ANNOUNCEMENTS

- Homework #12, due Friday, Nov. 16 before class

  Conceptual questions: Chapter 12, #4 and #10
  Problems: Chapter 12, #32, #38

- Study Chapter 13, sections 1 through 5. We won't have a quiz or homework on chapter 13, but this will be in the final exam!

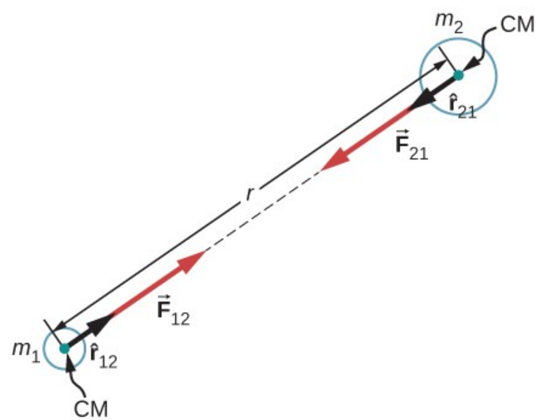
- Quiz #12, Friday November 16 at the beginning of class

NEWTON'S LAW OF GRAVITATION

Newton's law of gravitation can be expressed as

$$\vec{\mathbf{F}}_{12} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12} \tag{13.1}$$

where $\vec{\mathbf{F}}_{12}$ is the force on object 1 exerted by object 2 and $\hat{\mathbf{r}}_{12}$ is a unit vector that points from object 1 toward object 2.



$$U = -\frac{GM_E m}{r}. \tag{13.4}$$

Conservation of Energy

In Potential Energy and Conservation of Energy, we described how to apply conservation of energy for systems with conservative forces. We were able to solve many problems, particularly those involving gravity, more simply using conservation of energy. Those principles and problem-solving strategies apply equally well here. The only change is to place the new expression for potential energy into the conservation of energy equation, $E = K_1 + U_1 = K_2 + U_2$.

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2} \tag{13.5}$$

Escape velocity

Escape velocity is often defined to be the *minimum* initial velocity of an object that is required to escape the surface of a planet (or any large body like a moon) and never return. As usual, we assume no energy lost to an atmosphere, should there be any.

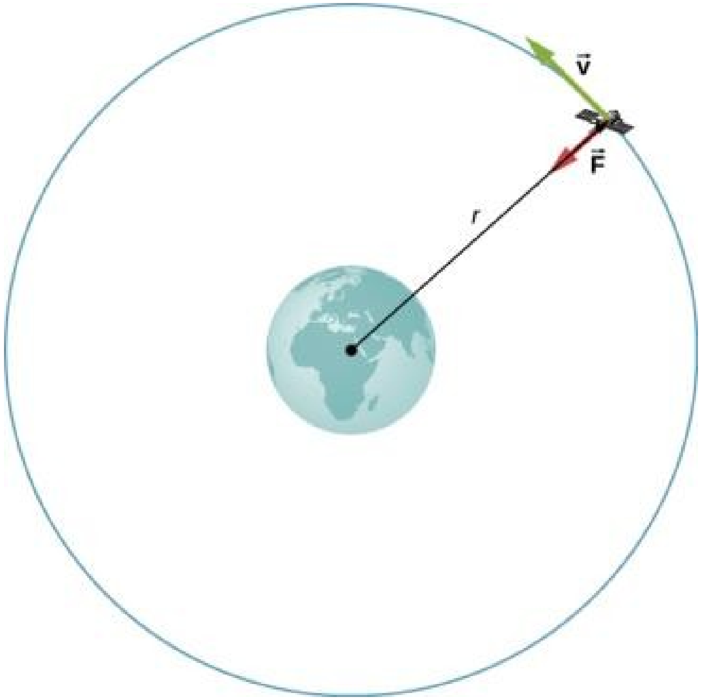
Consider the case where an object is launched from the surface of a planet with an initial velocity directed away from the planet. With the *minimum* velocity needed to escape, the object would *just* come to rest infinitely far away, that is, the object gives up the last of its kinetic energy just as it reaches infinity, where the force of gravity becomes zero. Since $U \to 0$ as $r \to \infty$, this means the total energy is zero. Thus, we find the escape velocity from the surface of an astronomical body of mass *M* and radius *R* by setting the total energy equal to zero. At the surface of the body, the object is located at $r_1 = R$ and it has escape velocity $v_1 = v_{\text{esc}}$. It reaches $r_2 = \infty$ with velocity $v_2 = 0$. Substituting into Equation 13.5, we have

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R} = \frac{1}{2}m0^2 - \frac{GMm}{\infty} = 0.$$

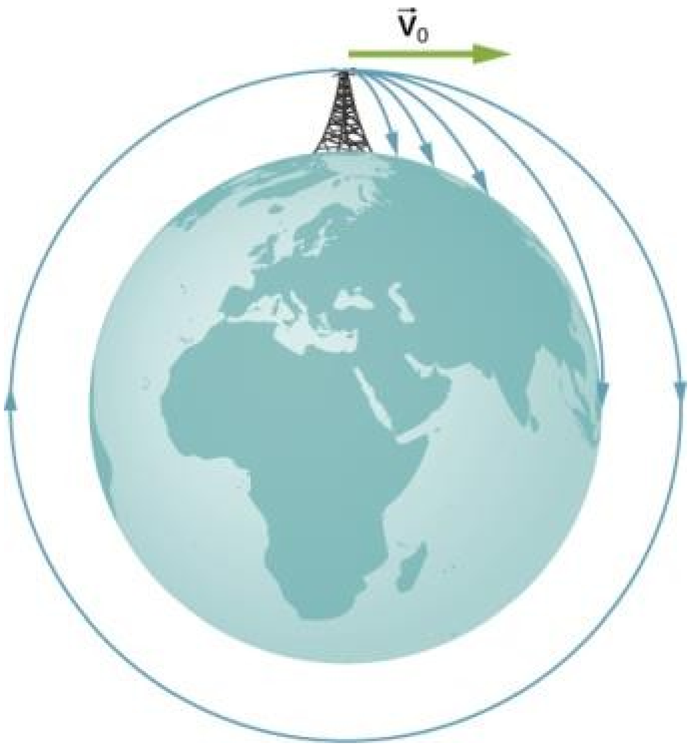
Solving for the escape velocity,

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}. \tag{13.6}$$

A satellite of mass *m* orbiting at radius *r* from the center of Earth. The gravitational force supplies the centripetal acceleration.



$$\frac{GmM_E}{r^2} = ma_c = \frac{mv_{orbit}^2}{r}. \longrightarrow v_{orbit} = \sqrt{\frac{GM_E}{r}}. \longrightarrow T = 2\pi\sqrt{\frac{r^3}{GM_E}}.$$



A circular orbit is the result of choosing a tangential velocity such that Earth’s surface curves away at the same rate as the object falls toward Earth.

Energy in Circular Orbits

In Gravitational Potential Energy and Total Energy, we argued that objects are gravitationally bound if their total energy is negative. The argument was based on the simple case where the velocity was directly away or toward the planet. We now examine the total energy for a circular orbit and show that indeed, the total energy is negative. As we did earlier, we start with Newton's second law applied to a circular orbit,

$$\begin{aligned} \frac{GmM_E}{r^2} &= ma_c = \frac{mv^2}{r} \\ \frac{GmM_E}{r} &= mv^2. \end{aligned}$$

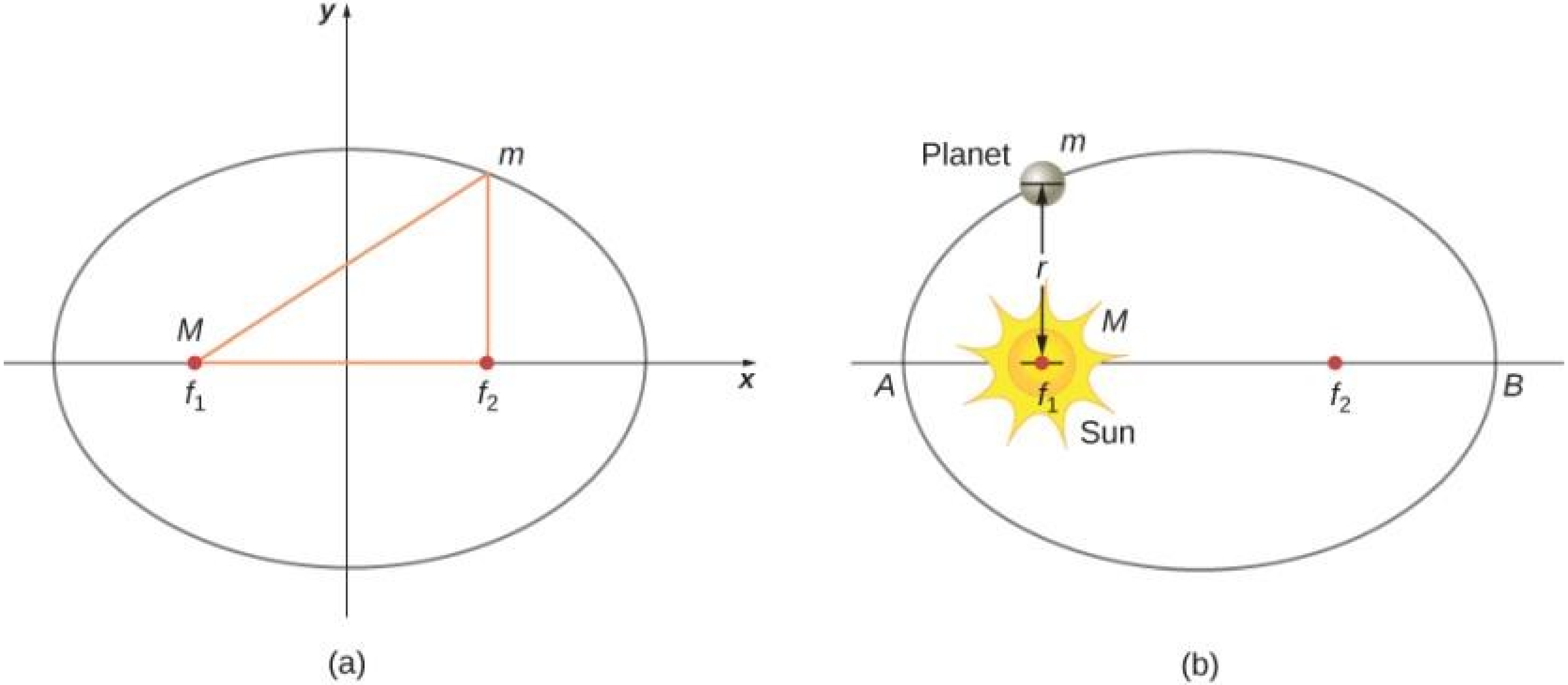
In the last step, we multiplied by *r* on each side. The right side is just twice the kinetic energy, so we have

$$K = \frac{1}{2}mv^2 = \frac{GmM_E}{2r}.$$

The total energy is the sum of the kinetic and potential energies, so our final result is

$$E = K + U = \frac{GmM_E}{2r} - \frac{GmM_E}{r} = -\frac{GmM_E}{2r}. \tag{13.9}$$

KEPLERIAN ORBITS

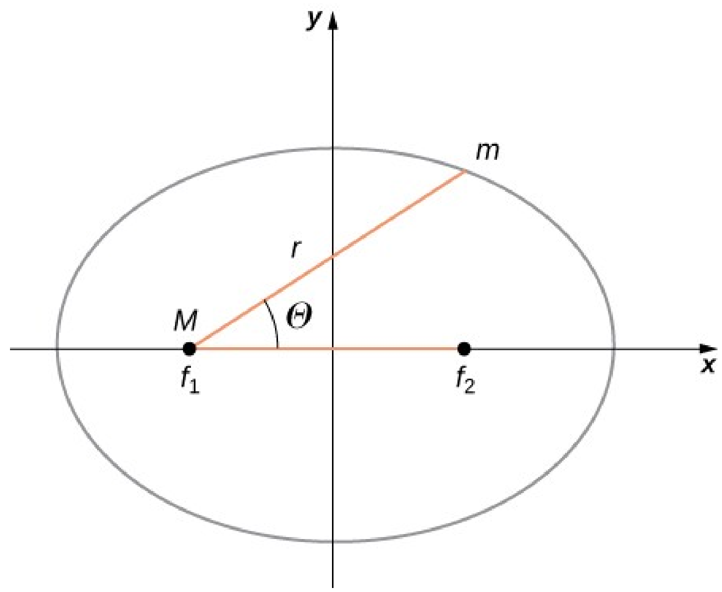


(a) An ellipse is a curve in which the sum of the distances from a point on the curve to two foci (f_1 and f_2) is a constant. From this definition, you can see that an ellipse can be created in the following way. Place a pin at each focus, then place a loop of string around a pencil and the pins. Keeping the string taught, move the pencil around in a complete circuit. If the two foci occupy the same place, the result is a circle—a special case of an ellipse.

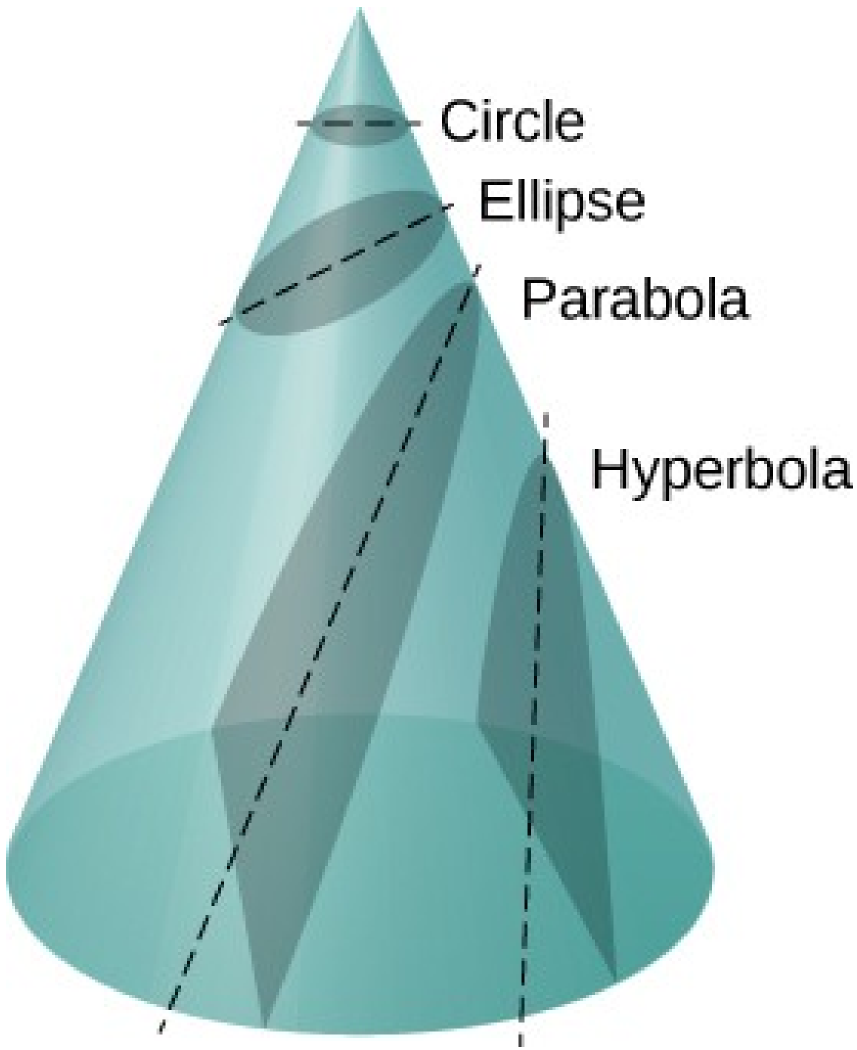
(b) For an elliptical orbit, if m $\ll$ *M*, then *m* follows an elliptical path with *M* at one focus. More exactly, both *m* and *M* move in their own ellipse about the common center of mass.

Kepler's First Law

The prevailing view during the time of Kepler was that all planetary orbits were circular. The data for Mars presented the greatest challenge to this view and that eventually encouraged Kepler to give up the popular idea. **Kepler's first law** states that every planet moves along an ellipse, with the Sun located at a focus of the ellipse. An ellipse is defined as the set of all points such that the sum of the distance from each point to two foci is a constant. Figure 13.16 shows an ellipse and describes a simple way to create it.



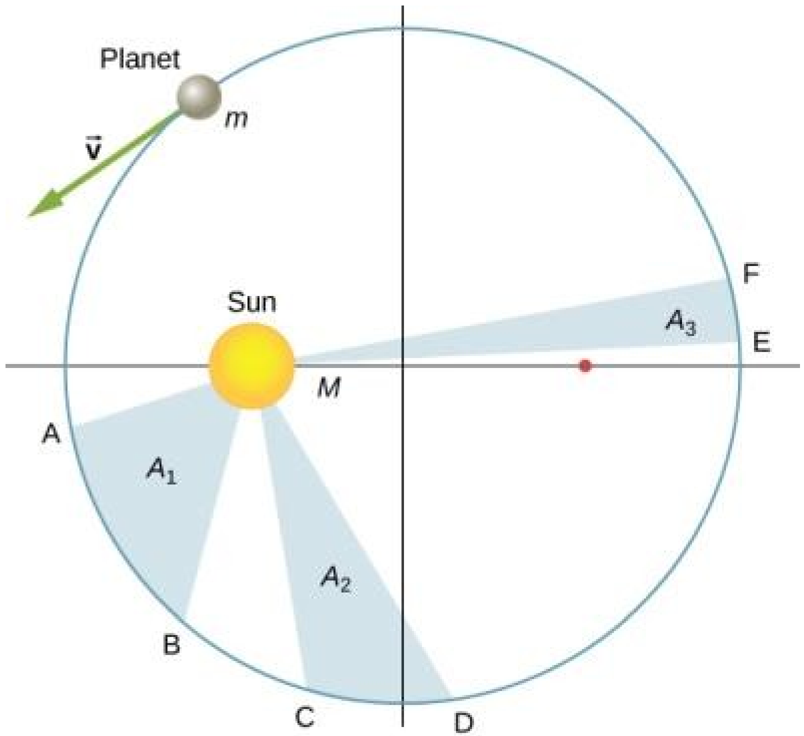
As before, the distance between the planet and the Sun is r, and the angle measured from the x-axis, which is along the major axis of the ellipse, is θ.



All motion caused by an inverse square force is one of the four conic sections and is determined by the energy and direction of the moving body.

Kepler’s Second Law

Kepler’s second law states that a planet sweeps out equal areas in equal times, that is, the area divided by time, called the areal velocity, is constant. Consider Figure 13.20. The time it takes a planet to move from position *A* to *B*, sweeping out area A_1, is exactly the time taken to move from position *C* to *D*, sweeping area A_2, and to move from *E* to *F*, sweeping out area A_3. These areas are the same: $A_1 = A_2 = A_3$.



The shaded regions shown have equal areas and represent the same time interval.

Kepler’s Third Law

Kepler’s third law states that the square of the period is proportional to the cube of the semi-major axis of the orbit. In Satellite Orbits and Energy, we derived Kepler’s third law for the special case of a circular orbit. Equation 13.8 gives us the period of a circular orbit of radius *r* about Earth:

$$T = 2\pi\sqrt{\frac{r^3}{GM_E}}.$$

For an ellipse, recall that the semi-major axis is one-half the sum of the perihelion and the aphelion. For a circular orbit, the semi-major axis (*a*) is the same as the radius for the orbit. In fact, Equation 13.8 gives us Kepler’s third law if we simply replace *r* with *a* and square both sides.

$$T^2 = \frac{4\pi^2}{GM}a^3 \tag{13.11}$$

We have changed the mass of Earth to the more general *M*, since this equation applies to satellites orbiting any large mass.

EXAMPLE 13.13

Orbit of Halley's Comet

Determine the semi-major axis of the orbit of Halley's comet, given that it arrives at perihelion every 75.3 years. If the perihelion is 0.586 AU, what is the aphelion?

Strategy

We are given the period, so we can rearrange Equation 13.11, solving for the semi-major axis. Since we know the value for the perihelion, we can use the definition of the semi-major axis, given earlier in this section, to find the aphelion. We note that 1 Astronomical Unit (AU) is the average radius of Earth's orbit and is defined to be $1 \text{ AU} = 1.50 \times 10^{11} \text{ m}$.

Solution

Rearranging Equation 13.11 and inserting the values of the period of Halley's comet and the mass of the Sun, we have

$$\begin{aligned} a &= \left(\frac{GM}{4\pi^2}T^2\right)^{1/3} \\ &= \left(\frac{(6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2)(2.00 \times 10^{30} \text{ kg})}{4\pi^2}(75.3 \text{ yr} \times 365 \text{ days/yr} \times 24 \text{ hr/day} \times 3600 \text{ s/hr})^2\right)^{1/3}. \end{aligned}$$

This yields a value of $2.67 \times 10^{12} \text{ m}$ or 17.8 AU for the semi-major axis.

The semi-major axis is one-half the sum of the aphelion and perihelion, so we have

$$\begin{aligned} a &= \frac{1}{2}(\text{aphelion} + \text{perihelion}) \\ \text{aphelion} &= 2a - \text{perihelion}. \end{aligned}$$

Substituting for the values, we found for the semi-major axis and the value given for the perihelion, we find the value of the aphelion to be 35.0 AU.

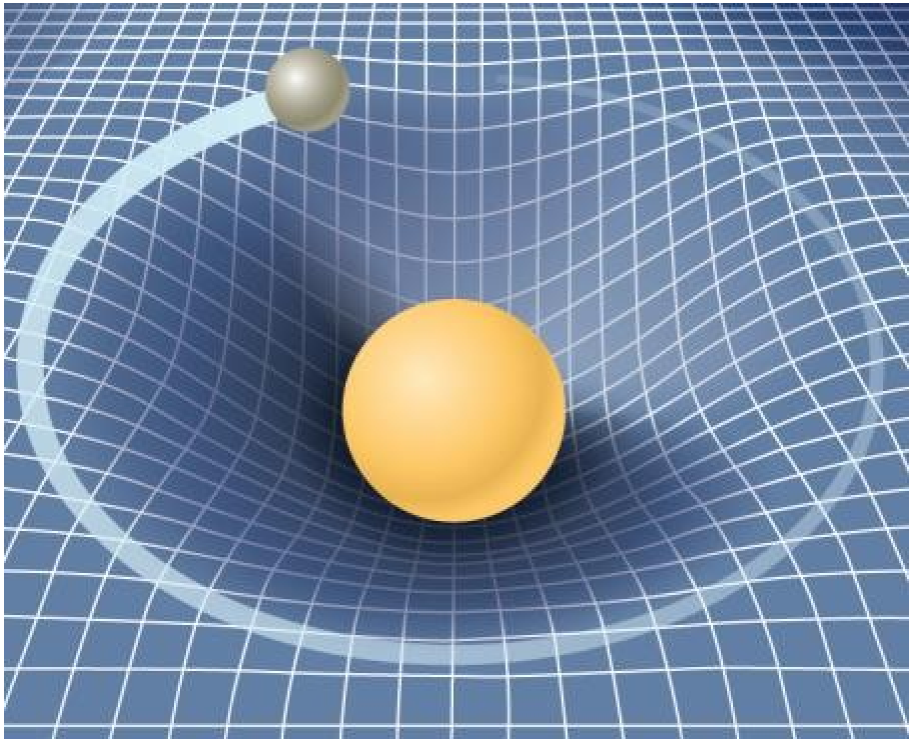
GENERAL RELATIVITY

The Principle of Equivalence

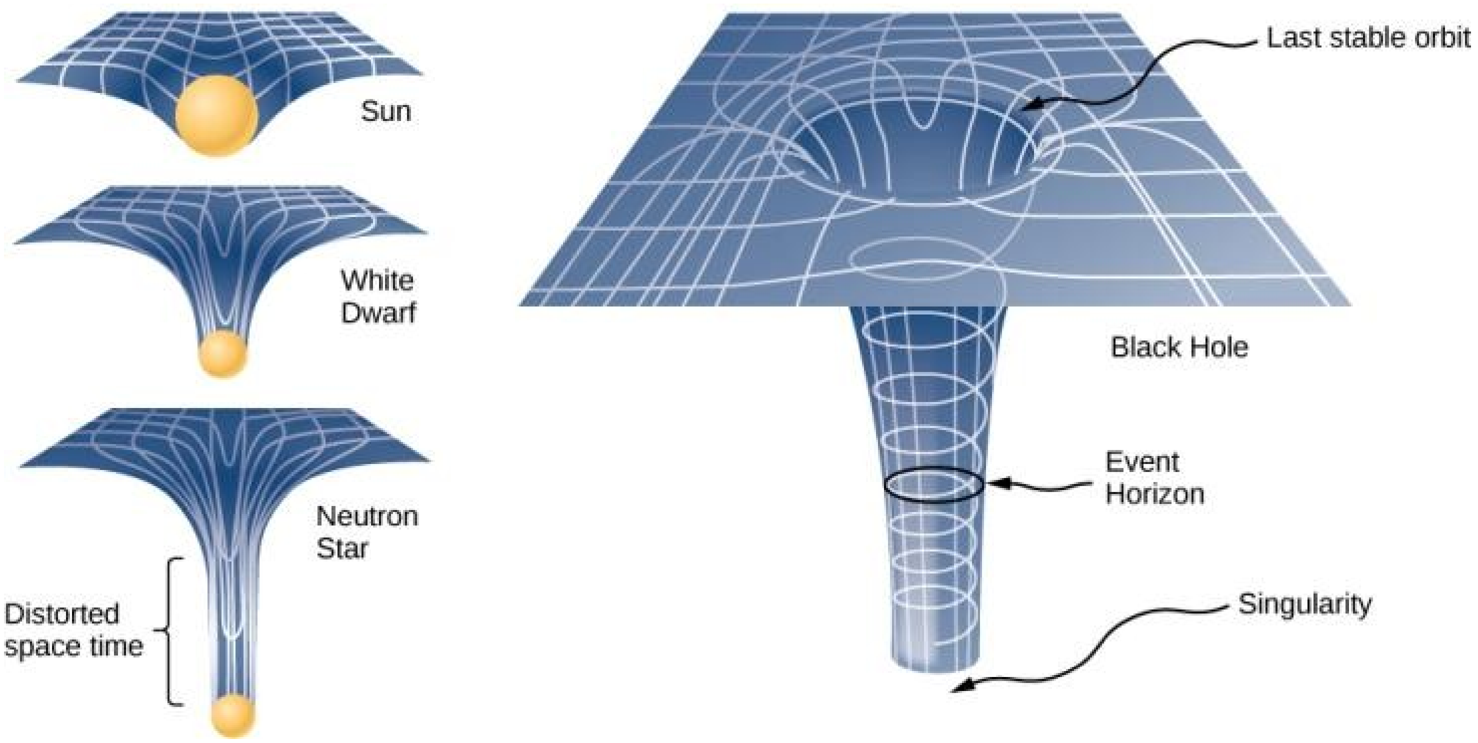
Einstein came to his general theory in part by wondering why someone who was free falling did not feel his or her weight. Indeed, it is common to speak of astronauts orbiting Earth as being weightless, despite the fact that Earth's gravity is still quite strong there. In Einstein's general theory, there is no difference between free fall and being weightless. This is called the **principle of equivalence**. The equally surprising corollary to this is that there is no difference between a uniform gravitational field and a uniform acceleration in the absence of gravity. Let's focus on this last statement. Although a perfectly uniform gravitational field is not feasible, we can approximate it very well.

Within a reasonably sized laboratory on Earth, the gravitational field $\vec{\mathbf{g}}$ is essentially uniform. The corollary states that any physical experiments performed there have the identical results as those done in a laboratory accelerating at $\vec{\mathbf{a}} = \vec{\mathbf{g}}$ in deep space, well away from all other masses. Figure 13.28 illustrates the concept.

GRAVITY AS GEOMETRY

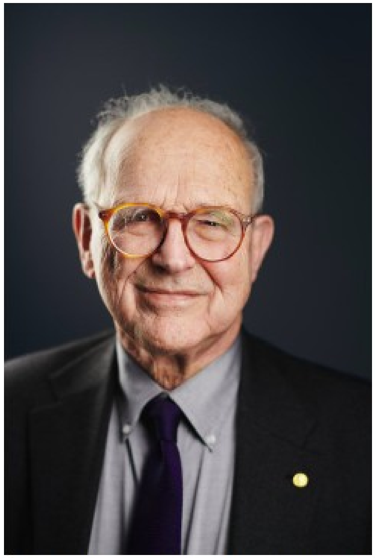


A smaller mass orbiting in the distorted space-time of a larger mass. In fact, all mass or energy distorts space-time.



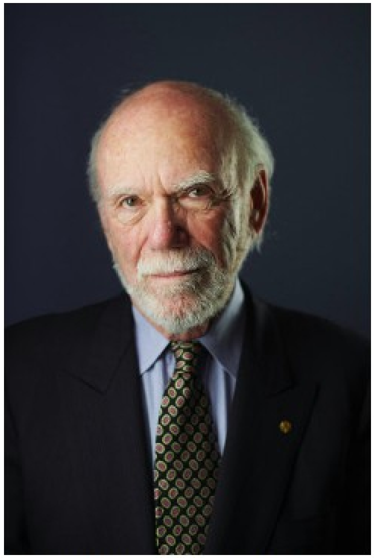
The space distortion becomes more noticeable around increasingly larger masses. Once the mass density reaches a critical level, a black hole forms and the fabric of space-time is torn. The curvature of space is greatest at the surface of each of the first three objects shown and is finite. The curvature then decreases (not shown) to zero as you move to the center of the object. But the black hole is different. The curvature becomes infinite: The surface has collapsed to a singularity, and the cone extends to infinity. (Note: These diagrams are not to any scale.)

The Nobel Prize in Physics 2017



Rainer Weiss

Prize share: 1/2



Barry C. Barish

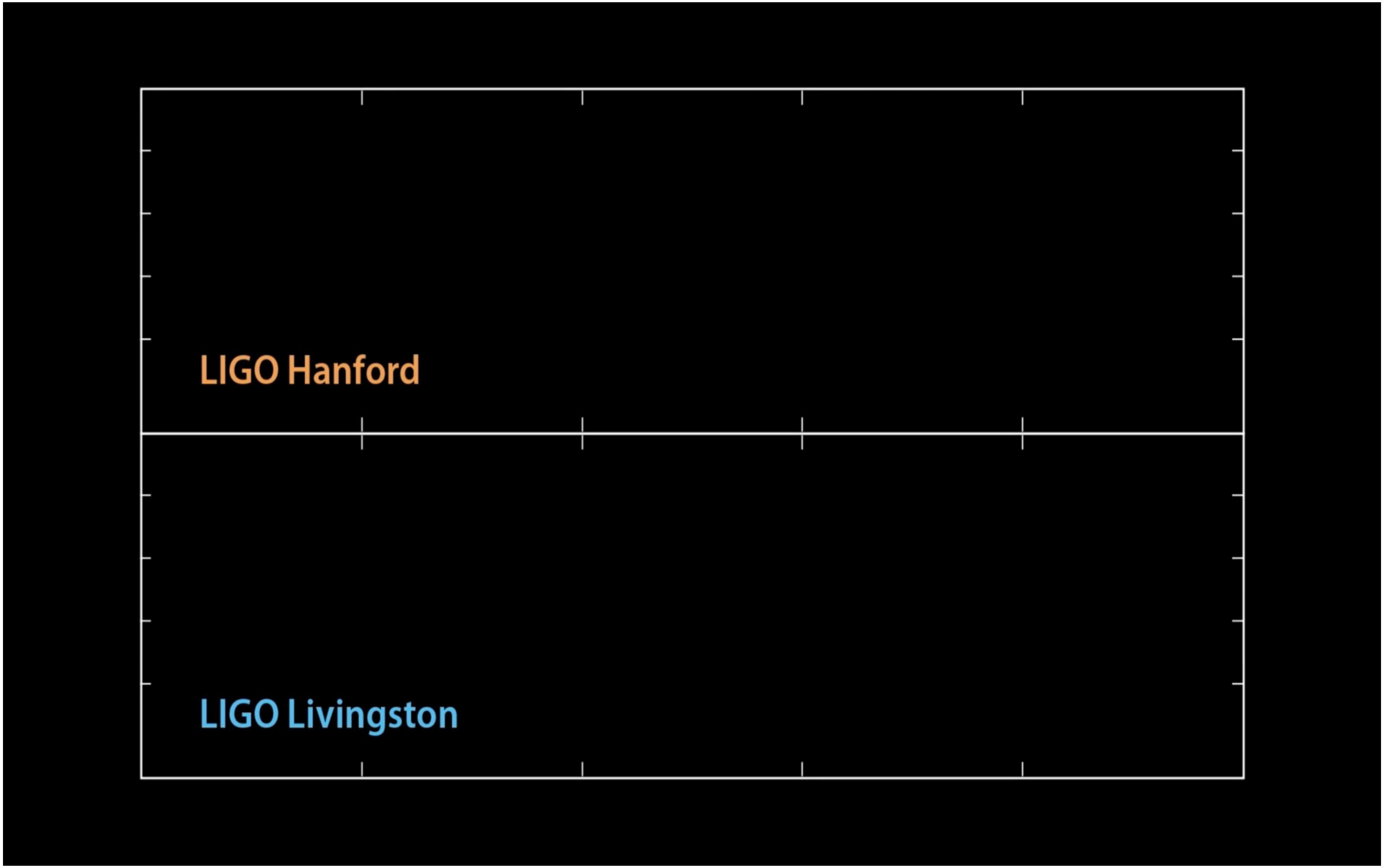
Prize share: 1/4

Kip S. Thorne

Prize share: 1/4

The Nobel Prize in Physics 2017 was divided, one half awarded to Rainer Weiss, the other half jointly to Barry C. Barish and Kip S. Thorne "for decisive contributions to the LIGO detector and the observation of gravitational waves."

FIRST DIRECT DETECTION OF GRAVITATIONAL WAVES AND COLLIDING BLACK HOLES


LIGO Hanford
LIGO Livingston