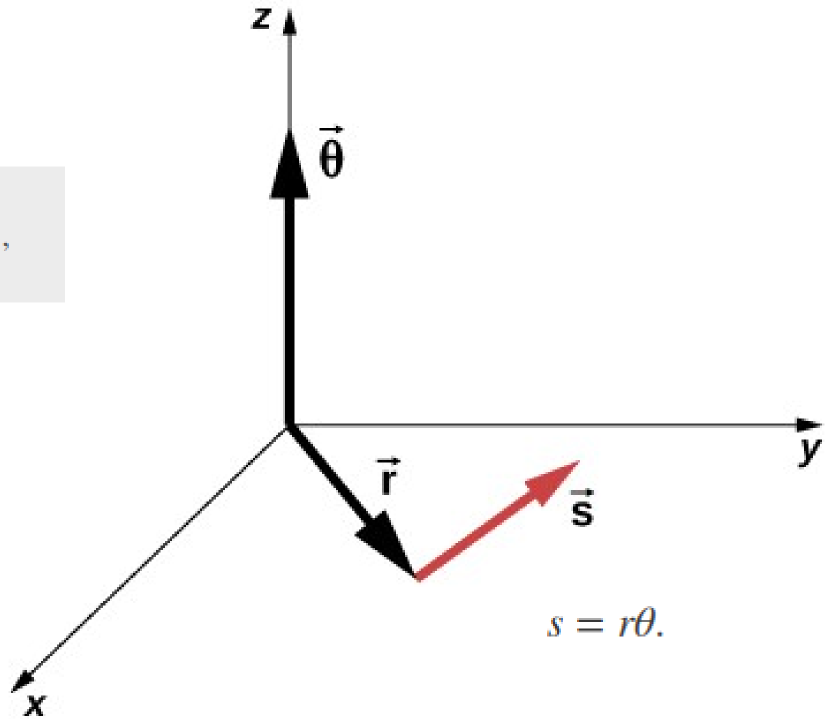
A particle follows a circular path. As it moves counterclockwise, it sweeps out a positive angle θ with respect to the *x*-axis and traces out an arc length *s*.

We can assign vectors to the quantities in Equation 10.1. The angle $\vec{\boldsymbol{\theta}}$ is a vector out of the page in Figure 10.2. The angular position vector $\vec{\mathbf{r}}$ and the arc length $\vec{\mathbf{s}}$ both lie in the plane of the page. These three vectors are related to each other by

$$\vec{\mathbf{s}} = \vec{\boldsymbol{\theta}} \times \vec{\mathbf{r}}. \tag{10.2}$$

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt},$$

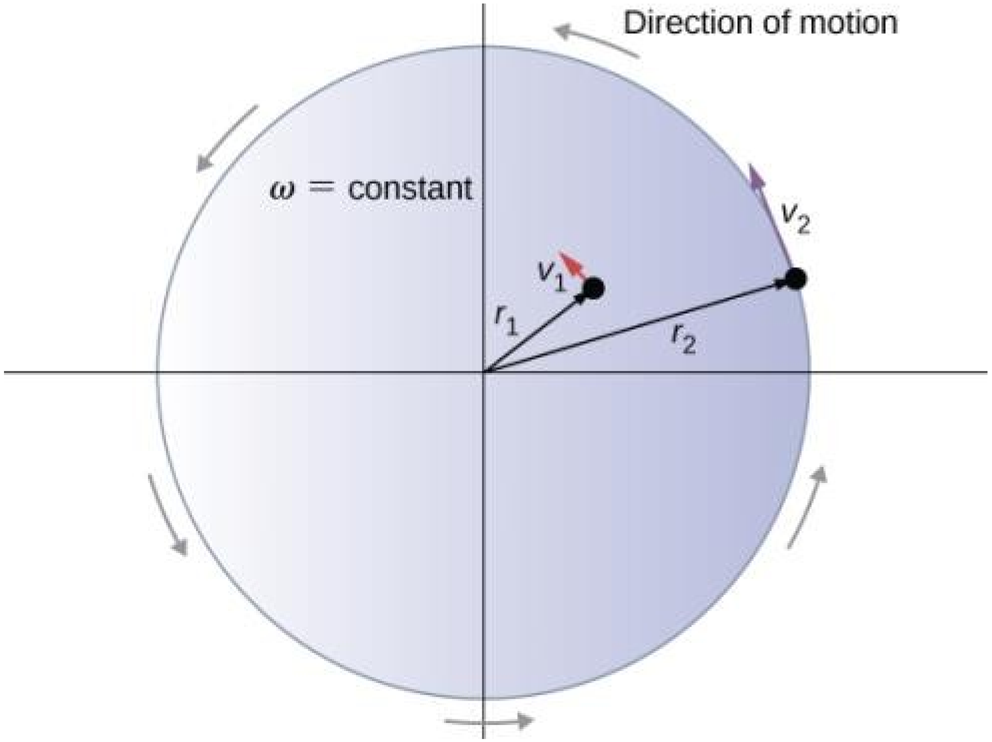
$$v_{\mathrm{t}} = r\omega.$$



$$s = r\theta.$$

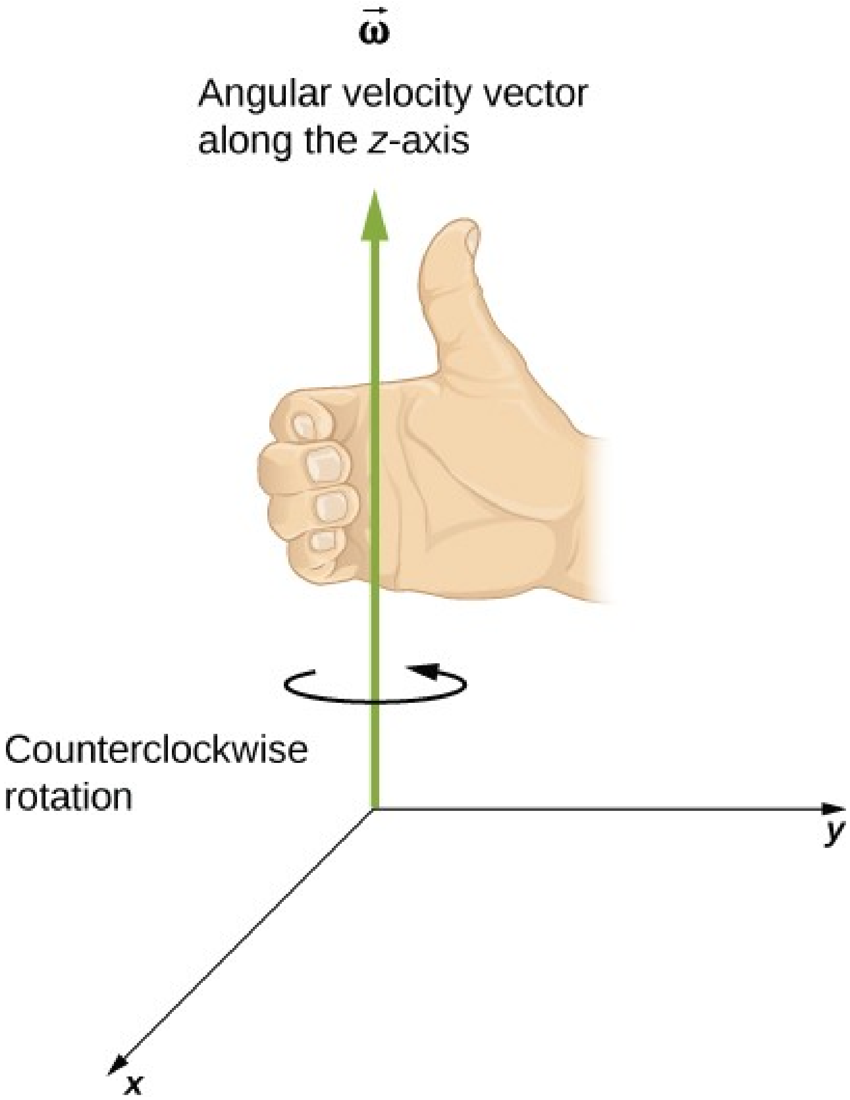
The angle vector points along the *z*-axis and the position vector and arc length vector both lie in the *xy*-plane. We see that $\vec{s} = \vec{\theta} \times \vec{r}$. All three vectors are perpendicular to each other.

TANGENTIAL SPEED



Two particles on a rotating disk have different tangential speeds, depending on their distance to the axis of rotation.

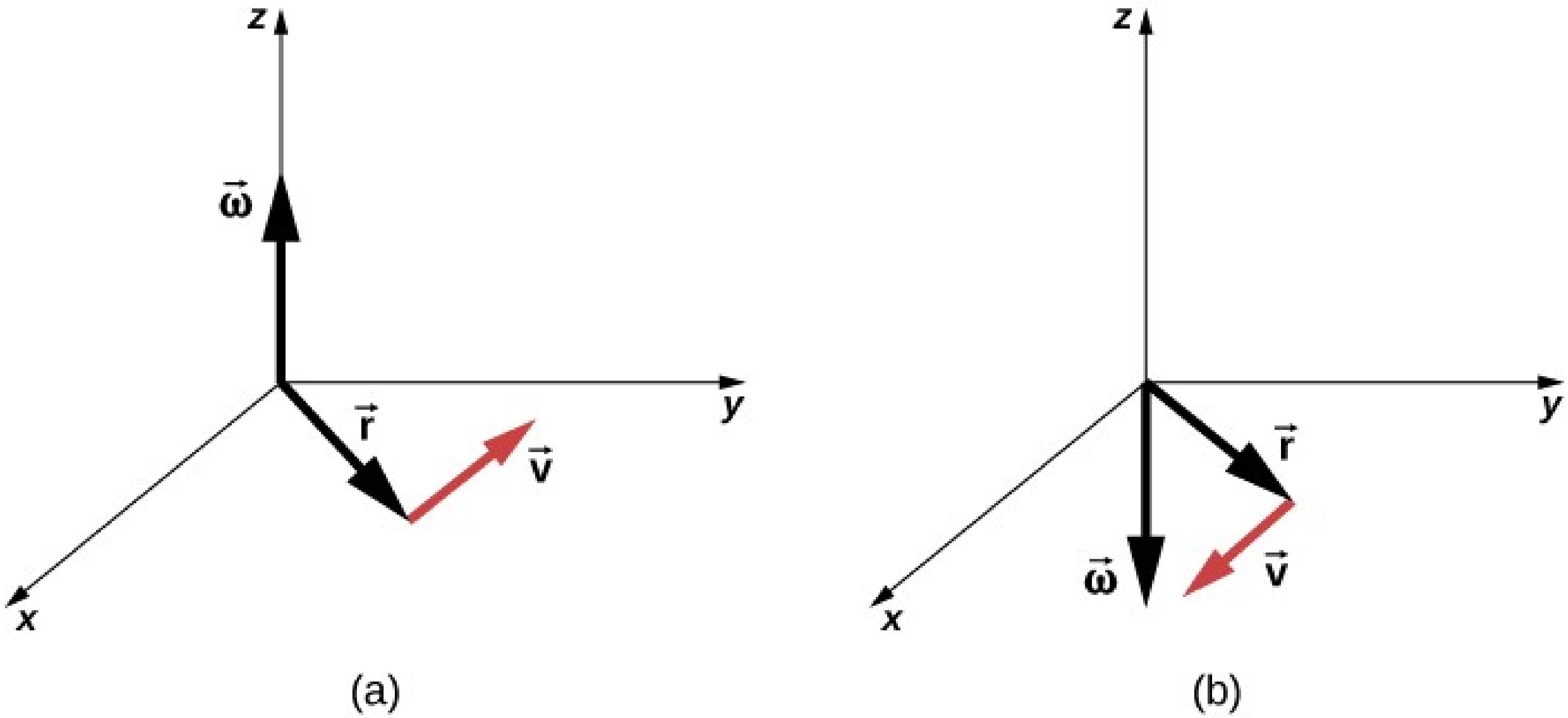
FIGURE 10.5



For counterclockwise rotation in the coordinate system shown, the angular velocity points in the positive z-direction by the right-hand-rule.

$$\vec{v} = \vec{\omega} \times \vec{r}.$$

FIGURE 10.6



The vectors shown are the angular velocity, position, and tangential velocity.

(a) The angular velocity points in the positive *z*-direction, giving a counterclockwise rotation in the *xy*-plane.

(b) The angular velocity points in the negative *z*-direction, giving a clockwise rotation.

EXAMPLE 10.1

Rotation of a Flywheel

A flywheel rotates such that it sweeps out an angle at the rate of $\theta = \omega t = (45.0 \text{ rad/s})t$ radians. The wheel rotates counterclockwise when viewed in the plane of the page. (a) What is the angular velocity of the flywheel? (b) What direction is the angular velocity? (c) How many radians does the flywheel rotate through in 30 s? (d) What is the tangential speed of a point on the flywheel 10 cm from the axis of rotation?

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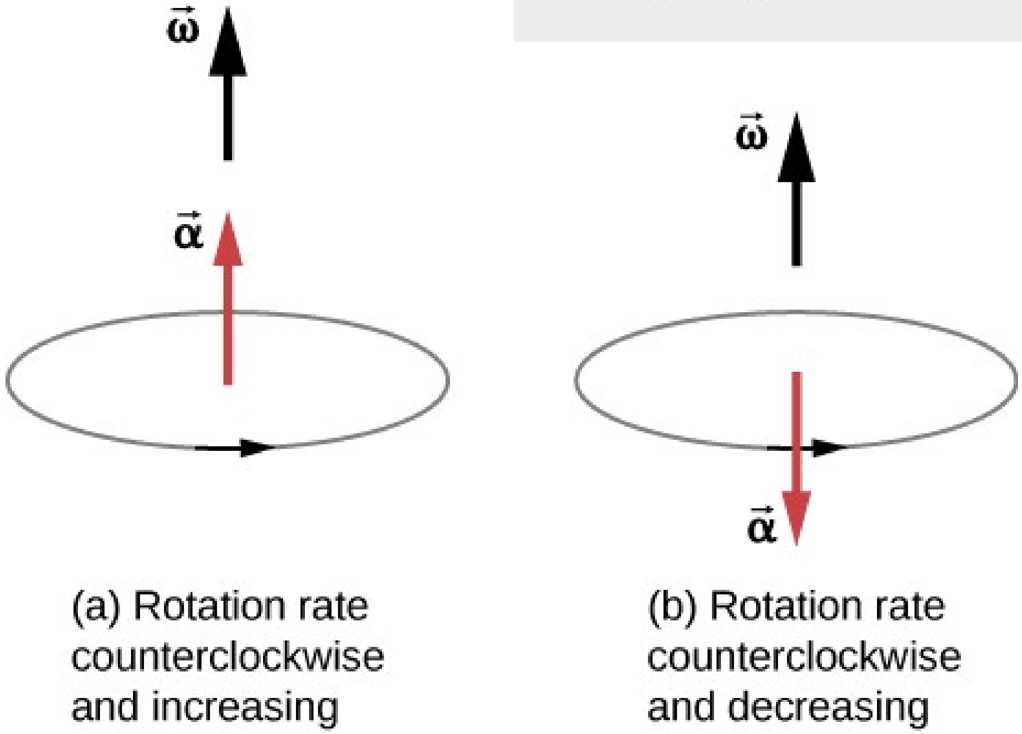
Angular position	$\theta = \frac{s}{r}$
Angular velocity	$\omega = \lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t} = \frac{d\theta}{dt}$
Tangential speed	$v_t = r\omega$

Solution

a. $\omega = \frac{d\theta}{dt} = 45 \text{ rad/s}$. We see that the angular velocity is a constant.
b. By the right-hand rule, we curl the fingers in the direction of rotation, which is counterclockwise in the plane of the page, and the thumb points in the direction of the angular velocity, which is out of the page.
c. $\Delta\theta = \theta(30 \text{ s}) - \theta(0 \text{ s}) = 45.0(30.0 \text{ s}) - 45.0(0 \text{ s}) = 1350.0 \text{ rad}$.
d. $v_t = r\omega = (0.1 \text{ m})(45.0 \text{ rad/s}) = 4.5 \text{ m/s}$.

ANGULAR ACCELERATION

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2},$$



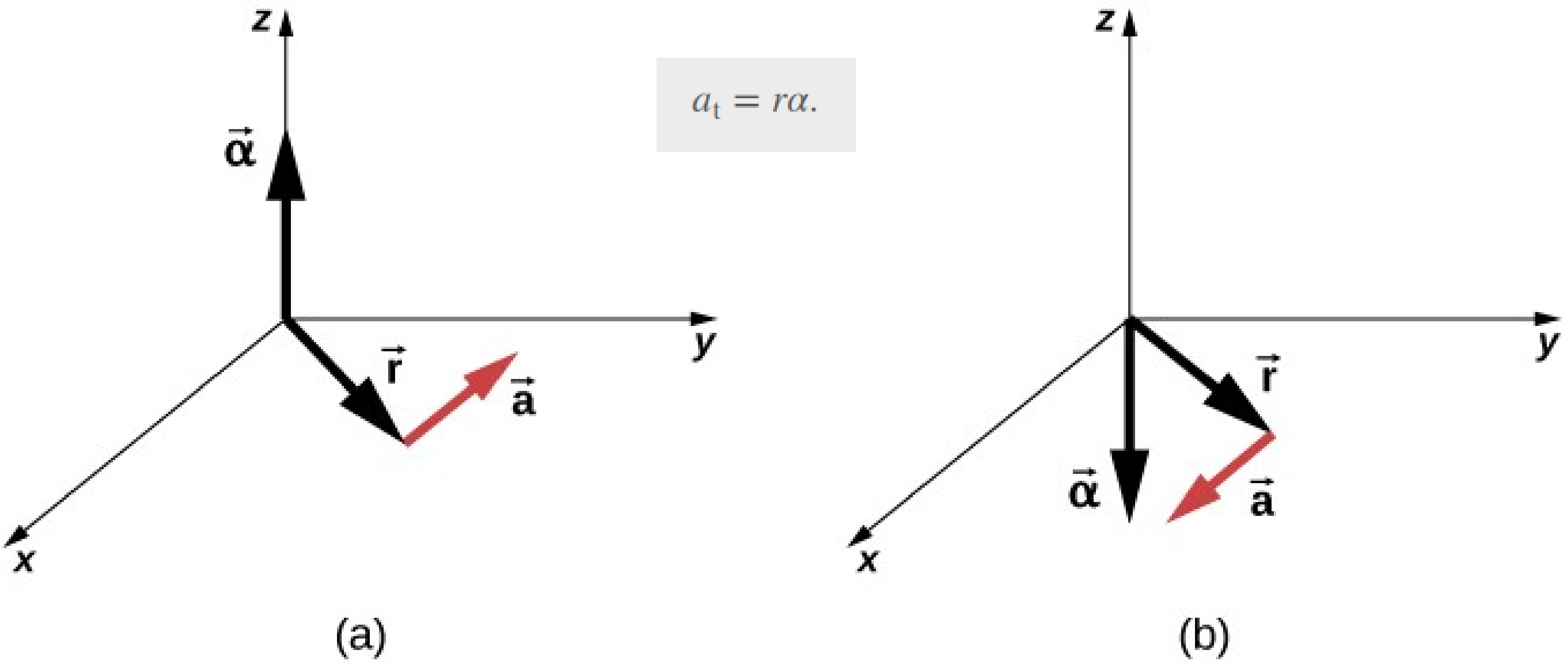
The rotation is counterclockwise in both (a) and (b) with the angular velocity in the same direction.

(a) The angular acceleration is in the same direction as the angular velocity, which increases the rotation rate.

(b) The angular acceleration is in the opposite direction to the angular velocity, which decreases the rotation rate.

$$\vec{\mathbf{a}} = \vec{\boldsymbol{\alpha}} \times \vec{\mathbf{r}}.$$

$$a_t = r\alpha.$$



(a) The angular acceleration is the positive *z*-direction and produces a tangential acceleration in a counterclockwise sense.

(b) The angular acceleration is in the negative *z*-direction and produces a tangential acceleration in the clockwise sense.

EXAMPLE

A Spinning Bicycle Wheel

A bicycle mechanic mounts a bicycle on the repair stand and starts the rear wheel spinning from rest to a final angular velocity of 250 rpm in 5.00 s. (a) Calculate the average angular acceleration in $\mathrm{rad/s^2}$. (b) If she now hits the brakes, causing an angular acceleration of $-87.3\ \mathrm{rad/s^2}$, how long does it take the wheel to stop?

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Solution

a. Entering known information into the definition of angular acceleration, we get

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \frac{250\ \text{rpm}}{5.00\ \text{s}}.$$

Because $\Delta\omega$ is in revolutions per minute (rpm) and we want the standard units of rad/s^2 for angular acceleration, we need to convert from rpm to rad/s:

$$\Delta\omega = 250\frac{\text{rev}}{\text{min}} \cdot \frac{2\pi\ \text{rad}}{\text{rev}} \cdot \frac{1\ \text{min}}{60\ \text{s}} = 26.2\frac{\text{rad}}{\text{s}}.$$

Entering this quantity into the expression for α, we get

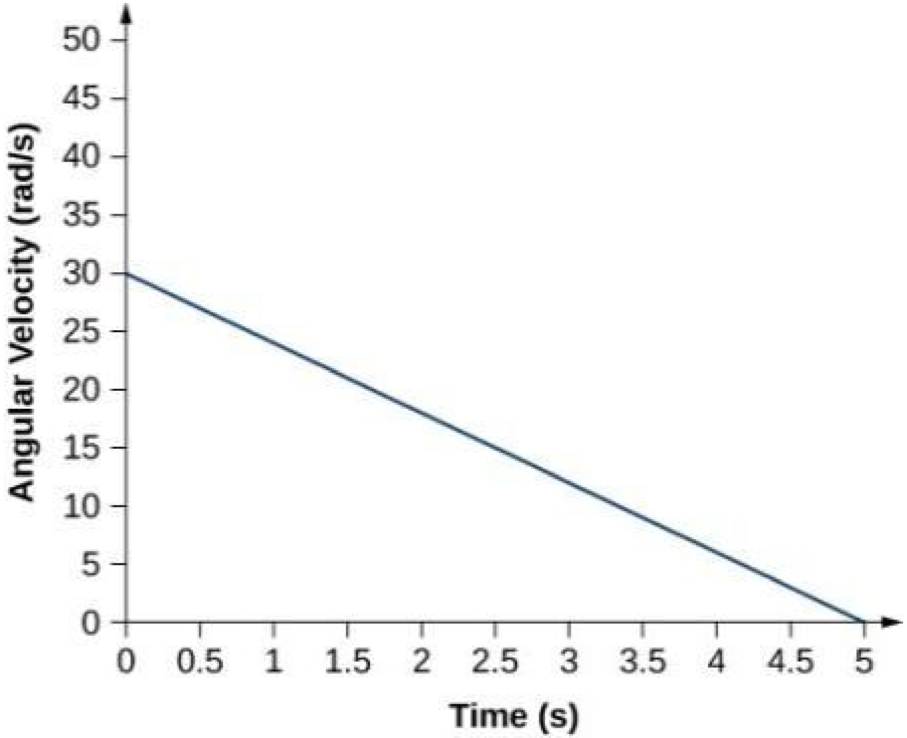
$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{26.2\ \text{rad/s}}{5.00\ \text{s}} = 5.24\ \text{rad/s}^2.$$

b. Here the angular velocity decreases from 26.2 rad/s (250 rpm) to zero, so that $\Delta\omega$ is −26.2 rad/s, and α is given to be –87.3 rad/s^2. Thus,

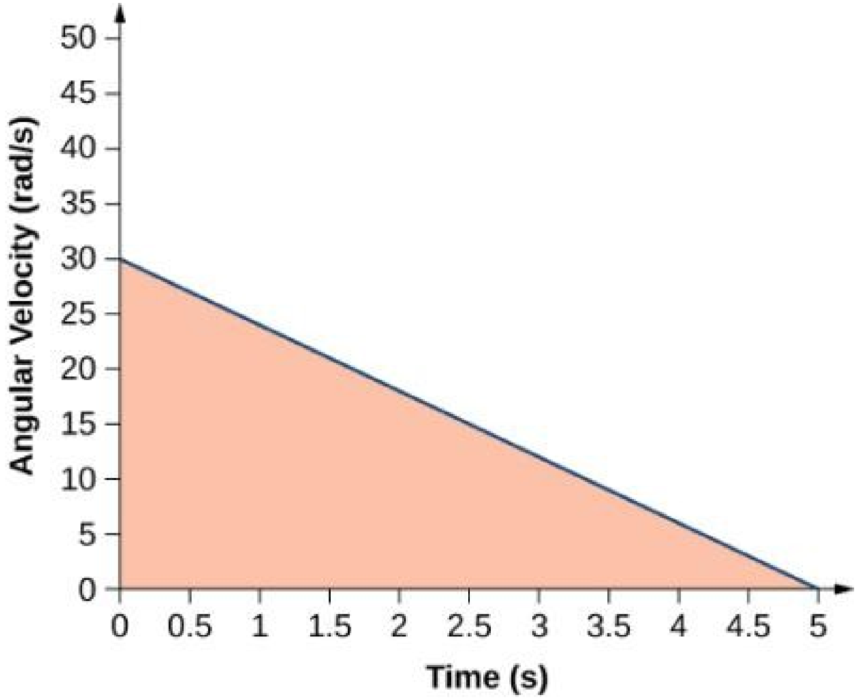
$$\Delta t = \frac{-26.2\ \text{rad/s}}{-87.3\ \text{rad/s}^2} = 0.300\ \text{s}.$$

CONSTANT ANGULAR DECELERATION

$$\omega_{\mathrm{f}} = \omega_0 + \alpha t,$$



CONSTANT ANGULAR DECELERATION



$$\omega_f = \omega_0 + \alpha t,$$

$$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2.$$

$$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta).$$

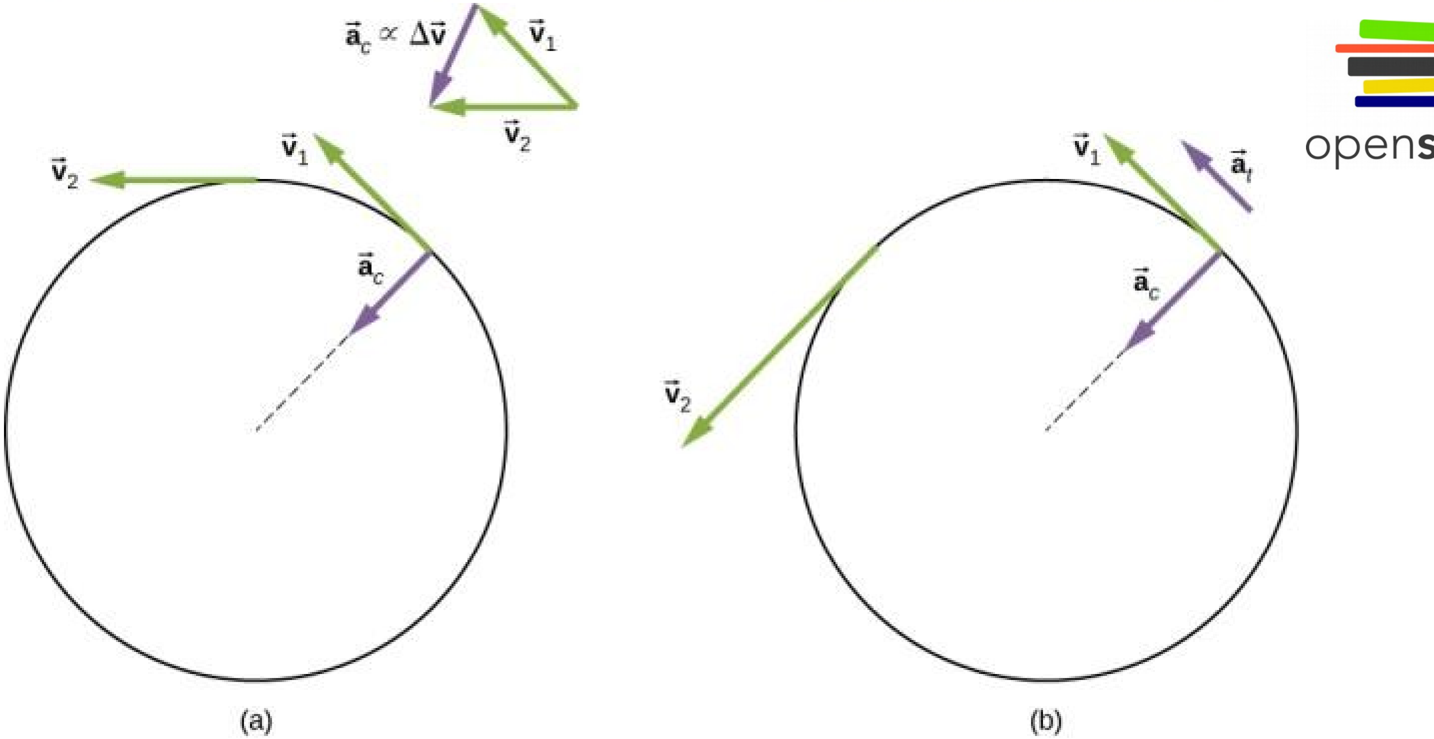
Angular displacement from average angular velocity	$\theta_f = \theta_0 + \bar{\omega} t$
Angular velocity from angular acceleration	$\omega_f = \omega_0 + \alpha t$
Angular displacement from angular velocity and angular acceleration	$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
Angular velocity from angular displacement and angular acceleration	$\omega_f^2 = {\omega_0}^2 + 2\alpha(\Delta\theta)$

ANGULAR VS LINEAR VARIABLES AND EQUATIONS

	Linear	Rotational
Position	x	θ
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

$\theta_f = \theta_0 + \bar{\omega}t$	$x = x_0 + \bar{v}t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + at$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

θ	s	$\theta = \frac{s}{r}$
ω	v_t	$\omega = \frac{v_t}{r}$
α	a_t	$\alpha = \frac{a_t}{r}$
	a_c	$a_c = \frac{v_t^2}{r}$

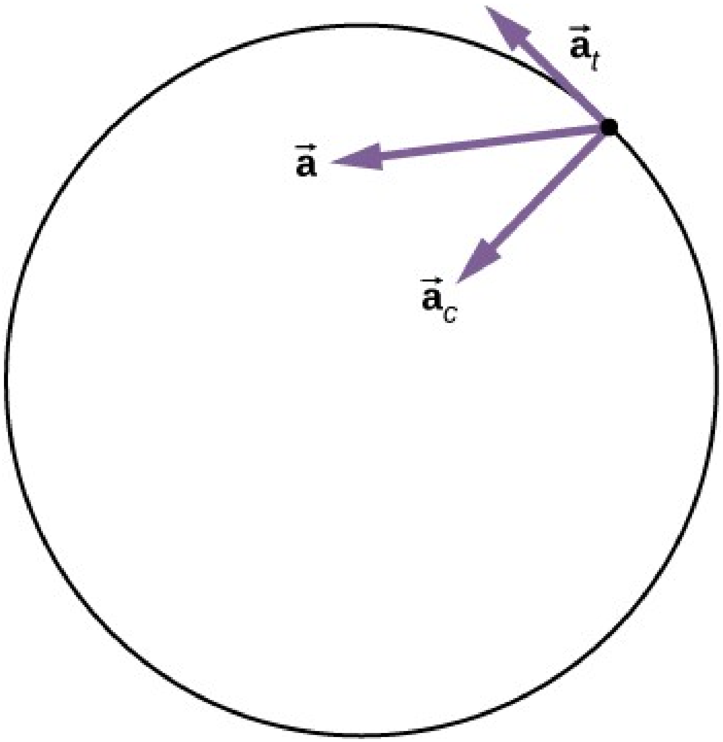


(a) Uniform circular motion: The centripetal acceleration a_c has its vector inward toward the axis of rotation. There is no tangential acceleration.

(b) Nonuniform circular motion: An angular acceleration produces an inward centripetal acceleration that is changing in magnitude, plus a tangential acceleration a_t .

$$\vec{a} = \vec{a}_c + \vec{a}_t.$$

$$|\vec{a}| = \sqrt{a_c^2 + a_t^2}.$$

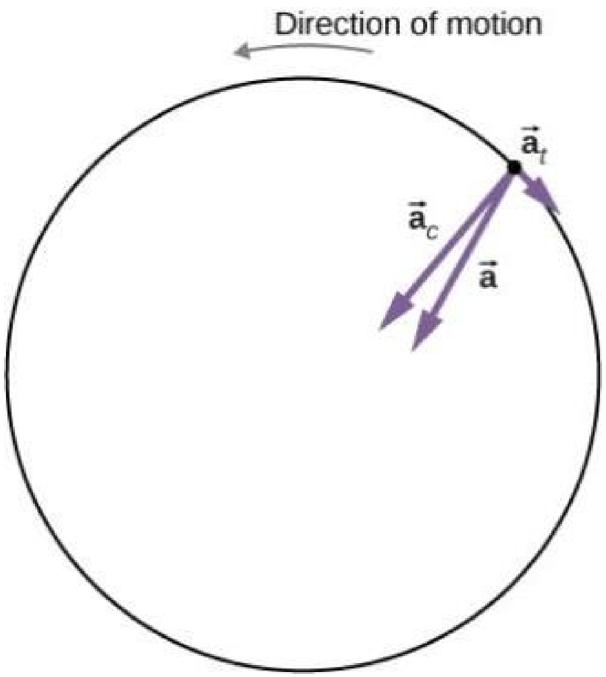


$$\vec{a} = \vec{a}_c + \vec{a}_t.$$

A particle is executing circular motion and has an angular acceleration. The total linear acceleration of the particle is the vector sum of the centripetal acceleration and tangential acceleration vectors. The total linear acceleration vector is at an angle in between the centripetal and tangential accelerations.

Linear Acceleration of a Centrifuge

A centrifuge has a radius of 20 cm and accelerates from a maximum rotation rate of 10,000 rpm to rest in 30 seconds under a constant angular acceleration. It is rotating counterclockwise. What is the magnitude of the total acceleration of a point at the tip of the centrifuge at $t = 29.0\text{s}$? What is the direction of the total acceleration vector?



The centripetal, tangential, and total acceleration vectors. The centrifuge is slowing down, so the tangential acceleration is clockwise, opposite the direction of rotation (counterclockwise).

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Solution

The angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - (1.0 \times 10^4)2\pi/60.0\text{ s(rad/s)}}{30.0\text{ s}} = -34.9\text{ rad/s}^2.$$

Therefore, the tangential acceleration is

$$a_t = r\alpha = 0.2\text{ m}(-34.9\text{ rad/s}^2) = -7.0\text{ m/s}^2.$$

The angular velocity at $t = 29.0$ s is

$$\begin{aligned}\omega &= \omega_0 + \alpha t = 1.0 \times 10^4\left(\frac{2\pi}{60.0\text{ s}}\right) + \left(-34.9\text{ rad/s}^2\right)(29.0\text{ s}) \\ &= 1047.2\text{ rad/s} - 1012.71 = 35.1\text{ rad/s}.\end{aligned}$$

Thus, the tangential speed at $t = 29.0$ s is

$$v_t = r\omega = 0.2\text{ m}(35.1\text{ rad/s}) = 7.0\text{ m/s}.$$

We can now calculate the centripetal acceleration at $t = 29.0$ s:

$$a_c = \frac{v^2}{r} = \frac{(7.0\text{ m/s})^2}{0.2\text{ m}} = 245.0\text{ m/s}^2.$$

Since the two acceleration vectors are perpendicular to each other, the magnitude of the total linear acceleration is

$$|\vec{\mathbf{a}}| = \sqrt{a_c^2 + a_t^2} = \sqrt{(245.0)^2 + (-7.0)^2} = 245.1\text{ m/s}^2.$$

Since the centrifuge has a negative angular acceleration, it is slowing down. The total acceleration vector is as shown in Figure 10.16. The angle with respect to the centripetal acceleration vector is

$$\theta = \tan^{-1}\frac{-7.0}{245.0} = -1.6°.$$

The negative sign means that the total acceleration vector is angled toward the clockwise direction.