ANNOUNCEMENTS

- <u>Homework #12, due Friday, Nov. 16 before class</u>

  Conceptual questions: Chapter 12, #4 and #10
  Problems: Chapter 12, #32, #38

- Study Chapter 12, sections 1 and 2

- Quiz #12, Friday November 16 at the beginning of class

FIRST EQUILIBRIUM CONDITION

The first equilibrium condition for the static equilibrium of a rigid body expresses *translational* equilibrium:

$$\sum_k \vec{\mathbf{F}}_k = \vec{\mathbf{0}}. \tag{12.2}$$

$$\sum_k F_{kx} = 0, \quad \sum_k F_{ky} = 0, \quad \sum_k F_{kz} = 0.$$

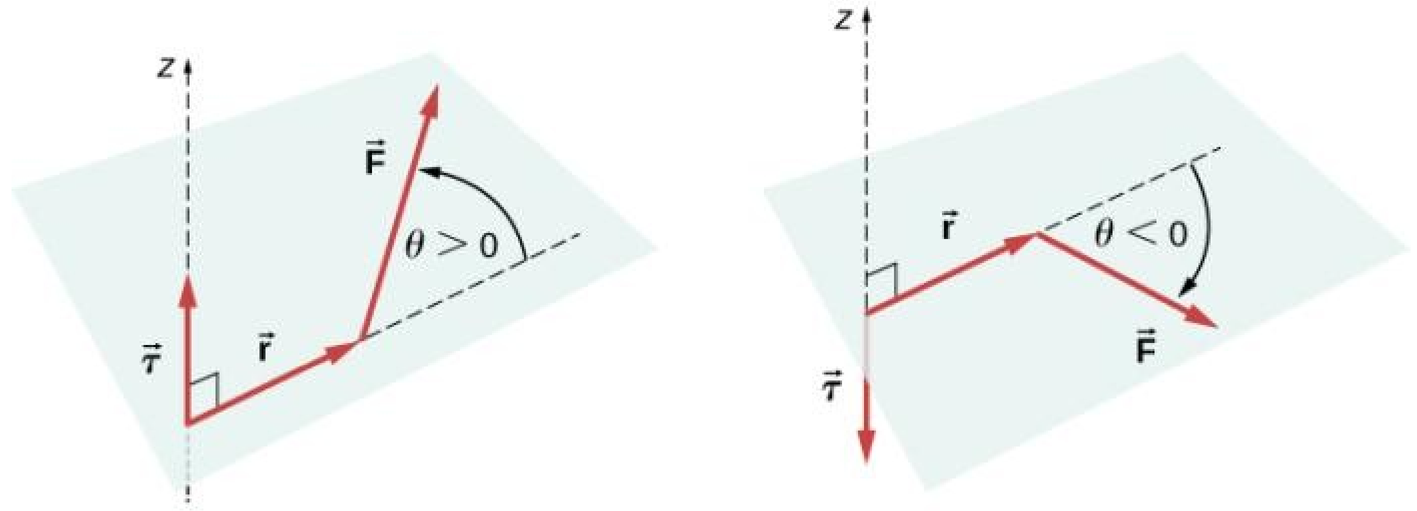
SECOND EQUILIBRIUM CONDITION

The second equilibrium condition for the static equilibrium of a rigid body expresses *rotational* equilibrium:

$$\sum_k \vec{\boldsymbol{\tau}}_k = \vec{\mathbf{0}}. \tag{12.5}$$

$$\sum_k \tau_{kx} = 0, \quad \sum_k \tau_{ky} = 0, \quad \sum_k \tau_{kz} = 0.$$

FIGURE 12.2



Counterclockwise rotation
the sense of torque:
$+\tau$

(a)

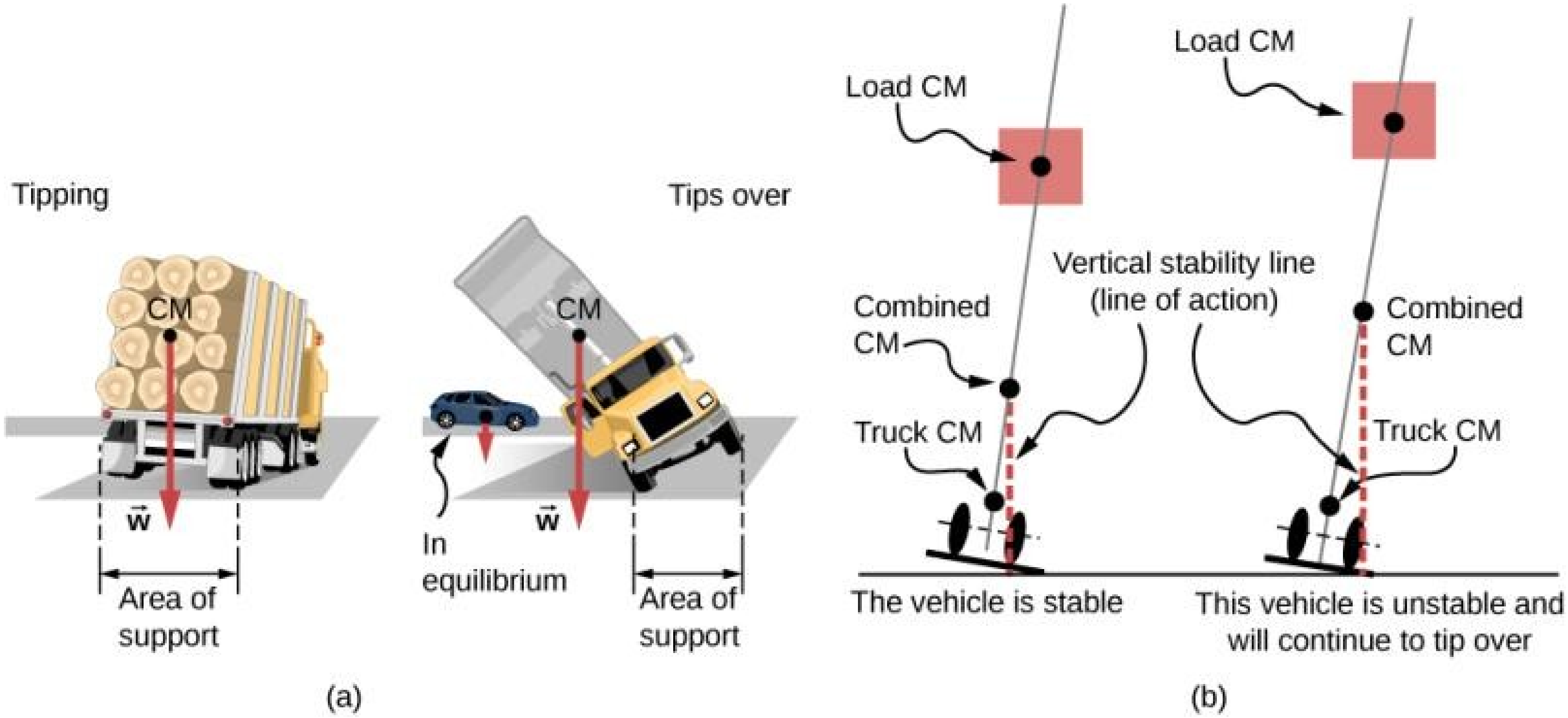
Clockwise rotation
the sense of torque:
$-\tau$

(b)

Torque of a force:

(a) When the torque of a force causes counterclockwise rotation about the axis of rotation, we say that its *sense* is positive, which means the torque vector is parallel to the axis of rotation.

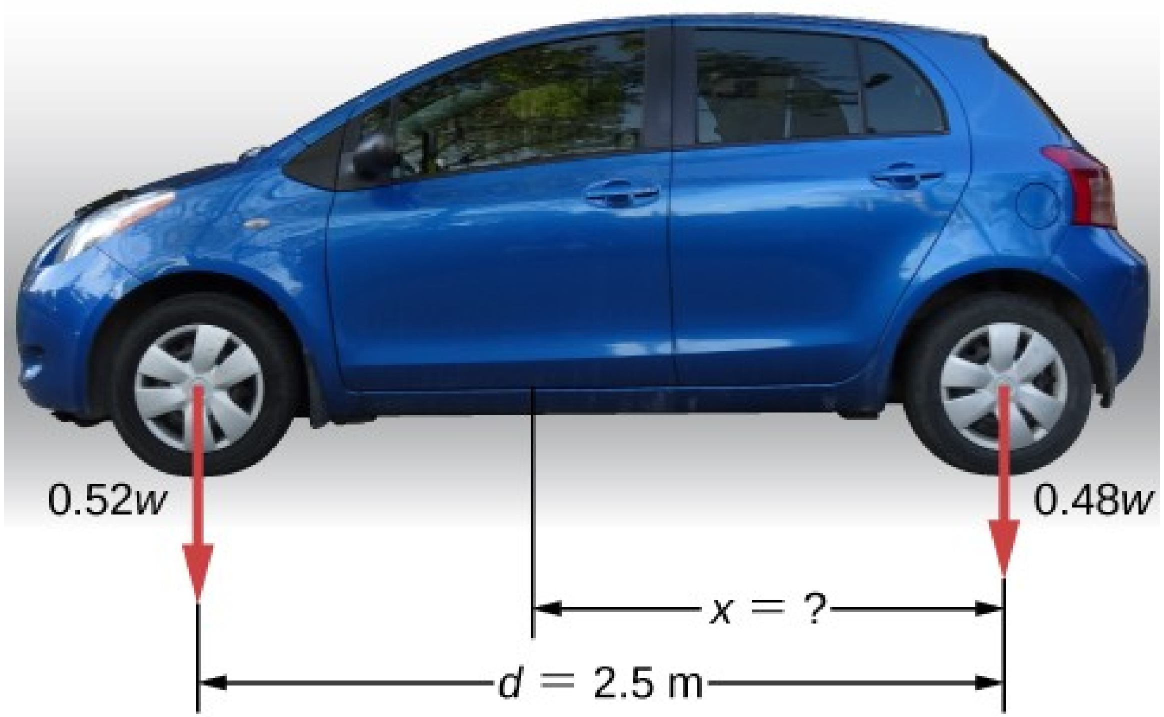
(b) When torque of a force causes clockwise rotation about the axis, we say that its sense is negative, which means the torque vector is antiparallel to the axis of rotation.

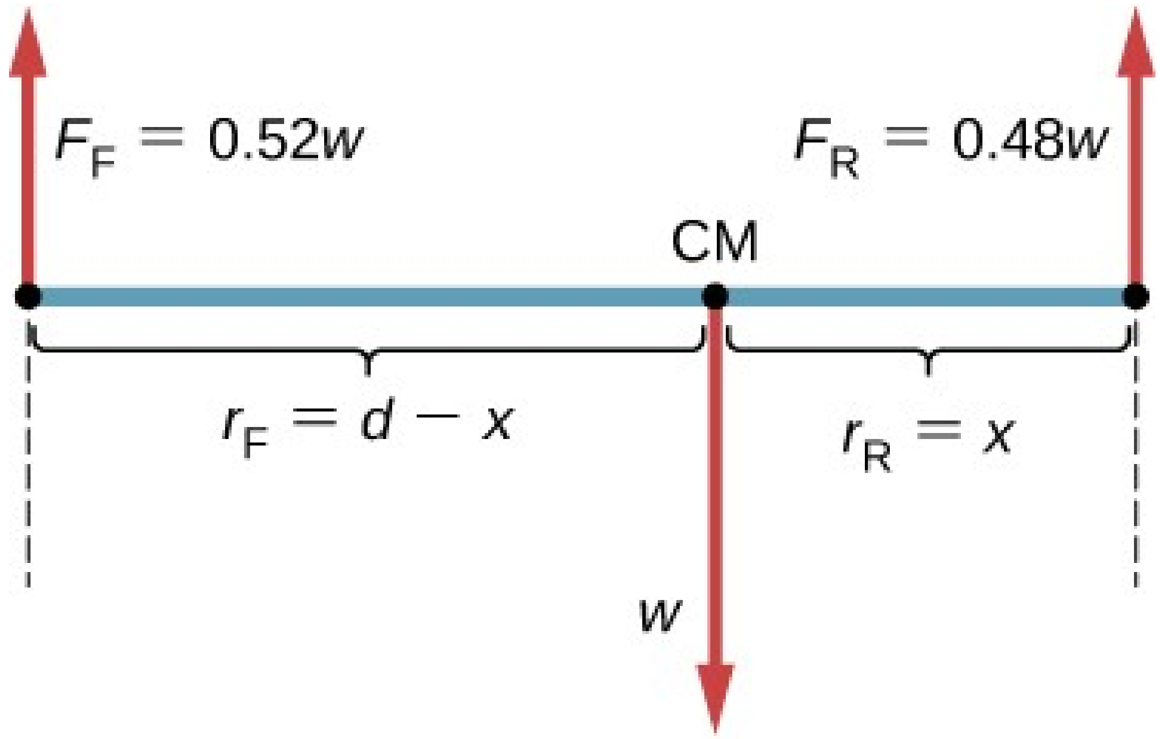


The distribution of mass affects the position of the center of mass (CM), where the weight vector $\vec{\mathbf{w}}$ is attached. If the center of gravity is within the area of support, the truck returns to its initial position after tipping [see the left panel in (b)]. But if the center of gravity lies outside the area of support, the truck turns over [see the right panel in (b)]. Both vehicles in (b) are out of equilibrium. Notice that the car in (a) is in equilibrium: The low location of its center of gravity makes it hard to tip over.

Center of Gravity of a Car

A passenger car with a 2.5-m wheelbase has 52% of its weight on the front wheels on level ground, as illustrated in Figure 12.4. Where is the CM of this car located with respect to the rear axle?





The free-body diagram for the car clearly indicates force vectors acting on the car and distances to the center of mass (CM). When CM is selected as the pivot point, these distances are lever arms of normal reaction forces. Notice that vector magnitudes and lever arms do not need to be drawn to scale, but all quantities of relevance must be clearly labeled.

Solution

Each equilibrium condition contains only three terms because there are $N = 3$ forces acting on the car. The first equilibrium condition, Equation 12.7, reads

$$+F_{\mathrm{F}} - w + F_{\mathrm{R}} = 0. \tag{12.11}$$

This condition is trivially satisfied because when we substitute the data, Equation 12.11 becomes $+0.52w - w + 0.48w = 0$. The second equilibrium condition, Equation 12.9, reads

$$\tau_{\mathrm{F}} + \tau_w + \tau_{\mathrm{R}} = 0 \tag{12.12}$$

where τ_{F} is the torque of force $F_{\mathrm{F}},\ \tau_w$ is the gravitational torque of force *w*, and τ_{R} is the torque of force F_{R}. When the pivot is located at CM, the gravitational torque is identically zero because the lever arm of the weight with respect to an axis that passes through CM is zero. The lines of action of both normal reaction forces are perpendicular to their lever arms, so in Equation 12.10, we have $|\sin\theta| = 1$ for both forces. From the free-body diagram, we read that torque τ_{F} causes clockwise rotation about the pivot at CM, so its sense is negative; and torque τ_{R} causes counterclockwise rotation about the pivot at CM, so its sense is positive. With this information, we write the second equilibrium condition as

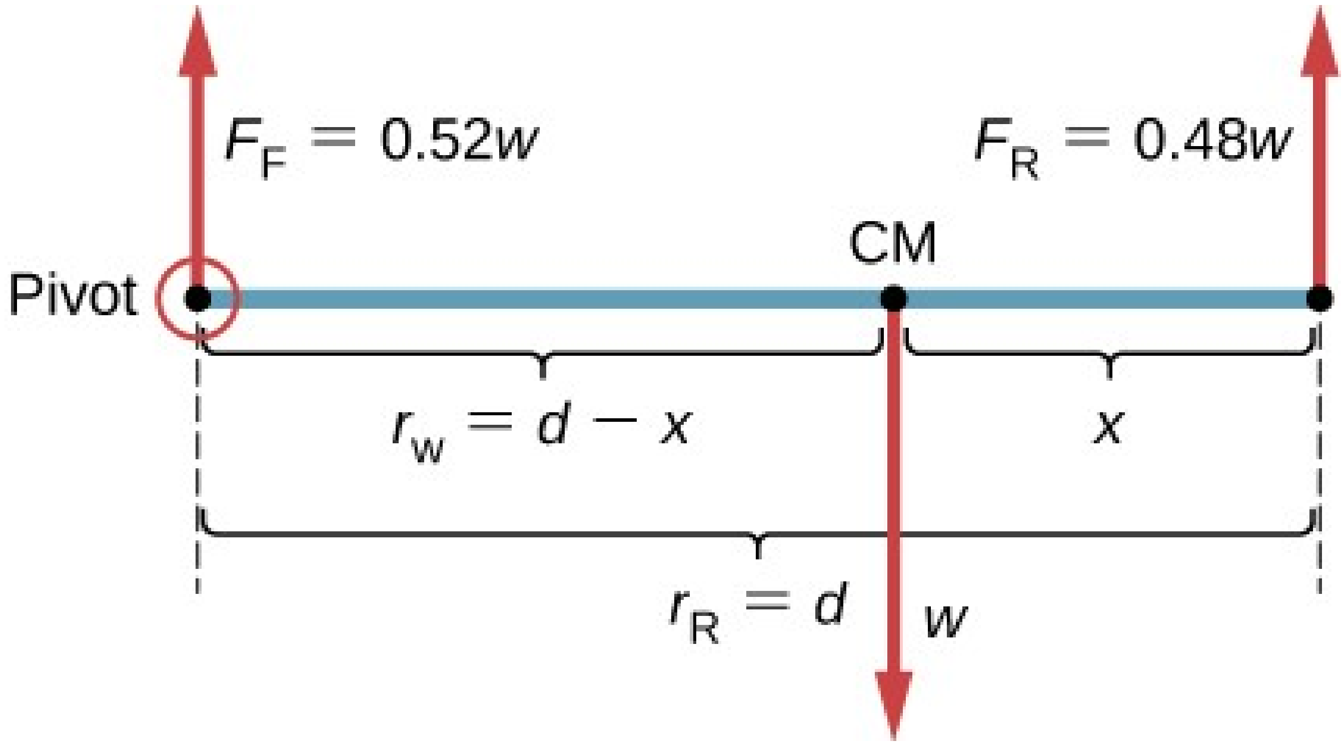
$$-r_{\mathrm{F}}F_{\mathrm{F}} + r_{\mathrm{R}}F_{\mathrm{R}} = 0. \tag{12.13}$$

With the help of the free-body diagram, we identify the force magnitudes $F_{\mathrm{R}} = 0.48w$ and $F_{\mathrm{F}} = 0.52w$, and their corresponding lever arms $r_{\mathrm{R}} = x$ and $r_{\mathrm{F}} = d - x$. We can now write the second equilibrium condition, Equation 12.13, explicitly in terms of the unknown distance *x*:

$$-0.52(d - x)w + 0.48xw = 0. \tag{12.14}$$

Here the weight *w* cancels and we can solve the equation for the unknown position *x* of the CM. The answer is $x = 0.52d = 0.52(2.5\text{ m}) = 1.3\text{ m}$.

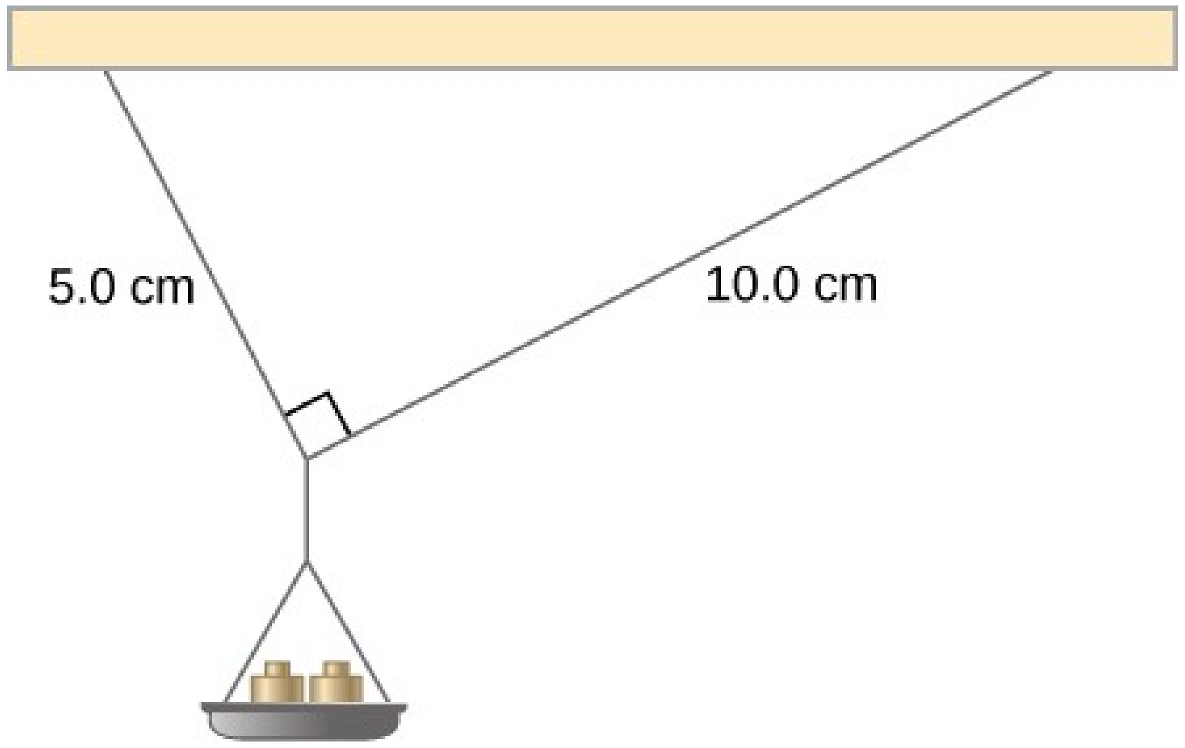
DIY



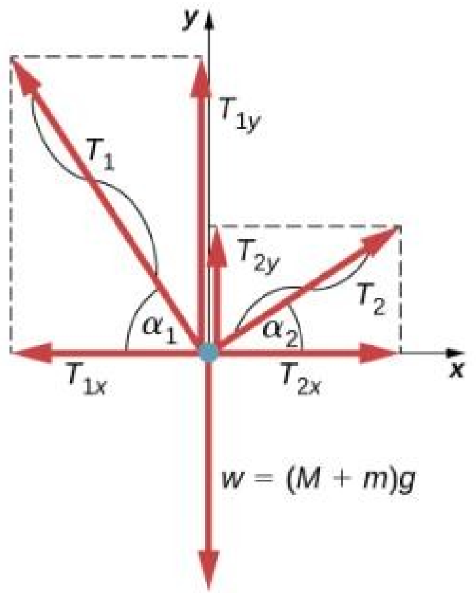
This example shows that when solving static equilibrium problems, we are free to choose the pivot location. For different choices of the pivot point we have different sets of equilibrium conditions to solve. However, all choices lead to the same solution to the problem.

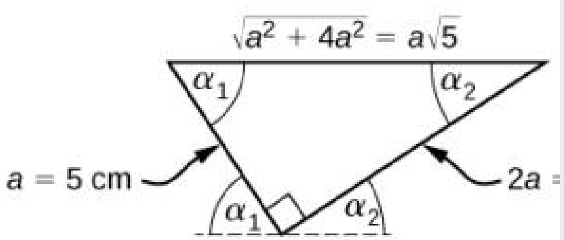
A Breaking Tension

A small pan of mass 42.0 g is supported by two strings, as shown in Figure 12.7. The maximum tension that the string can support is 2.80 N. Mass is added gradually to the pan until one of the strings snaps. Which string is it? How much mass must be added for this to occur?



SOLUTION





$$\cos\alpha_1 = \sin\alpha_2 = \frac{1}{\sqrt{5}}$$

$$\cos\alpha_2 = \sin\alpha_1 = \frac{2}{\sqrt{5}}$$

The weight *w* pulling on the knot is due to the mass *M* of the pan and mass *m* added to the pan, or $w = (M + m)g$. With the help of the free-body diagram in Figure 12.8, we can set up the equilibrium conditions for the knot:

$$\begin{aligned} &\text{in the } x\text{-direction,} & -T_{1x} + T_{2x} &= 0 \\ &\text{in the } y\text{-direction,} & +T_{1y} + T_{2y} - w &= 0. \end{aligned}$$

From the free-body diagram, the magnitudes of components in these equations are

$$\begin{aligned} T_{1x} &= T_1 \cos\alpha_1 = T_1/\sqrt{5}, & T_{1y} &= T_1 \sin\alpha_1 = 2T_1/\sqrt{5} \\ T_{2x} &= T_2 \cos\alpha_2 = 2T_2/\sqrt{5}, & T_{2y} &= T_2 \sin\alpha_2 = T_2/\sqrt{5}. \end{aligned}$$

We substitute these components into the equilibrium conditions and simplify. We then obtain two equilibrium equations for the tensions:

$$\begin{aligned} &\text{in } x\text{-direction,} & T_1 &= 2T_2 \\ &\text{in } y\text{-direction,} & \frac{2T_1}{\sqrt{5}} + \frac{T_2}{\sqrt{5}} &= (M + m)g. \end{aligned}$$

The equilibrium equation for the *x*-direction tells us that the tension T_1 in the 5.0-cm string is twice the tension T_2 in the 10.0-cm string. Therefore, the shorter string will snap. When we use the first equation to eliminate T_2 from the second equation, we obtain the relation between the mass m on the pan and the tension T_1 in the shorter string:

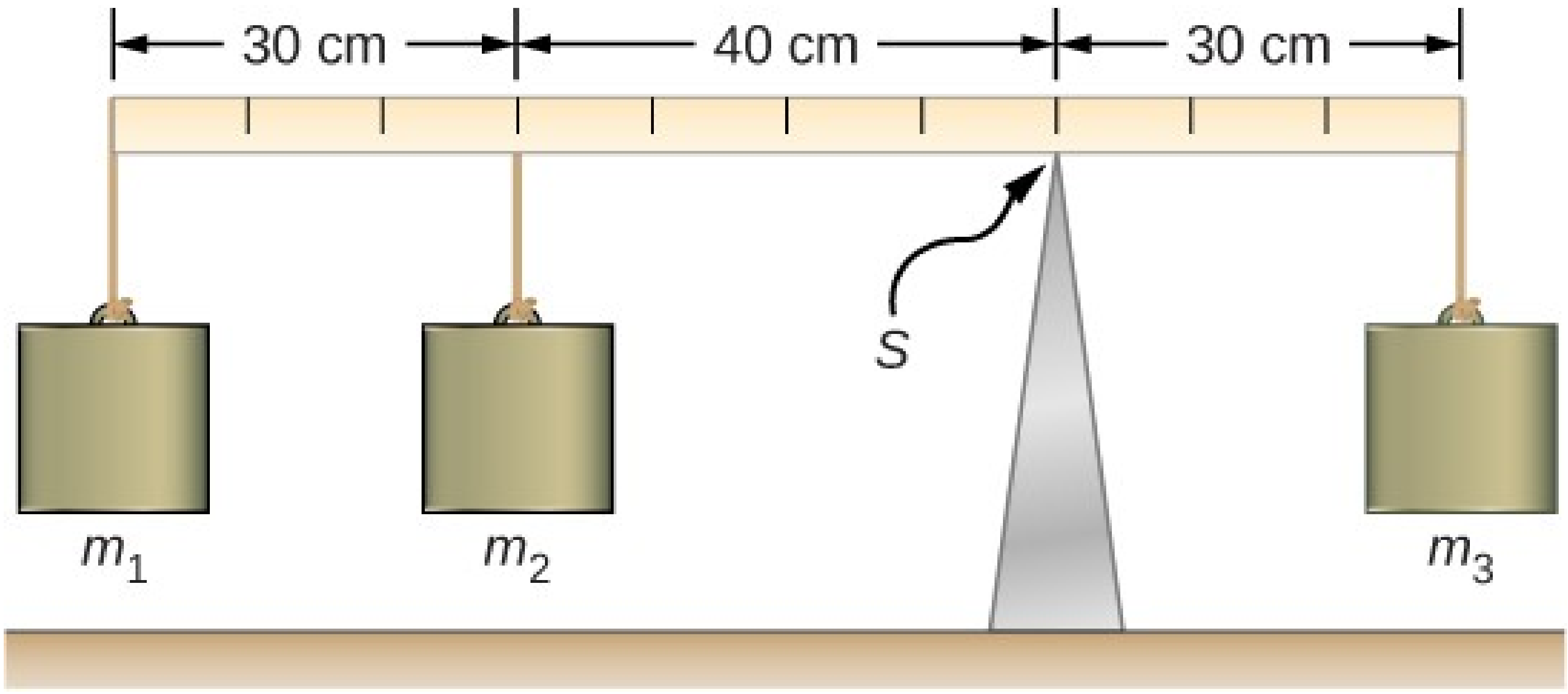
$$2.5T_1/\sqrt{5} = (M + m)g.$$

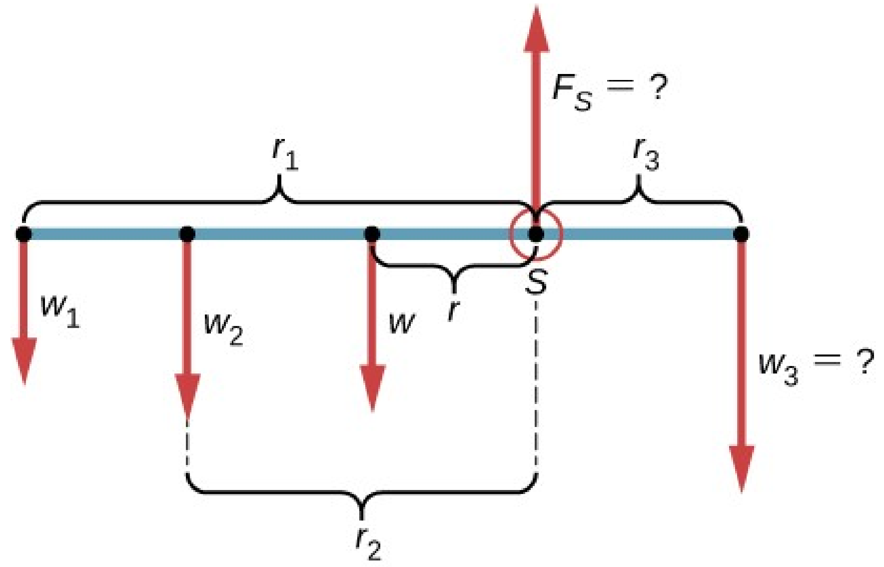
The string breaks when the tension reaches the critical value of $T_1 = 2.80\ \text{N}$. The preceding equation can be solved for the critical mass *m* that breaks the string:

$$m = \frac{2.5}{\sqrt{5}} \frac{T_1}{g} - M = \frac{2.5}{\sqrt{5}} \frac{2.80\ \text{N}}{9.8\ \text{m/s}^2} - 0.042\ \text{kg} = 0.277\ \text{kg} = 277.0\ \text{g}.$$

The Torque Balance

Three masses are attached to a uniform meter stick, as shown in Figure 12.9. The mass of the meter stick is 150.0 g and the masses to the left of the fulcrum are $m_1 = 50.0\ \text{g}$ and $m_2 = 75.0\ \text{g}$. Find the mass m_3 that balances the system when it is attached at the right end of the stick, and the normal reaction force at the fulcrum when the system is balanced.

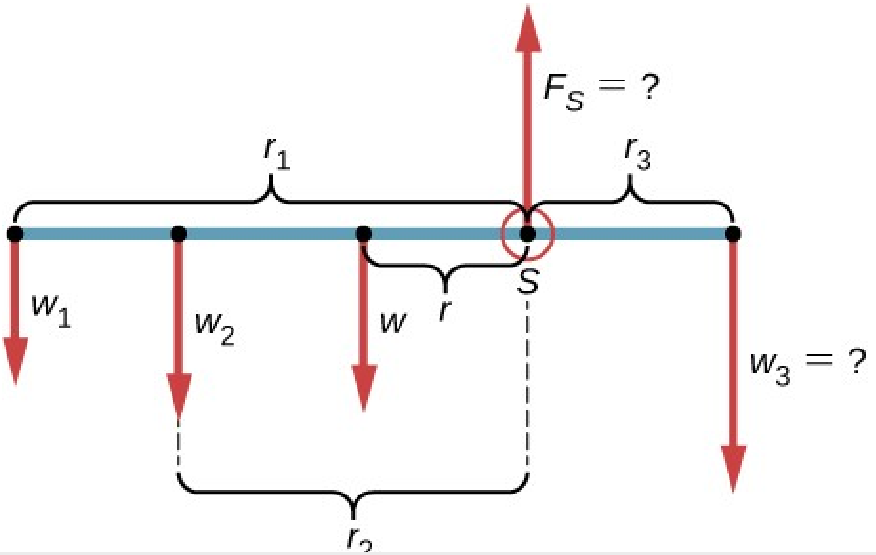




$$
\begin{aligned}
r_1 &= 30.0 \text{ cm} + 40.0 \text{ cm} = 70.0 \text{ cm} \\
r_2 &= 40.0 \text{ cm} \\
r &= 50.0 \text{ cm} - 30.0 \text{ cm} = 20.0 \text{ cm} \\
r_S &= 0.0 \text{ cm (because } F_S \text{ is attached at the pivot)} \\
r_3 &= 30.0 \text{ cm.}
\end{aligned}
$$

Now we can find the five torques with respect to the chosen pivot:

$$
\begin{aligned}
\tau_1 &= +r_1 w_1 \sin 90^\circ = +r_1 m_1 g && \text{(counterclockwise rotation, positive sense)} \\
\tau_2 &= +r_2 w_2 \sin 90^\circ = +r_2 m_2 g && \text{(counterclockwise rotation, positive sense)} \\
\tau &= +rw \sin 90^\circ = +rmg && \text{(gravitational torque)} \\
\tau_S &= r_S F_S \sin \theta_S = 0 && \text{(because } r_S = 0 \text{ cm)} \\
\tau_3 &= -r_3 w_3 \sin 90^\circ = -r_3 m_3 g && \text{(clockwise rotation, negative sense)}
\end{aligned}
$$



The second equilibrium condition (equation for the torques) for the meter stick is

$$\tau_1 + \tau_2 + \tau + \tau_S + \tau_3 = 0.$$

When substituting torque values into this equation, we can omit the torques giving zero contributions. In this way the second equilibrium condition is

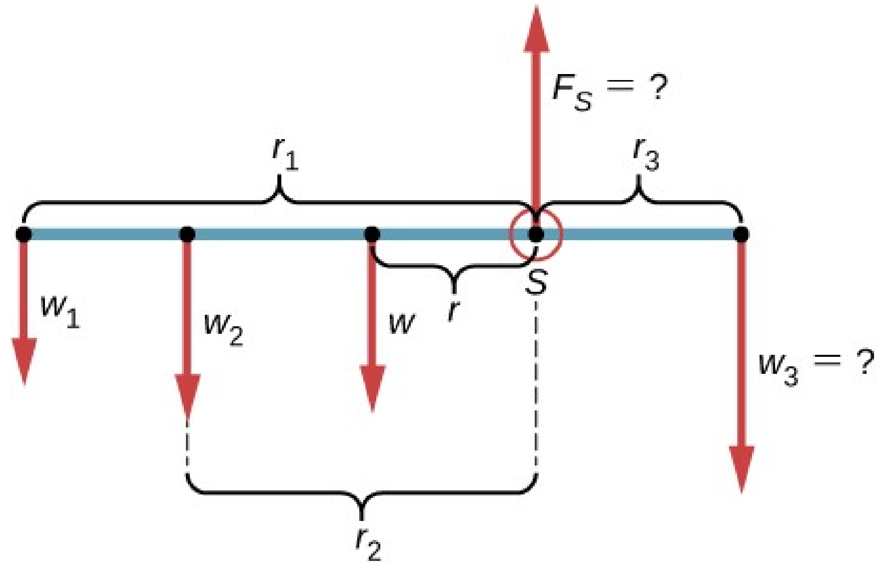
$$+r_1 m_1 g + r_2 m_2 g + rmg - r_3 m_3 g = 0. \tag{12.17}$$

Selecting the $+y$-direction to be parallel to $\vec{F}_S$, the first equilibrium condition for the stick is

$$-w_1 - w_2 - w + F_S - w_3 = 0.$$

Substituting the forces, the first equilibrium condition becomes

$$-m_1 g - m_2 g - mg + F_S - m_3 g = 0. \tag{12.18}$$



We solve these equations simultaneously for the unknown values m_3 and F_S. In Equation 12.17, we cancel the *g* factor and rearrange the terms to obtain

$$r_3 m_3 = r_1 m_1 + r_2 m_2 + rm.$$

To obtain m_3 we divide both sides by r_3, so we have

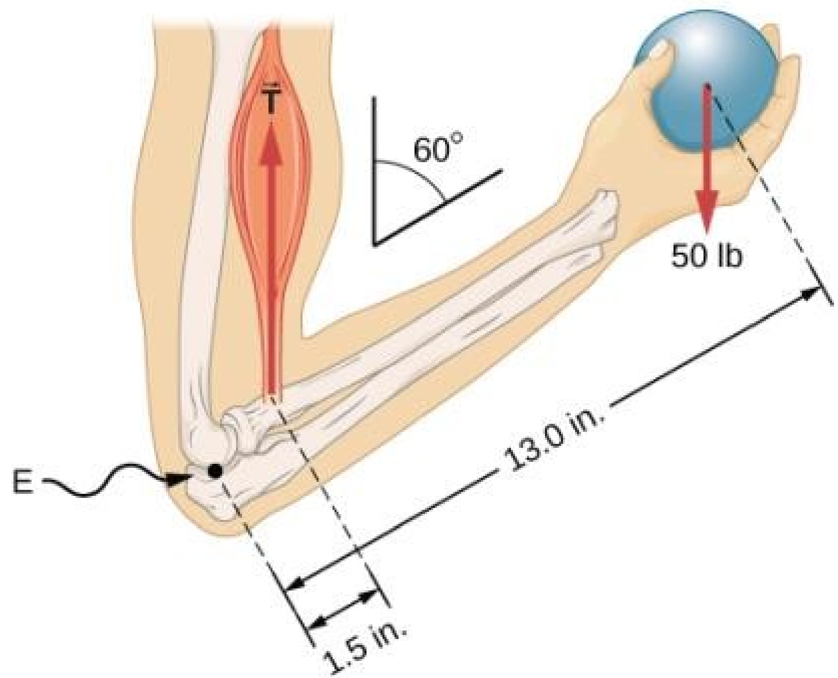
$$\begin{aligned} m_3 &= \frac{r_1}{r_3} m_1 + \frac{r_2}{r_3} m_2 + \frac{r}{r_3} m \\ &= \frac{70}{30}(50.0\text{ g}) + \frac{40}{30}(75.0\text{ g}) + \frac{20}{30}(150.0\text{ g}) = 316.0\tfrac{2}{3}\text{ g} \simeq 317\text{ g}. \end{aligned} \tag{12.19}$$

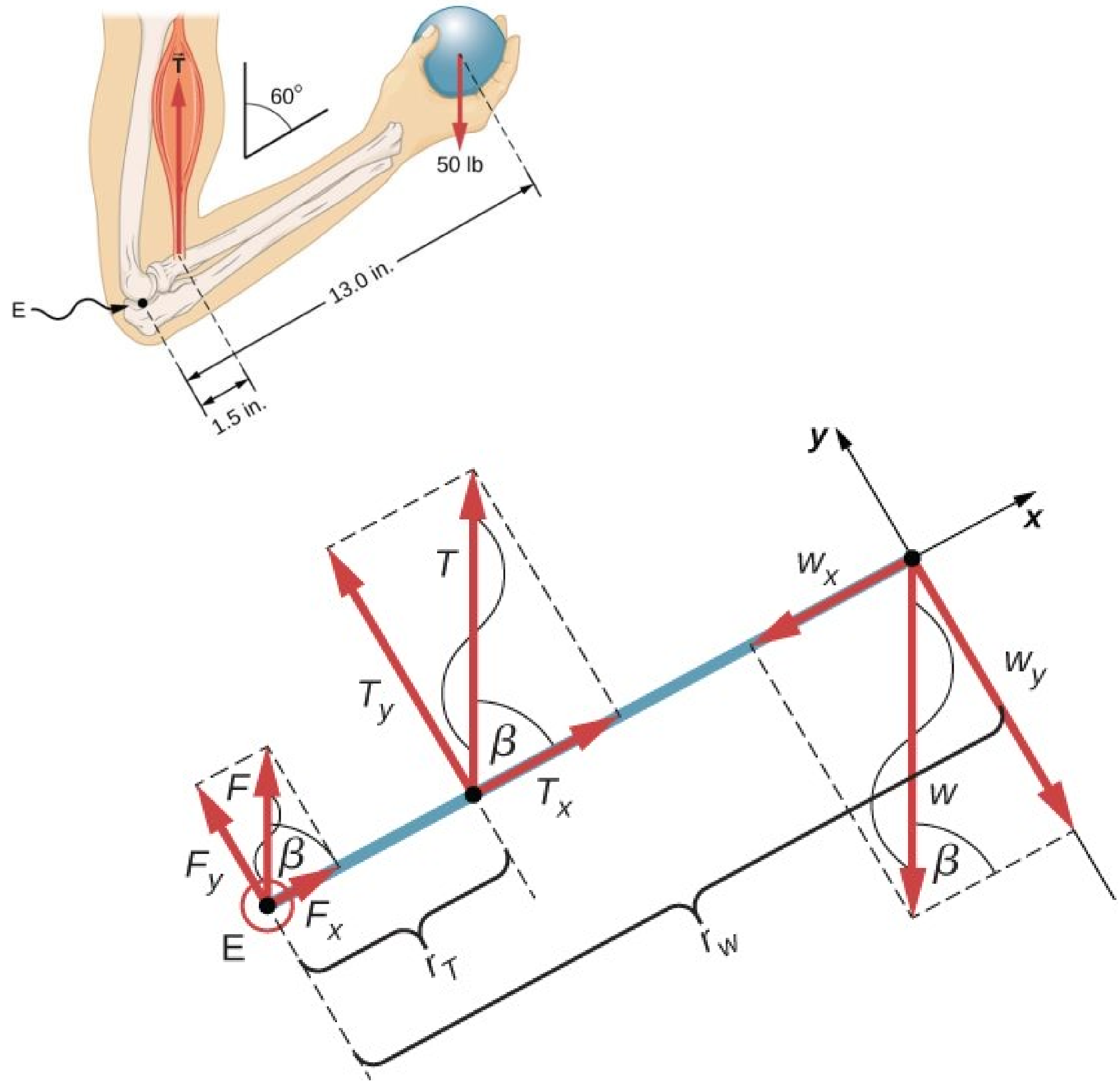
To find the normal reaction force, we rearrange the terms in Equation 12.18, converting grams to kilograms:

$$\begin{aligned} F_S &= (m_1 + m_2 + m + m_3)g \\ &= (50.0 + 75.0 + 150.0 + 316.7) \times 10^{-3}\text{kg} \times 9.8\,\frac{\text{m}}{\text{s}^2} = 5.8\text{ N}. \end{aligned} \tag{12.20}$$

Forces in the Forearm

A weightlifter is holding a 50.0-lb weight (equivalent to 222.4 N) with his forearm, as shown in Figure 12.11. His forearm is positioned at $\beta = 60°$ with respect to his upper arm. The forearm is supported by a contraction of the biceps muscle, which causes a torque around the elbow. Assuming that the tension in the biceps acts along the vertical direction given by gravity, what tension must the muscle exert to hold the forearm at the position shown? What is the force on the elbow joint? Assume that the forearm's weight is negligible. Give your final answers in SI units.

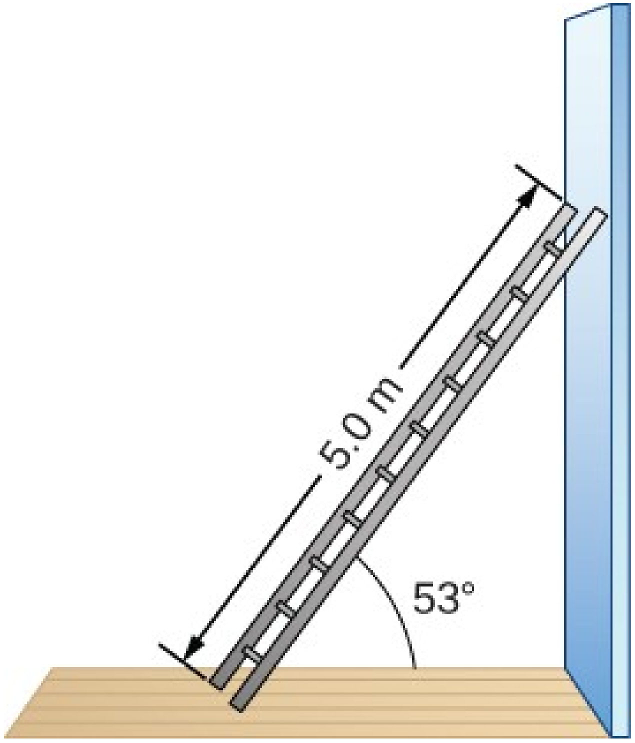


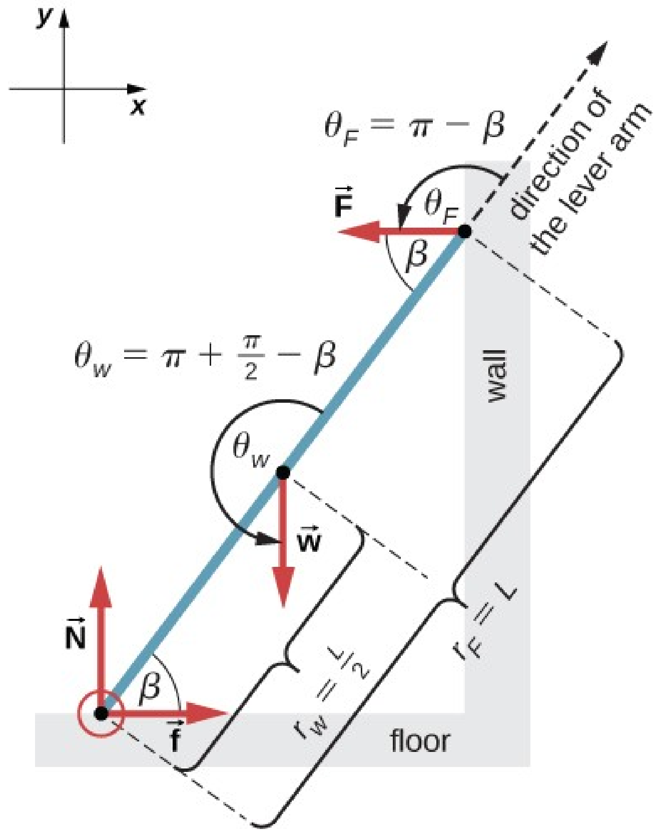


Free-body diagram for the forearm: The pivot is located at point *E* (elbow).

A Ladder Resting Against a Wall

A uniform ladder is $L = 5.0\ \mathrm{m}$ long and weighs 400.0 N. The ladder rests against a slippery vertical wall, as shown in Figure 12.14. The inclination angle between the ladder and the rough floor is $\beta = 53°$. Find the reaction forces from the floor and from the wall on the ladder and the coefficient of static friction μ_s at the interface of the ladder with the floor that prevents the ladder from slipping.





From the free-body diagram, the net force in the x-direction is

$$+f - F = 0$$

the net force in the y-direction is

$$+N - w = 0$$

and the net torque along the rotation axis at the pivot point is

$$\tau_w + \tau_F = 0.$$

$$\tau_w = r_w w \sin\theta_w = r_w w \sin(180° + 90° - \beta) = -\frac{L}{2} w \sin(90° - \beta) = -\frac{L}{2} w \cos\beta$$
$$\tau_F = r_F F \sin\theta_F = r_F F \sin(180° - \beta) = LF \sin\beta.$$

We substitute the torques into Equation 12.30 and solve for F :

$$\begin{aligned} -\frac{L}{2} w \cos\beta + LF \sin\beta &= 0 \\ F = \frac{w}{2} \cot\beta = \frac{400.0\text{ N}}{2} \cot 53° &= 150.7\text{ N} \end{aligned} \tag{12.31}$$

We obtain the normal reaction force with the floor by solving Equation 12.29: $N = w = 400.0\text{ N}$. The magnitude of friction is obtained by solving Equation 12.28: $f = F = 150.7\text{ N}$. The coefficient of static friction is $\mu_s = f/N = 150.7/400.0 = 0.377$.