ANNOUNCEMENTS

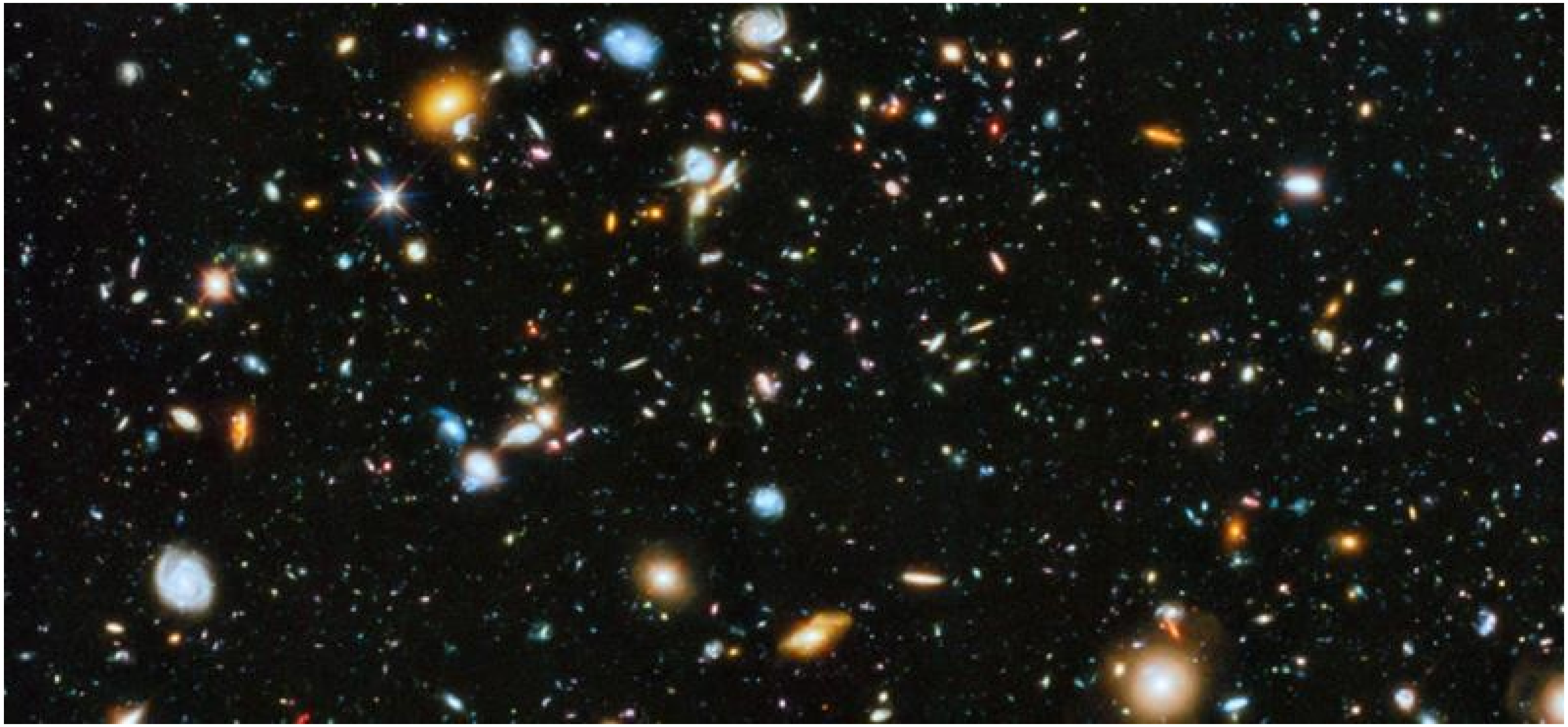
- Homework #12, due Friday, Nov. 16 before class

  Conceptual questions: Chapter 12, #4 and #10
  Problems: Chapter 12, #32, #38

- Study Chapter 12, sections 1 and 2

- Quiz #12, Friday November 16 at the beginning of class

GRAVITATION



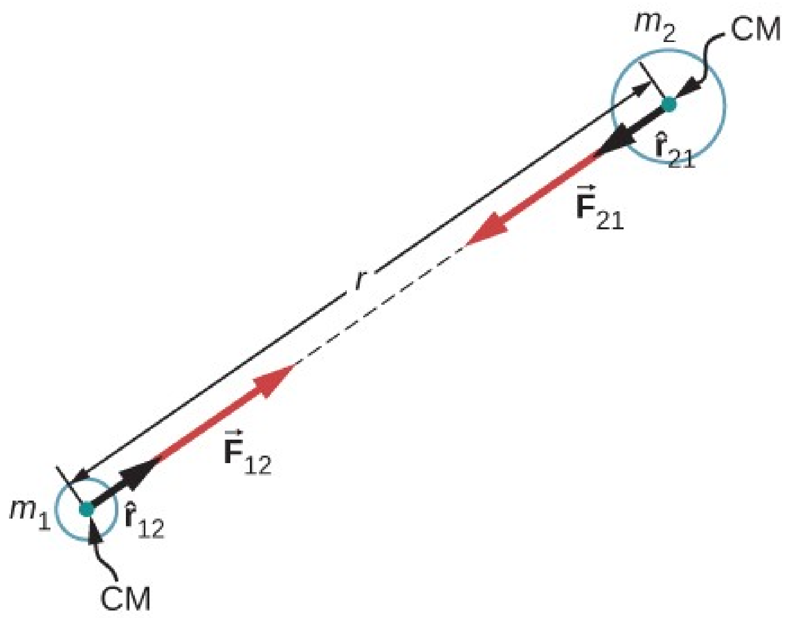
Our visible Universe contains billions of galaxies, whose very existence is due to the force of gravity. Gravity is ultimately responsible for the energy output of all stars—initiating thermonuclear reactions in stars, allowing the Sun to heat Earth, and making galaxies visible from unfathomable distances. Most of the dots you see in this image are not stars, but galaxies. (credit: modification of work by NASA)

NEWTON'S LAW OF GRAVITATION

Newton's law of gravitation can be expressed as

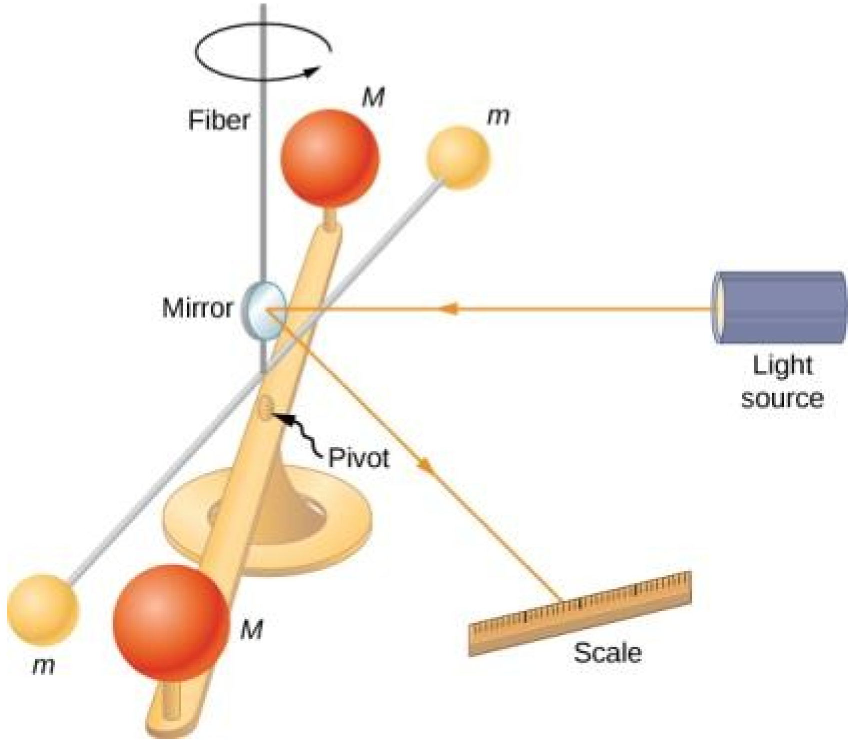
$$\vec{\mathbf{F}}_{12} = G\frac{m_1 m_2}{r^2}\hat{\mathbf{r}}_{12} \tag{13.1}$$

where $\vec{\mathbf{F}}_{12}$ is the force on object 1 exerted by object 2 and $\hat{\mathbf{r}}_{12}$ is a unit vector that points from object 1 toward object 2.



Gravitational force acts along a line joining the centers of mass of two objects.

FIGURE 13.3



Cavendish used an apparatus similar to this to measure the gravitational attraction between two spheres (*m*) suspended from a wire and two stationary spheres (*M*). This is a common experiment performed in undergraduate laboratories, but it is quite challenging. Passing trucks outside the laboratory can create vibrations that overwhelm the gravitational forces.

A Collision in Orbit

Consider two nearly spherical *Soyuz* payload vehicles, in orbit about Earth, each with mass 9000 kg and diameter 4.0 m. They are initially at rest relative to each other, 10.0 m from center to center. (As we will see in Kepler's Laws of Planetary Motion, both orbit Earth at the same speed and interact nearly the same as if they were isolated in deep space.) Determine the gravitational force between them and their initial acceleration. Estimate how long it takes for them to drift together, and how fast they are moving upon impact.

Solution

The magnitude of the force is

$$\left|\vec{\mathbf{F}}_{12}\right| = F_{12} = G\frac{m_1 m_2}{r^2} = 6.67 \times 10^{-11}\ \mathrm{N\cdot m^2/kg^2}\frac{(9000\ \mathrm{kg})(9000\ \mathrm{kg})}{(10\ \mathrm{m})^2} = 5.4 \times 10^{-5}\mathrm{N}.$$

The initial acceleration of each payload is

$$a = \frac{F}{m} = \frac{5.4 \times 10^{-5}\ \mathrm{N}}{9000\ \mathrm{kg}} = 6.0 \times 10^{-9}\mathrm{m/s^2}.$$

The vehicles are 4.0 m in diameter, so the vehicles move from 10.0 m to 4.0 m apart, or a distance of 3.0 m each. A similar calculation to that above, for when the vehicles are 4.0 m apart, yields an acceleration of $3.8 \times 10^{-8}\ \mathrm{m/s^2}$, and the average of these two values is $2.2 \times 10^{-8}\ \mathrm{m/s^2}$. If we assume a constant acceleration of this value and they start from rest, then the vehicles collide with speed given by

$$v^2 = v_0^2 + 2a(x - x_0),\ \text{where } v_0 = 0,$$

so

$$v = \sqrt{2(2.2 \times 10^{-9}\ \mathrm{N})(3.0\ \mathrm{m})} = 3.6 \times 10^{-4}\ \mathrm{m/s}.$$

We use $v = v_0 + at$ to find $t = v/a = 1.7 \times 10^4$ s or about 4.6 hours.

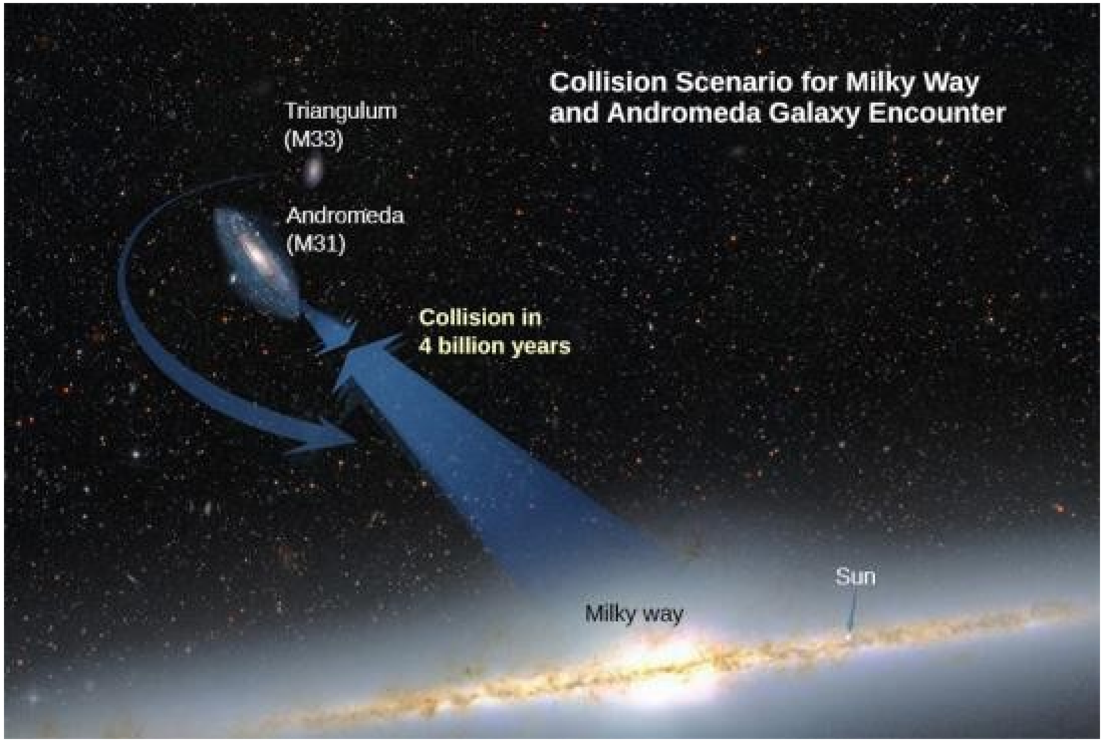
FIGURE 13.4

This photo shows Ed White tethered to the Space Shuttle during a spacewalk. (credit: NASA)

Attraction between Galaxies

Find the acceleration of our galaxy, the Milky Way, due to the nearest comparably sized galaxy, the Andromeda galaxy (Figure 13.5). The approximate mass of each galaxy is 800 billion solar masses (a solar mass is the mass of our Sun), and they are separated by 2.5 million light-years. (Note that the mass of Andromeda is not so well known but is believed to be slightly larger than our galaxy.) Each galaxy has a diameter of roughly 100,000 light-years $(1 \text{ light-year} = 9.5 \times 10^{15} \text{ m})$.

Galaxies interact gravitationally over immense distances. The Andromeda galaxy is the nearest spiral galaxy to the Milky Way, and they will eventually collide. (credit: Boris Štromar)



Based on the results of this example, plus what astronomers have observed elsewhere in the Universe, our galaxy will collide with the Andromeda Galaxy in about 4 billion years. (credit: NASA)

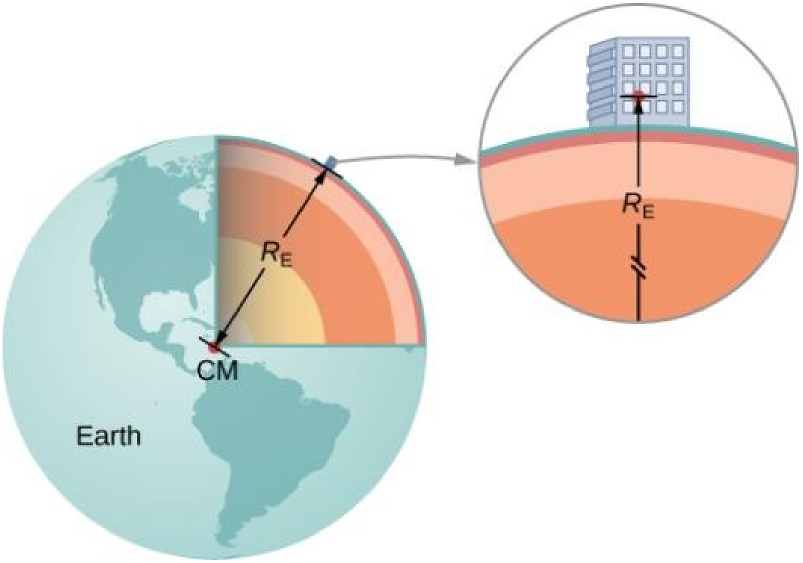
The acceleration of the Milky Way is

$$a = \frac{F}{m} = \frac{3.0 \times 10^{29}\ \text{N}}{(800 \times 10^{9})(2.0 \times 10^{30}\ \text{kg})} = 1.9 \times 10^{-13}\,\text{m/s}^2.$$

$$mg = G\frac{mM_{\mathrm{E}}}{r^2}$$

where *r* is the distance between the centers of mass of the object and Earth. The average radius of Earth is about 6370 km. Hence, for objects within a few kilometers of Earth's surface, we can take $r = R_{\mathrm{E}}$ (Figure 13.7). The mass *m* of the object cancels, leaving

$$g = G\frac{M_{\mathrm{E}}}{r^2}. \tag{13.2}$$



We can take the distance between the centers of mass of Earth and an object on its surface to be the radius of Earth, provided that its size is much less than the radius of Earth.

Masses of Earth and Moon

Have you ever wondered how we know the mass of Earth? We certainly can't place it on a scale. The values of *g* and the radius of Earth were measured with reasonable accuracy centuries ago.

a. Use the standard values of g, R_{E}, and Equation 13.2 to find the mass of Earth.
b. Estimate the value of g on the Moon. Use the fact that the Moon has a radius of about 1700 km (a value of this accuracy was determined many centuries ago) and assume it has the same average density as Earth, $5500\ \mathrm{kg/m^3}$.

Strategy

With the known values of g and R_{E}, we can use Equation 13.2 to find M_{E}. For the Moon, we use the assumption of equal average density to determine the mass from a ratio of the volumes of Earth and the Moon.

Solution

a. Rearranging Equation 13.2, we have

$$M_{\mathrm{E}} = \frac{gR_{\mathrm{E}}^2}{G} = \frac{9.80\ \mathrm{m/s^2}(6.37\ \times\ 10^6\ \mathrm{m})^2}{6.67\ \times\ 10^{-11}\ \mathrm{N \cdot m^2/kg^2}} = 5.95\ \times\ 10^{24}\ \mathrm{kg}.$$

b. The volume of a sphere is proportional to the radius cubed, so a simple ratio gives us

$$\frac{M_{\mathrm{M}}}{M_{\mathrm{E}}} = \frac{R_{\mathrm{M}}^3}{R_{\mathrm{E}}^3} \rightarrow M_{\mathrm{M}} = \left(\frac{(1.7\ \times\ 10^6\ \mathrm{m})^3}{(6.37\ \times\ 10^6\ \mathrm{m})^3}\right)(5.95\ \times\ 10^{24}\ \mathrm{kg}) = 1.1\ \times\ 10^{23}\ \mathrm{kg}.$$

We now use Equation 13.2.

$$g_{\mathrm{M}} = G\frac{M_{\mathrm{M}}}{r_{\mathrm{M}}^2} = (6.67\ \times\ 10^{-11}\ \mathrm{N \cdot m^2/kg^2})\frac{(1.1\ \times\ 10^{23}\ \mathrm{kg})}{(1.7\ \times\ 10^6\ \mathrm{m})^2} = 2.5\mathrm{m/s^2}$$

EXAMPLE 13.4

Gravity above Earth's Surface

What is the value of *g* 400 km above Earth's surface, where the International Space Station is in orbit?

Strategy

Using the value of M_{E} and noting the radius is $r = R_{\mathrm{E}} + 400\ \mathrm{km}$, we use Equation 13.2 to find *g*.

From Equation 13.2 we have

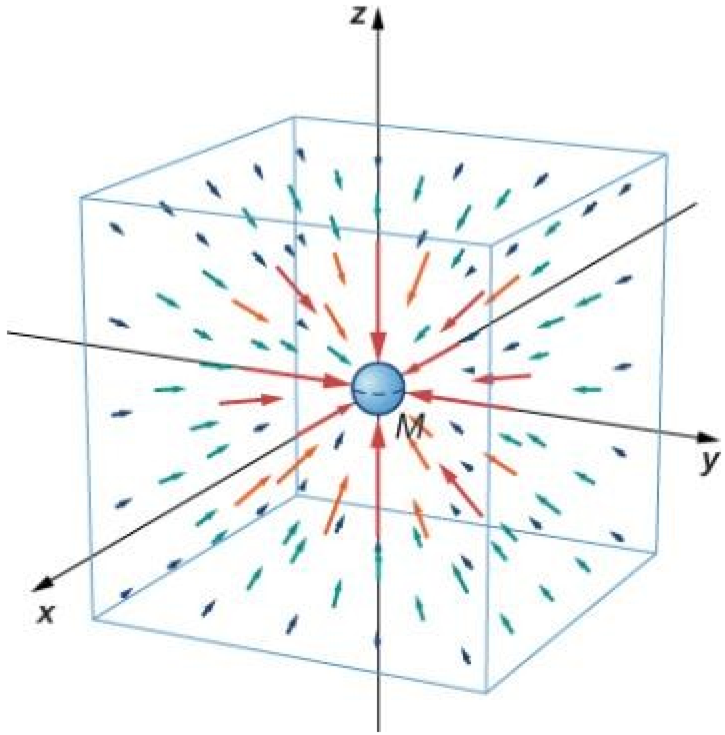
$$g = G\frac{M_{\mathrm{E}}}{r^2} = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}\,\frac{5.96 \times 10^{24}\ \mathrm{kg}}{(6.37 \times 10^6 + 400 \times 10^3\ \mathrm{m})^2} = 8.67\ \mathrm{m/s^2}.$$

The Gravitational Field

Equation 13.2 is a scalar equation, giving the magnitude of the gravitational acceleration as a function of the distance from the center of the mass that causes the acceleration. But we could have retained the vector form for the force of gravity in Equation 13.1, and written the acceleration in vector form as

$$\vec{\mathbf{g}} = G\frac{M}{r^2}\hat{\mathbf{r}}.$$

We identify the vector field represented by $\vec{\mathbf{g}}$ as the **gravitational field** caused by mass M. We can picture the field as shown Figure 13.8. The lines are directed radially inward and are symmetrically distributed about the mass.

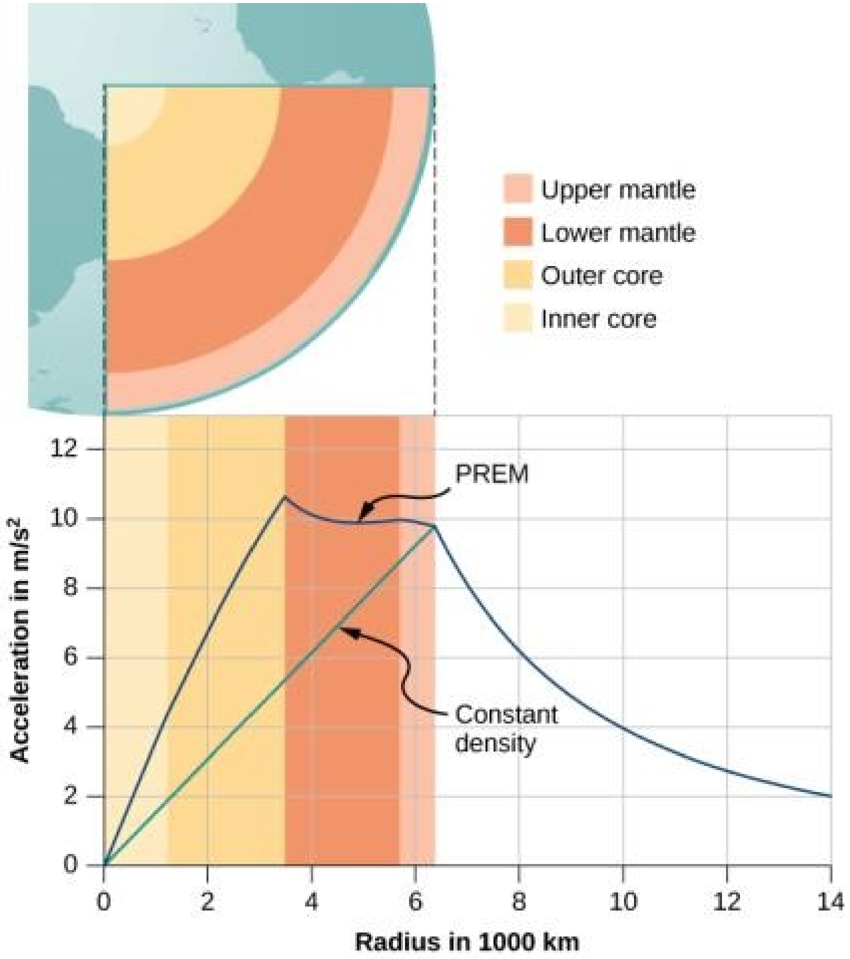


A three-dimensional representation of the gravitational field created by mass *M*. Note that the lines are uniformly distributed in all directions. (The box has been added only to aid in visualization.)

FIGURE 13.10

For $r < R_E$, the value of g for the case of constant density is the straight green line. The blue line from the PREM (Preliminary Reference Earth Model) is probably closer to the actual profile for g.

$$g = G\frac{M_E}{R_E^2} = G\frac{\rho\left(4/3\pi r^3\right)}{r^2} = \frac{4}{3}G\rho\pi r.$$



GRAVITATIONAL POTENTIAL ENERGY

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$$U = -\frac{GM_E m}{r}. \tag{13.4}$$

Lifting a Payload

How much energy is required to lift the 9000-kg *Soyuz* vehicle from Earth's surface to the height of the ISS, 400 km above the surface?

Strategy

Use Equation 13.2 to find the change in potential energy of the payload. That amount of work or energy must be supplied to lift the payload.

Solution

Paying attention to the fact that we start at Earth's surface and end at 400 km above the surface, the change in *U* is

$$\Delta U = U_{\text{orbit}} - U_{\text{Earth}} = -\frac{GM_E m}{R_E + 400\text{ km}} - \left(-\frac{GM_E m}{R_E}\right).$$

We insert the values

$$m = 9000\text{ kg}, \quad M_E = 5.96 \times 10^{24}\text{kg}, \quad R_E = 6.37 \times 10^6\text{ m}$$

and convert 400 km into 4.00×10^5 m. We find $\Delta U = 3.32 \times 10^{10}$ J. It is positive, indicating an increase in potential energy, as we would expect.

Conservation of Energy

In Potential Energy and Conservation of Energy, we described how to apply conservation of energy for systems with conservative forces. We were able to solve many problems, particularly those involving gravity, more simply using conservation of energy. Those principles and problem-solving strategies apply equally well here. The only change is to place the new expression for potential energy into the conservation of energy equation, $E = K_1 + U_1 = K_2 + U_2$.

$$\frac{1}{2}mv_1^2 - \frac{GMm}{r_1} = \frac{1}{2}mv_2^2 - \frac{GMm}{r_2} \tag{13.5}$$

Escape velocity

Escape velocity is often defined to be the *minimum* initial velocity of an object that is required to escape the surface of a planet (or any large body like a moon) and never return. As usual, we assume no energy lost to an atmosphere, should there be any.

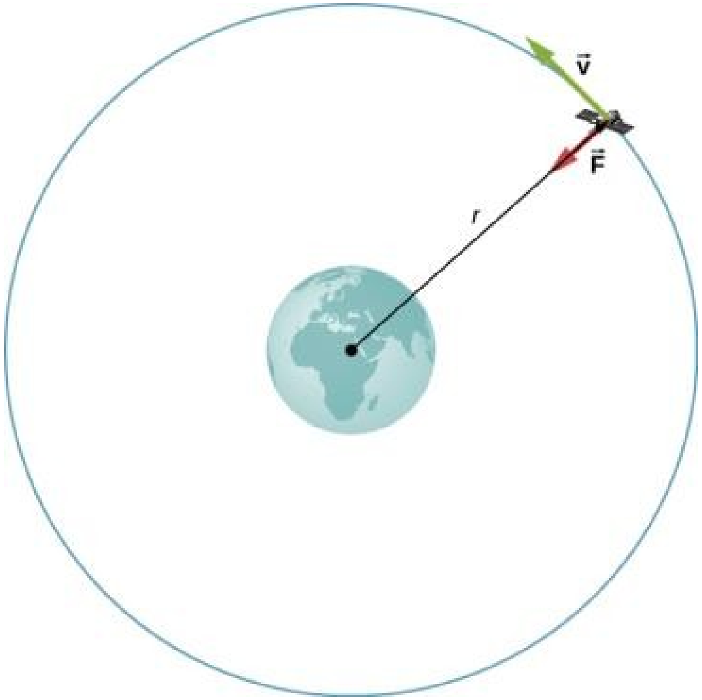
Consider the case where an object is launched from the surface of a planet with an initial velocity directed away from the planet. With the *minimum* velocity needed to escape, the object would *just* come to rest infinitely far away, that is, the object gives up the last of its kinetic energy just as it reaches infinity, where the force of gravity becomes zero. Since $U \to 0$ as $r \to \infty$, this means the total energy is zero. Thus, we find the escape velocity from the surface of an astronomical body of mass *M* and radius *R* by setting the total energy equal to zero. At the surface of the body, the object is located at $r_1 = R$ and it has escape velocity $v_1 = v_{\text{esc}}$. It reaches $r_2 = \infty$ with velocity $v_2 = 0$. Substituting into Equation 13.5, we have

$$\frac{1}{2}mv_{\text{esc}}^2 - \frac{GMm}{R} = \frac{1}{2}m0^2 - \frac{GMm}{\infty} = 0.$$

Solving for the escape velocity,

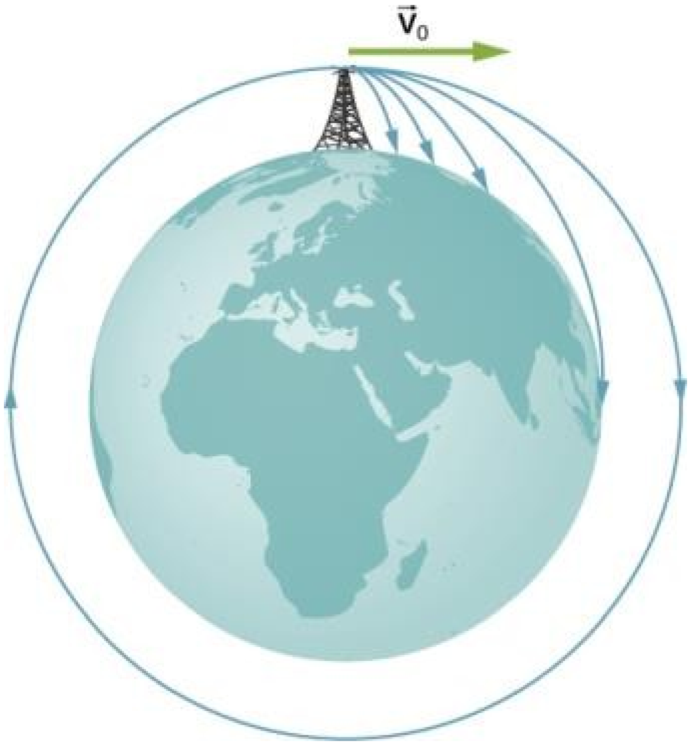
$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}}. \tag{13.6}$$

A satellite of mass *m* orbiting at radius *r* from the center of Earth. The gravitational force supplies the centripetal acceleration.



$$\frac{GmM_E}{r^2} = ma_c = \frac{mv_{orbit}^2}{r}.$$ → $$v_{orbit} = \sqrt{\frac{GM_E}{r}}.$$ → $$T = 2\pi\sqrt{\frac{r^3}{GM_E}}.$$

FIGURE 13.13



A circular orbit is the result of choosing a tangential velocity such that Earth’s surface curves away at the same rate as the object falls toward Earth.

Energy in Circular Orbits

In Gravitational Potential Energy and Total Energy, we argued that objects are gravitationally bound if their total energy is negative. The argument was based on the simple case where the velocity was directly away or toward the planet. We now examine the total energy for a circular orbit and show that indeed, the total energy is negative. As we did earlier, we start with Newton's second law applied to a circular orbit,

$$\begin{aligned} \frac{GmM_\mathrm{E}}{r^2} &= ma_c = \frac{mv^2}{r} \\ \frac{GmM_\mathrm{E}}{r} &= mv^2. \end{aligned}$$

In the last step, we multiplied by *r* on each side. The right side is just twice the kinetic energy, so we have

$$K = \frac{1}{2}mv^2 = \frac{GmM_\mathrm{E}}{2r}.$$

The total energy is the sum of the kinetic and potential energies, so our final result is

$$E = K + U = \frac{GmM_\mathrm{E}}{2r} - \frac{GmM_\mathrm{E}}{r} = -\frac{GmM_\mathrm{E}}{2r}. \tag{13.9}$$