ANNOUNCEMENTS

- Homework #11, due Friday, Nov. 9 before class

  Conceptual questions: Chapter 11, #8 and #14
  Problems: Chapter 11, #48, #56

- Study Chapter 11, sections 1 through 3

- Quiz #11, Friday November 9 at the beginning of class

ANGULAR MOMENTUM

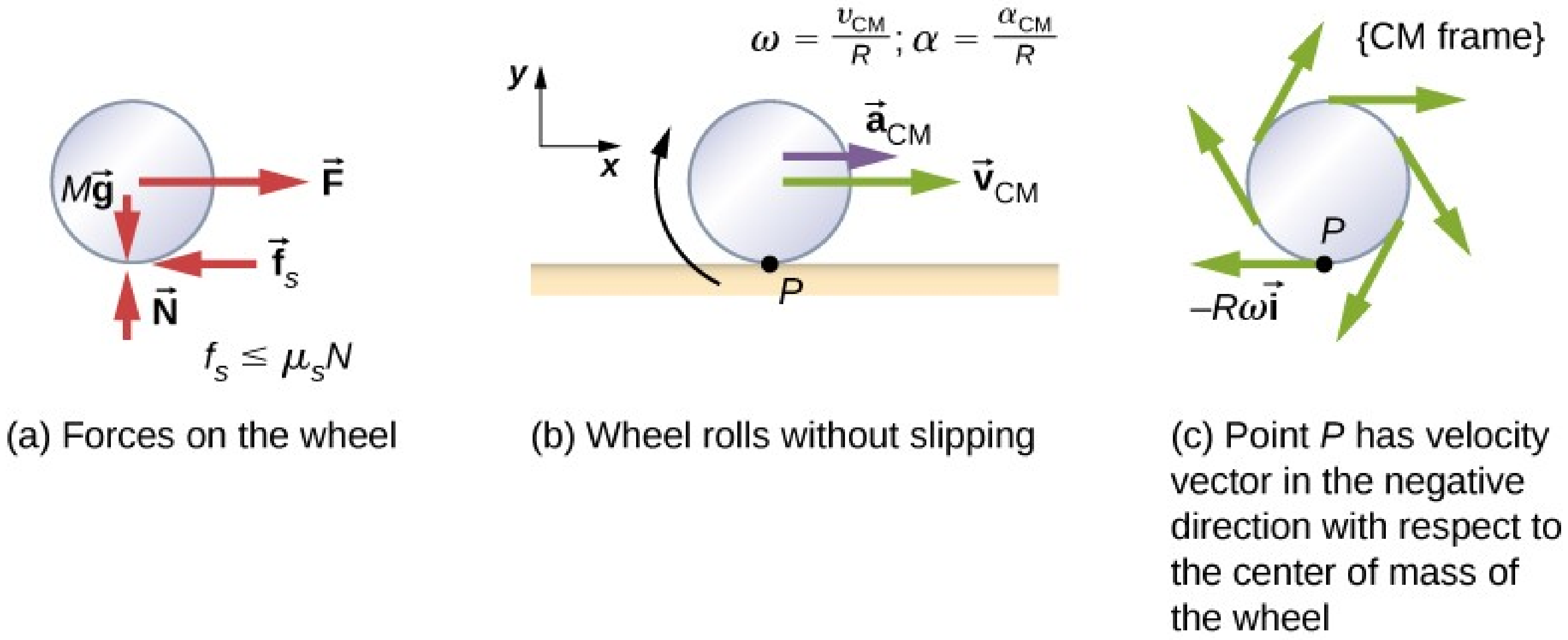
ROLLING MOTION

(a)

(b)

(a) The bicycle moves forward, and its tires do not slip. The bottom of the slightly deformed tire is at rest with respect to the road surface for a measurable amount of time.

(b) This image shows that the top of a rolling wheel appears blurred by its motion, but the bottom of the wheel is instantaneously at rest. (credit a: modification of work by Nelson Lourenço; credit b: modification of work by Colin Rose)

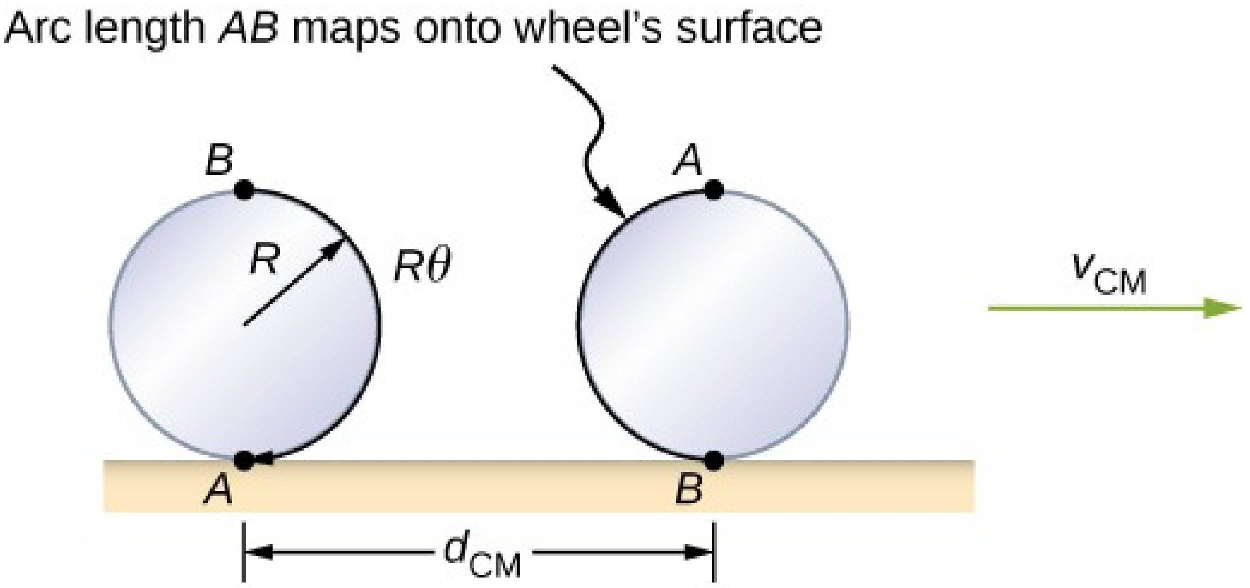


(a) A wheel is pulled across a horizontal surface by a force $\vec{\mathbf{F}}$. The force of static friction $\vec{\mathbf{f}}_s$, $|\vec{\mathbf{f}}_s| \leq \mu_s N$ is large enough to keep it from slipping.

(b) The linear velocity and acceleration vectors of the center of mass and the relevant expressions for ω and α. Point *P* is at rest relative to the surface.

(c) Relative to the center of mass (CM) frame, point *P* has linear velocity $-R\omega\hat{\mathbf{i}}$.

FIGURE 11.4

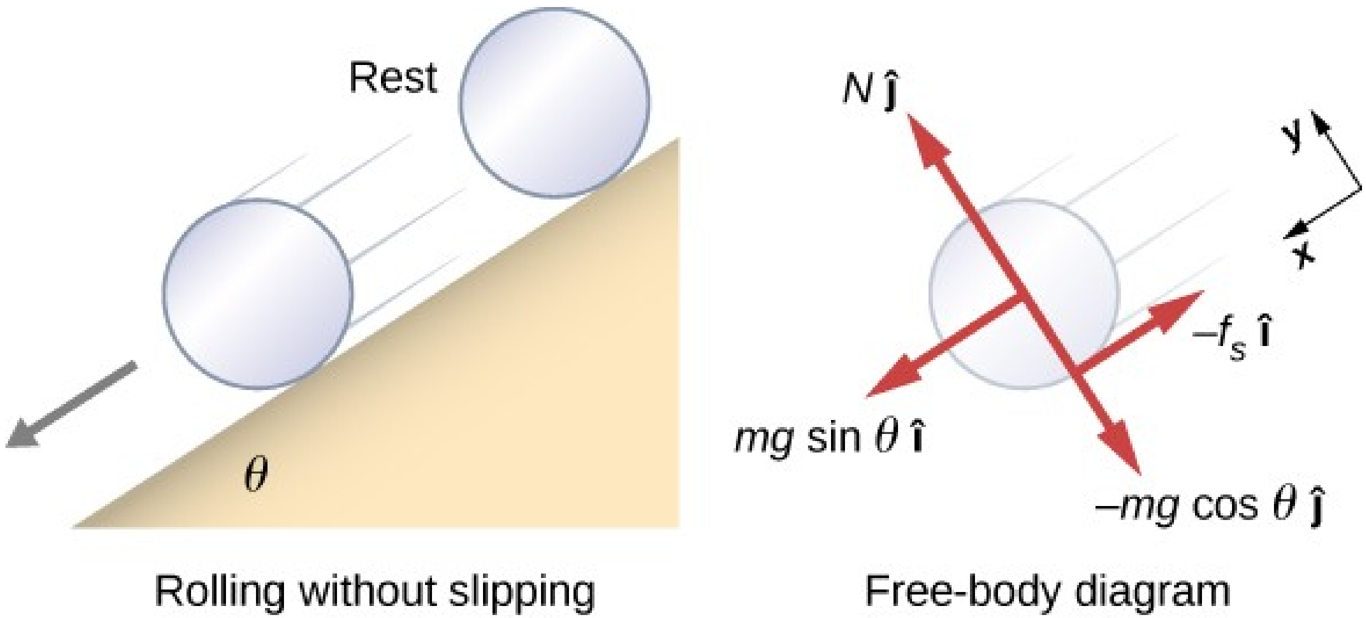


As the wheel rolls on the surface, the arc length $R\theta$ from A to B maps onto the surface, corresponding to the distance d_{CM} that the center of mass has moved.

$d_{CM} = R\theta.$ → $v_{CM} = R\omega.$ → $a_{CM} = R\alpha.$

Rolling Down an Inclined Plane

A solid cylinder rolls down an inclined plane without slipping, starting from rest. It has mass *m* and radius *r*. (a) What is its acceleration? (b) What condition must the coefficient of static friction μ_S satisfy so the cylinder does not slip?



A solid cylinder rolls down an inclined plane without slipping from rest. The coordinate system has *x* in the direction down the inclined plane and *y* perpendicular to the plane. The free-body diagram is shown with the normal force, the static friction force, and the components of the weight $m\vec{g}$. Friction makes the cylinder roll down the plane rather than slip.

Rolling Down an Inclined Plane

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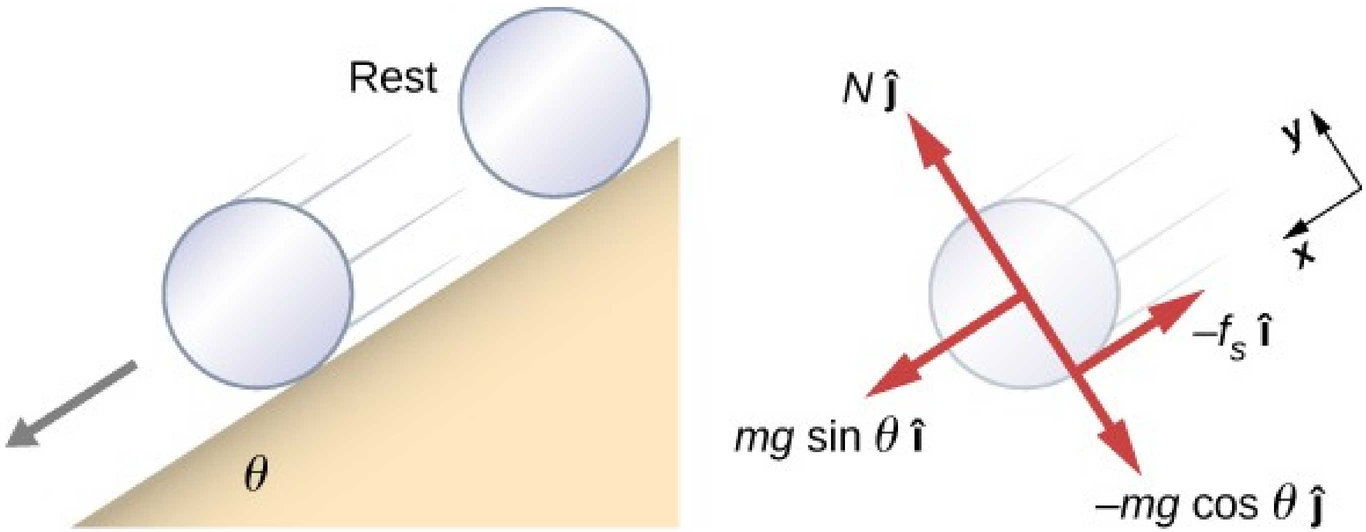


Rolling without slipping

Free-body diagram

$$\begin{aligned} mg\sin\theta - f_S &= m(a_{CM})_x, \\ N - mg\cos\theta &= 0, \\ f_S &\le \mu_S N, \end{aligned}$$

$$\sum \tau_{CM} = I_{CM}\alpha. \quad \rightarrow \quad f_S r = I_{CM}\alpha.$$

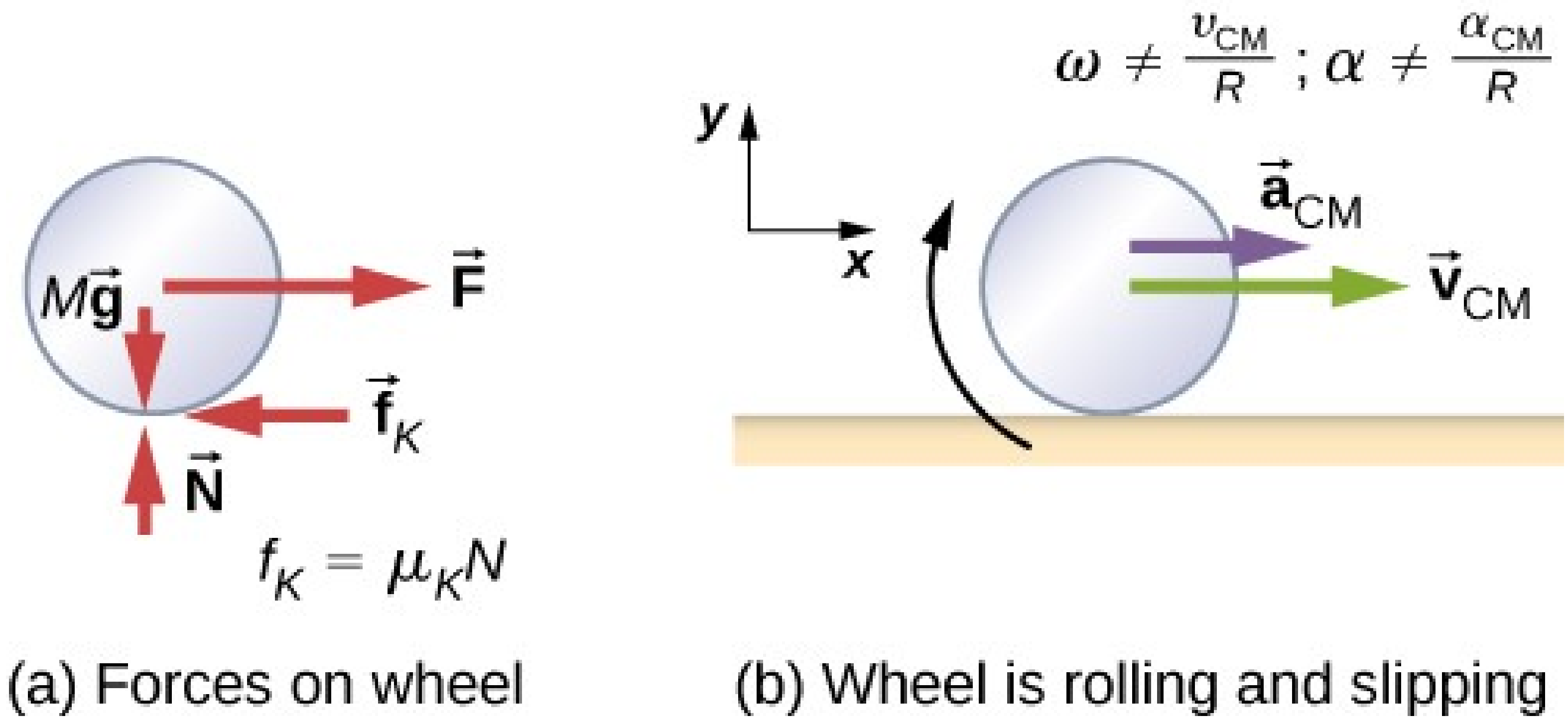
$$(a_{CM})_x = r\alpha.$$

$$I_{CM} = mr^2/2$$

$$a_{CM} = \frac{mg\sin\theta}{m + (mr^2/2r^2)} = \frac{2}{3}g\sin\theta.$$

$$\rightarrow \quad \alpha = \frac{a_{CM}}{r} = \frac{2}{3r}g\sin\theta.$$

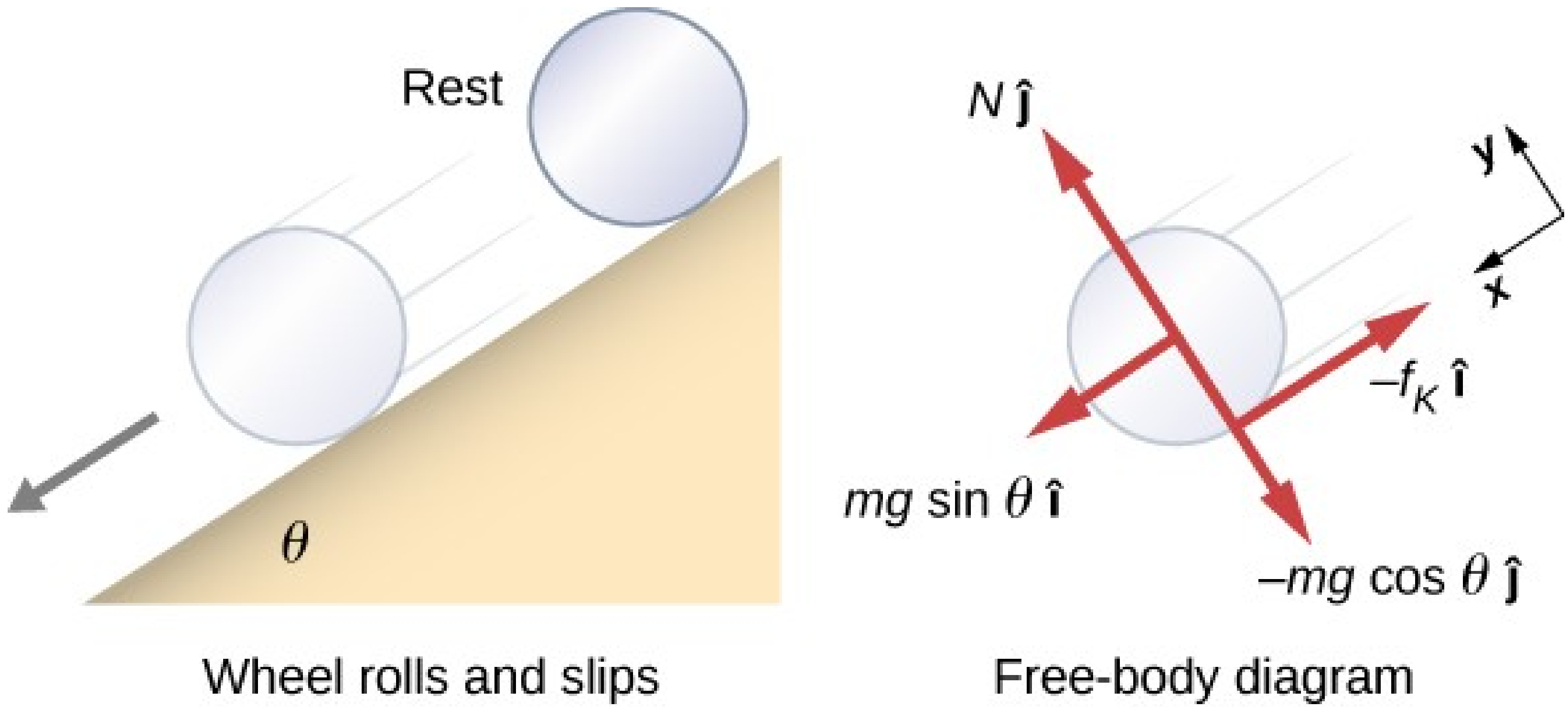
ROLLING AND SLIPPING



(a) Kinetic friction arises between the wheel and the surface because the wheel is slipping.

(b) The simple relationships between the linear and angular variables are no longer valid.

FIGURE 11.7



A solid cylinder rolls down an inclined plane from rest and undergoes slipping. The coordinate system has *x* in the direction down the inclined plane and *y* upward perpendicular to the plane. The free-body diagram shows the normal force, kinetic friction force, and the components of the weight $m\vec{g}$.

Conservation of Mechanical Energy in Rolling Motion

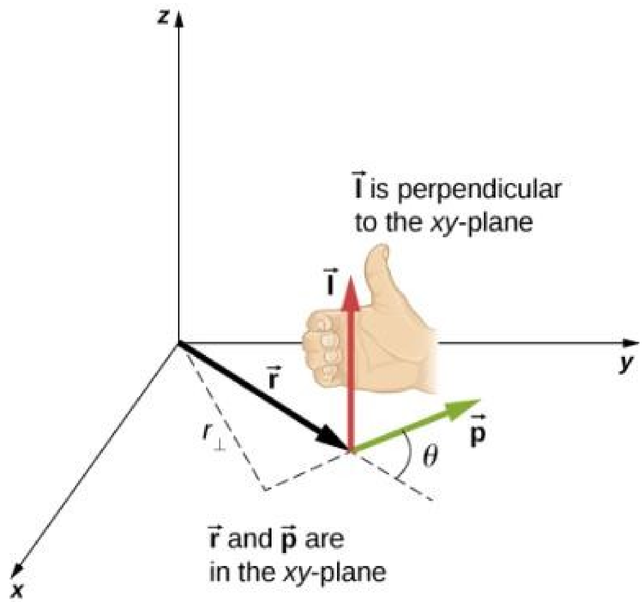
In the preceding chapter, we introduced rotational kinetic energy. Any rolling object carries rotational kinetic energy, as well as translational kinetic energy and potential energy if the system requires. Including the gravitational potential energy, the total mechanical energy of an object rolling is

$$E_{\text{T}} = \frac{1}{2}mv_{\text{CM}}^2 + \frac{1}{2}I_{\text{CM}}\omega^2 + mgh.$$

ANGULAR MOMENTUM OF A PARTICLE

The **angular momentum** $\vec{\mathbf{l}}$ of a particle is defined as the cross-product of $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$, and is perpendicular to the plane containing $\vec{\mathbf{r}}$ and $\vec{\mathbf{p}}$:

$$\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}. \tag{11.5}$$

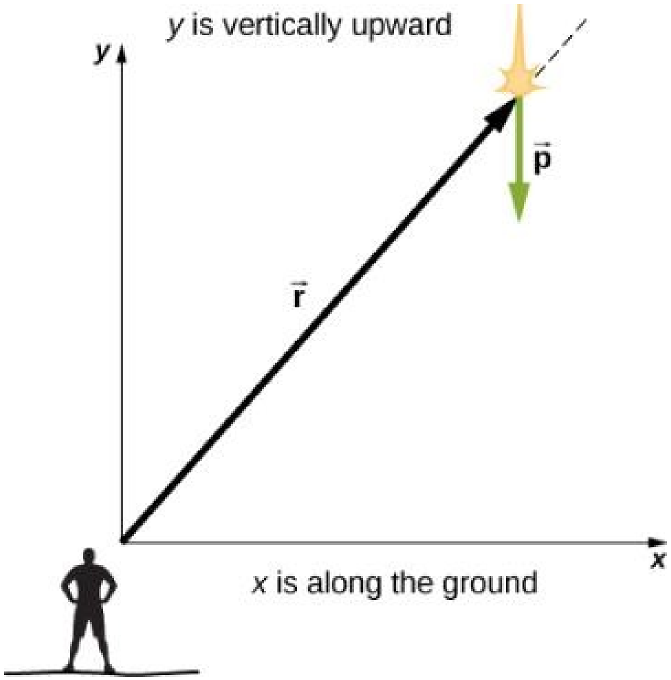


$$\frac{d\vec{\mathbf{l}}}{dt} = \sum \vec{\boldsymbol{\tau}}.$$

In three-dimensional space, the position vector $\vec{\mathbf{r}}$ locates a particle in the *xy*-plane with linear momentum $\vec{\mathbf{p}}$. The angular momentum with respect to the origin is $\vec{\mathbf{l}} = \vec{\mathbf{r}} \times \vec{\mathbf{p}}$, which is in the *z*-direction. The direction of $\vec{\mathbf{l}}$ is given by the right-hand rule, as shown.

Angular Momentum and Torque on a Meteor

A meteor enters Earth's atmosphere (Figure 11.10) and is observed by someone on the ground before it burns up in the atmosphere. The vector $\vec{\mathbf{r}} = 25\ \text{km}\hat{\mathbf{i}} + 25\ \text{km}\hat{\mathbf{j}}$ gives the position of the meteor with respect to the observer. At the instant the observer sees the meteor, it has linear momentum $\vec{\mathbf{p}} = 15.0\ \text{kg}(-2.0\text{km/s}\hat{\mathbf{j}})$, and it is accelerating at a constant $2.0\ \text{m/s}^2(-\hat{\mathbf{j}})$ along its path, which for our purposes can be taken as a straight line. (a) What is the angular momentum of the meteor about the origin, which is at the location of the observer? (b) What is the torque on the meteor about the origin?



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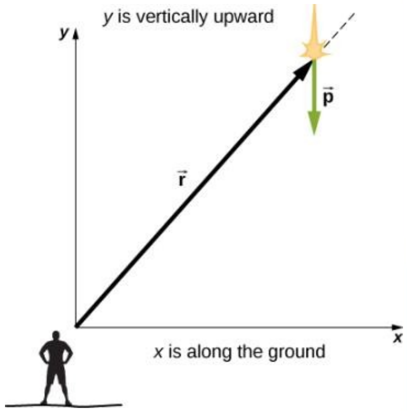


$$a_x = 0,\quad a_y = -2.0\ \text{m/s}^2.$$

We write the velocities using the kinematic equations.

$$v_x = 0,\quad v_y = -2.0 \times 10^3\ \text{m/s} - (2.0\ \text{m/s}^2)t.$$

a. The angular momentum is

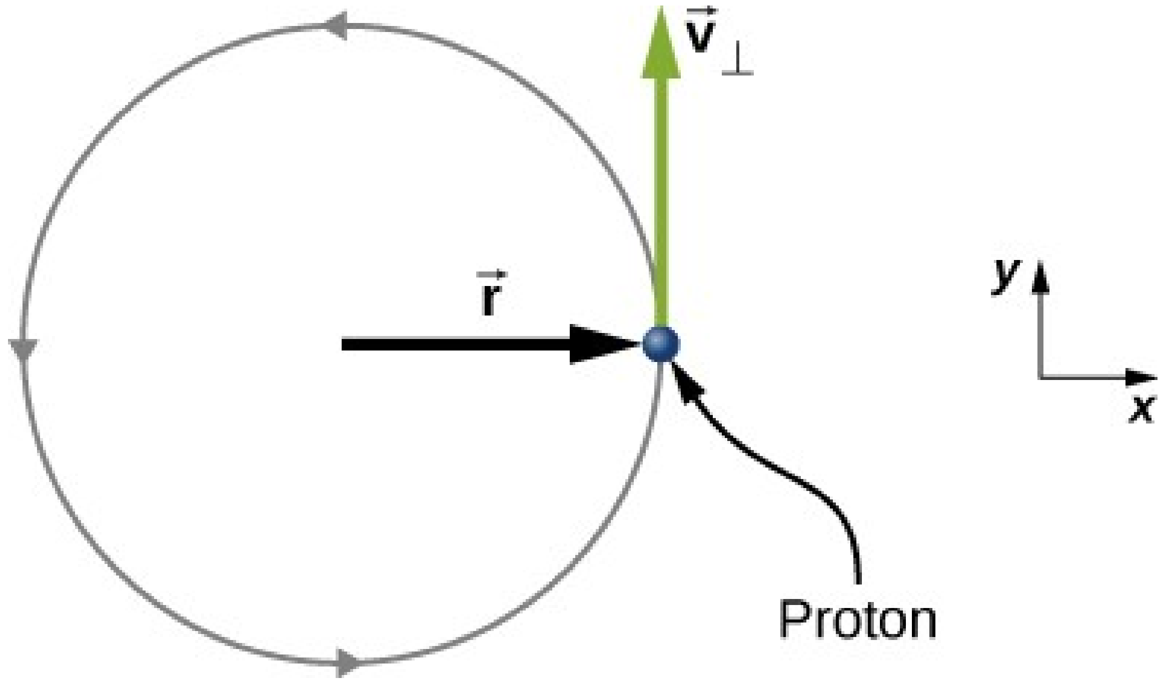
$$\begin{aligned}\vec{\mathbf{l}} &= \vec{\mathbf{r}} \times \vec{\mathbf{p}} = (25.0\ \text{km}\hat{\mathbf{i}} + 25.0\ \text{km}\hat{\mathbf{j}}) \times 15.0\ \text{kg}(0\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}) \\ &= 15.0\ \text{kg}[25.0\ \text{km}(v_y)\hat{\mathbf{k}}] \\ &= 15.0\ \text{kg}[2.50 \times 10^4\ \text{m}(-2.0 \times 10^3\ \text{m/s} - (2.0\ \text{m/s}^2)t)\hat{\mathbf{k}}].\end{aligned}$$

At $t = 0$, the angular momentum of the meteor about the origin is

$$\vec{\mathbf{l}}_0 = 15.0\ \text{kg}[2.50 \times 10^4\ \text{m}(-2.0 \times 10^3\ \text{m/s})\hat{\mathbf{k}}] = 7.50 \times 10^8\ \text{kg} \cdot \text{m}^2/\text{s}(-\hat{\mathbf{k}}).$$

DIY

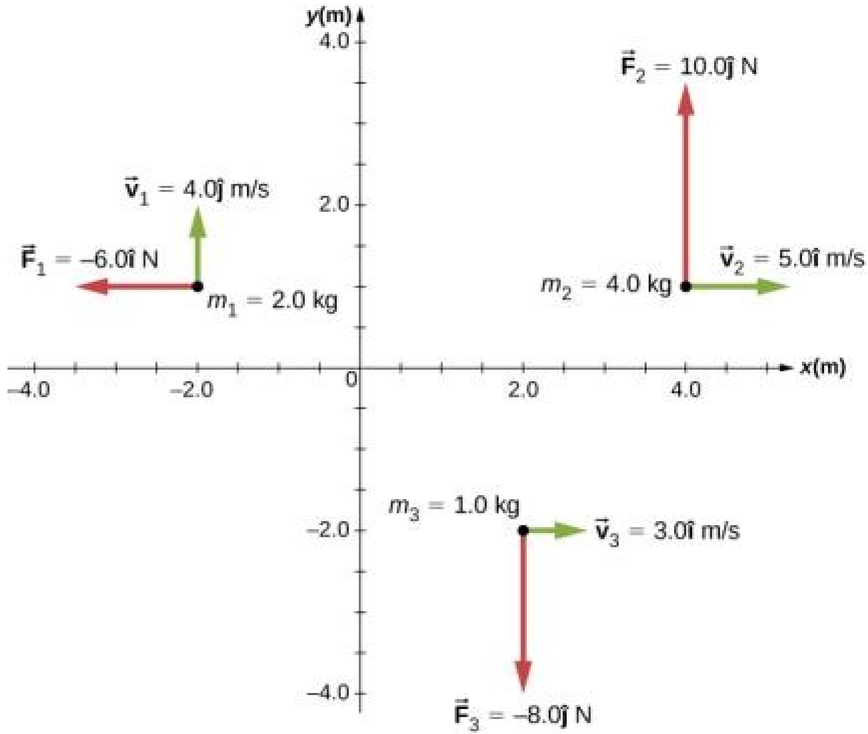
A proton spiraling around a magnetic field executes circular motion in the plane of the paper, as shown below. The circular path has a radius of 0.4 m and the proton has velocity 4.0×10^6 m/s. What is the angular momentum of the proton about the origin?



TOTAL ANGULAR MOMENTUM

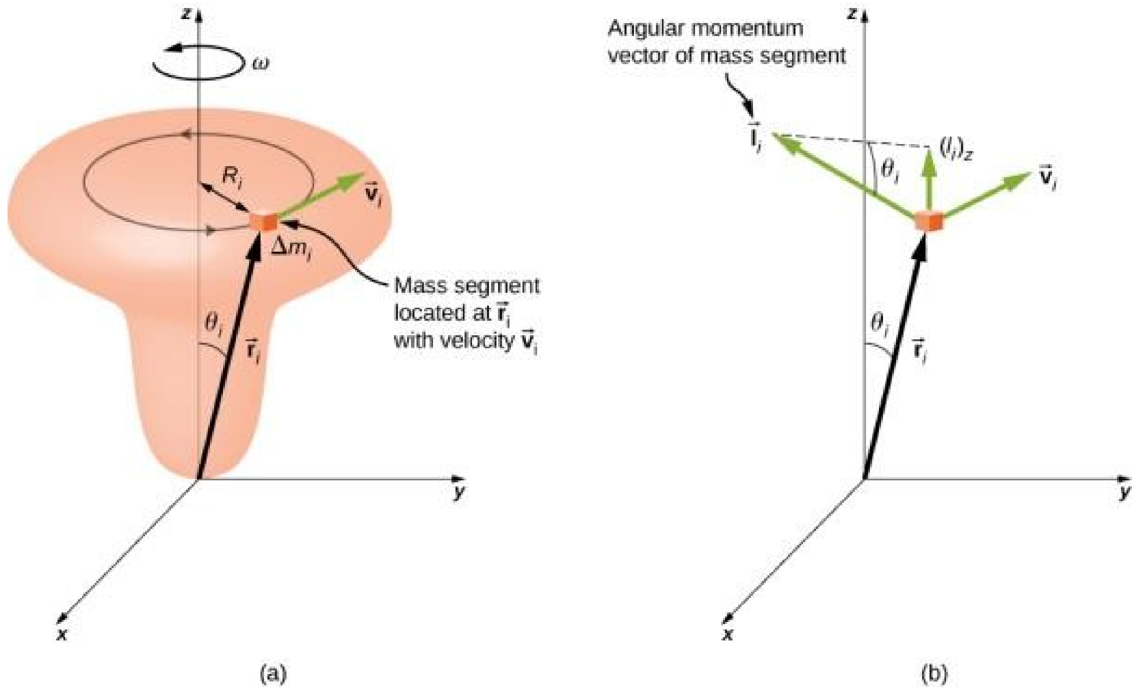
$$\vec{L} = \vec{l}_1 + \vec{l}_2 + \cdots + \vec{l}_N.$$

$$\frac{d\vec{L}}{dt} = \sum \vec{\tau}.$$



Three particles in the *xy*-plane with different position and momentum vectors.

ANGULAR MOMENTUM OF A RIGID BODY



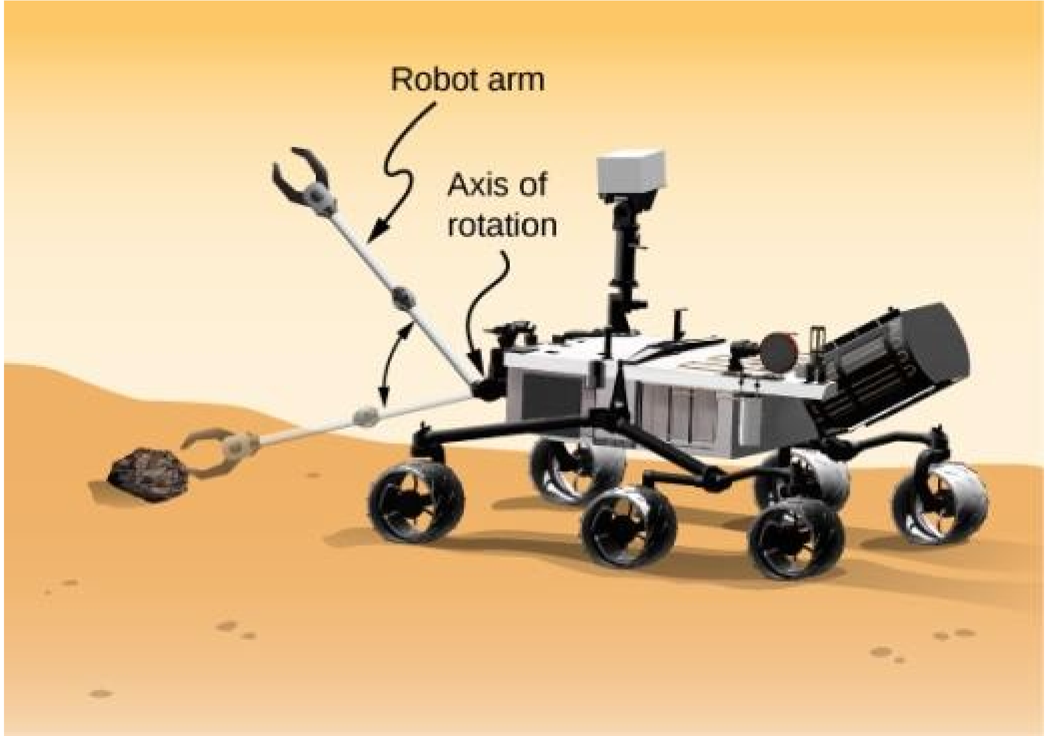
(a) A rigid body is constrained to rotate around the *z*-axis. The rigid body is symmetrical about the *z*-axis. A mass segment Δm_i is located at position $\vec{\mathbf{r}}_i$, which makes angle θ_i with respect to the *z*-axis. The circular motion of an infinitesimal mass segment is shown.

(b) $\vec{\mathbf{l}}_i$ is the angular momentum of the mass segment and has a component along the *z*-axis $(\vec{\mathbf{l}}_i)_z$.

$$L = I\omega.$$

Angular Momentum of a Robot Arm

A robot arm on a Mars rover like *Curiosity* shown in Figure 11.8 is 1.0 m long and has forceps at the free end to pick up rocks. The mass of the arm is 2.0 kg and the mass of the forceps is 1.0 kg. See Figure 11.13. The robot arm and forceps move from rest to $\omega = 0.1\pi \text{ rad/s}$ in 0.1 s. It rotates down and picks up a Mars rock that has mass 1.5 kg. The axis of rotation is the point where the robot arm connects to the rover. (a) What is the angular momentum of the robot arm by itself about the axis of rotation after 0.1 s when the arm has stopped accelerating? (b) What is the angular momentum of the robot arm when it has the Mars rock in its forceps and is rotating upwards? (c) When the arm does not have a rock in the forceps, what is the torque about the point where the arm connects to the rover when it is accelerating from rest to its final angular velocity?



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Robot arm: $I_R = \frac{1}{3} m_R r^2 = \frac{1}{3}(2.00\text{ kg})(1.00\text{ m})^2 = \frac{2}{3}\text{ kg}\cdot\text{m}^2$.

Forceps: $I_F = m_F r^2 = (1.0\text{ kg})(1.0\text{ m})^2 = 1.0\text{ kg}\cdot\text{m}^2$.

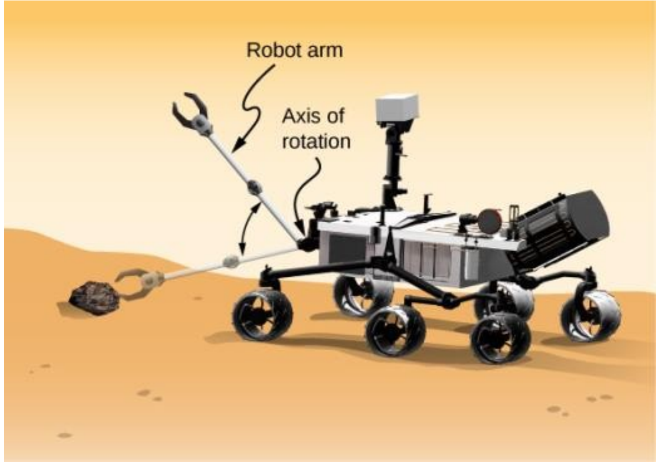
Mars rock: $I_{MR} = m_{MR} r^2 = (1.5\text{ kg})(1.0\text{ m})^2 = 1.5\text{ kg}\cdot\text{m}^2$.

Therefore, without the Mars rock, the total moment of inertia is

$$I_{\text{Total}} = I_R + I_F = 1.67\text{ kg}\cdot\text{m}^2$$

and the magnitude of the angular momentum is

$$L = I\omega = 1.67\text{ kg}\cdot\text{m}^2(0.1\pi\text{ rad/s}) = 0.17\pi\text{ kg}\cdot\text{m}^2/\text{s}.$$



b. We must include the Mars rock in the calculation of the moment of inertia, so we have

$$I_{\text{Total}} = I_R + I_F + I_{MR} = 3.17\text{ kg}\cdot\text{m}^2$$

and

$$L = I\omega = 3.17\text{ kg}\cdot\text{m}^2(0.1\pi\text{ rad/s}) = 0.32\pi\text{ kg}\cdot\text{m}^2/\text{s}.$$

$$\sum \tau = I\alpha = 1.67\text{ kg}\cdot\text{m}^2(\pi\text{ rad/s}^2) = 1.67\pi\text{ N}\cdot\text{m}.$$

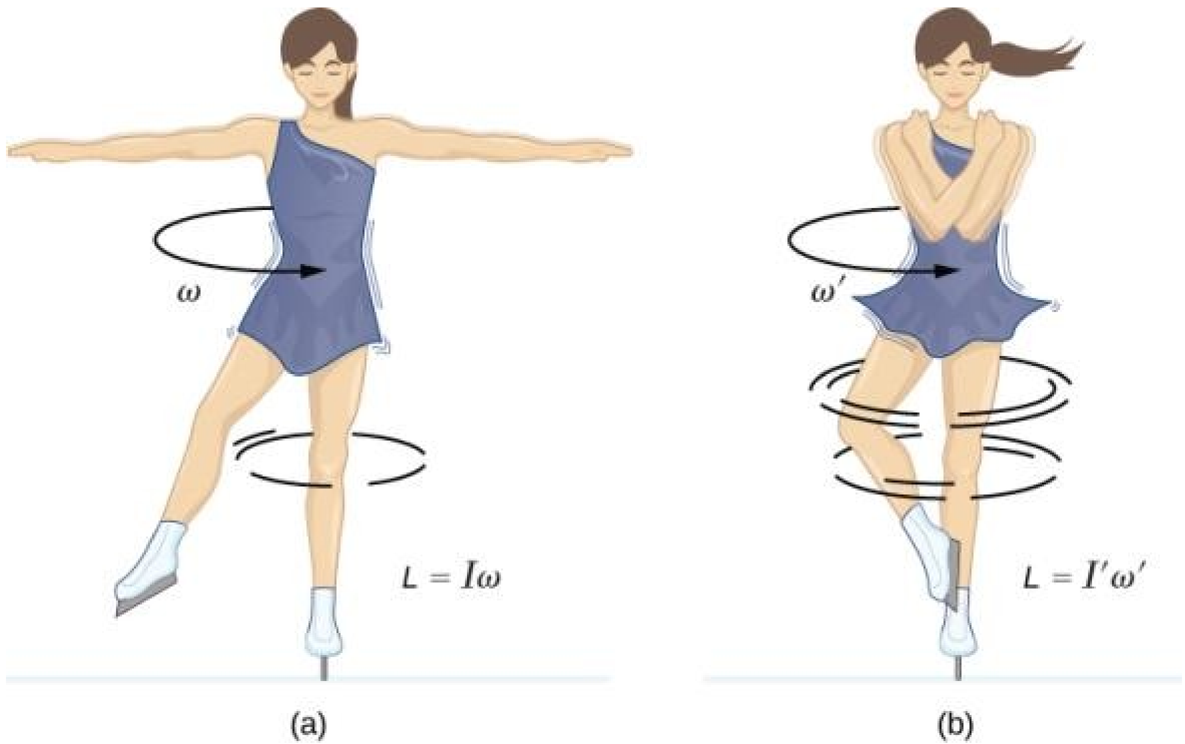
LAW OF CONSERVATION OF ANGULAR MOMENTUM

The angular momentum of a system of particles around a point in a fixed inertial reference frame is conserved if there is no net external torque around that point:

$$\frac{d\vec{\mathbf{L}}}{dt} = 0 \tag{11.10}$$

or

$$\vec{\mathbf{L}} = \vec{\mathbf{l}}_1 + \vec{\mathbf{l}}_2 + \cdots + \vec{\mathbf{l}}_N = \text{constant}. \tag{11.11}$$



$$L' = L$$

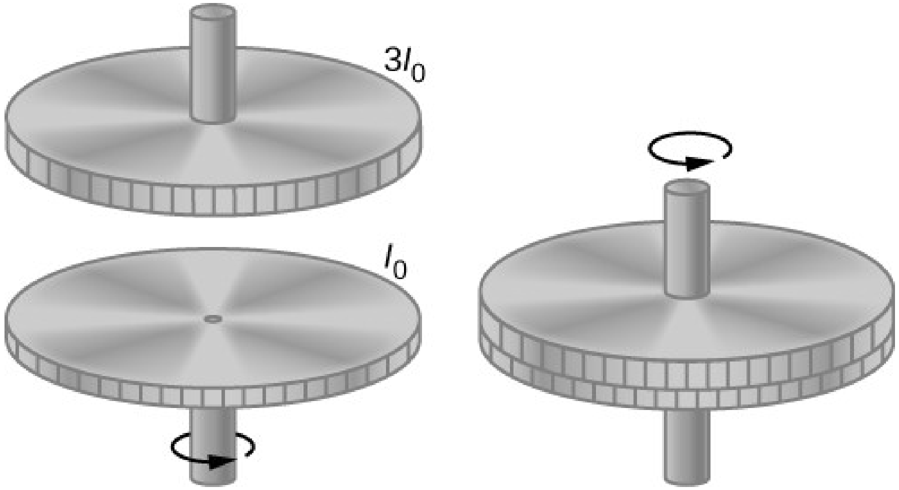
$$I'\omega' = I\omega,$$

(a) An ice skater is spinning on the tip of her skate with her arms extended. Her angular momentum is conserved because the net torque on her is negligibly small.

(b) Her rate of spin increases greatly when she pulls in her arms, decreasing her moment of inertia. The work she does to pull in her arms results in an increase in rotational kinetic energy.

Coupled Flywheels

A flywheel rotates without friction at an angular velocity $\omega_0 = 600\ \text{rev/min}$ on a frictionless, vertical shaft of negligible rotational inertia. A second flywheel, which is at rest and has a moment of inertia three times that of the rotating flywheel, is dropped onto it (Figure 11.16). Because friction exists between the surfaces, the flywheels very quickly reach the same rotational velocity, after which they spin together. (a) Use the law of conservation of angular momentum to determine the angular velocity ω of the combination. (b) What fraction of the initial kinetic energy is lost in the coupling of the flywheels?



$$I_0\omega_0 = (I_0 + 3I_0)\omega,$$
$$\omega = \frac{1}{4}\omega_0 = 150\ \text{rev/min} = 15.7\ \text{rad/s}.$$

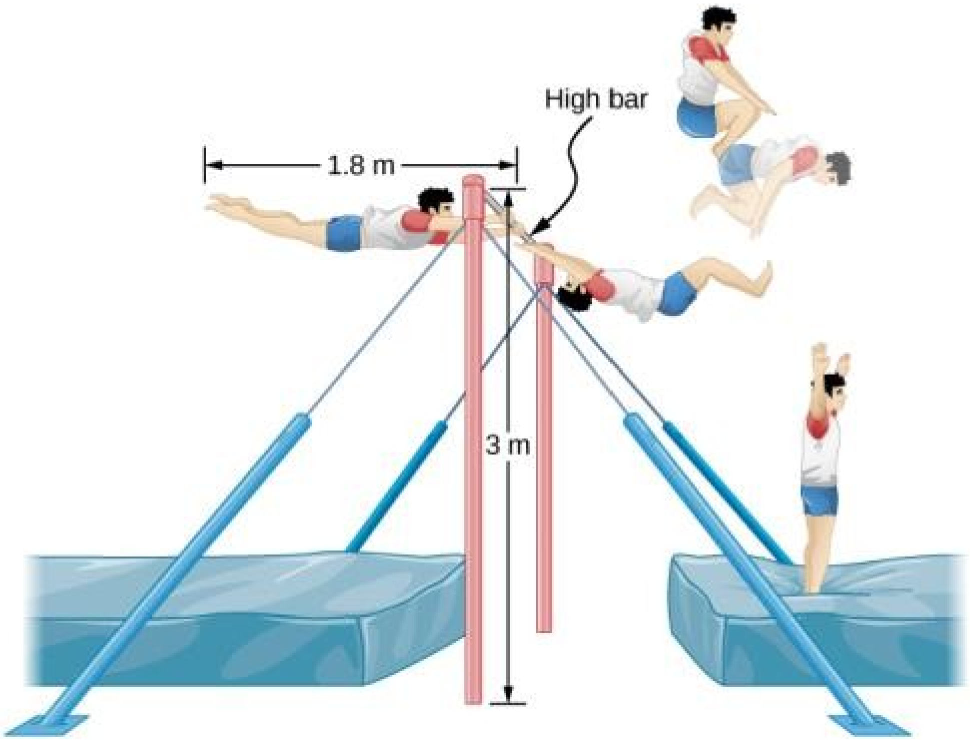
b. Before contact, only one flywheel is rotating. The rotational kinetic energy of this flywheel is the initial rotational kinetic energy of the system, $\frac{1}{2}I_0\omega_0^2$. The final kinetic energy is $\frac{1}{2}(4I_0)\omega^2 = \frac{1}{2}(4I_0)\left(\frac{\omega_0}{4}\right)^2 = \frac{1}{8}I_0\omega_0^2$.

Therefore, the ratio of the final kinetic energy to the initial kinetic energy is

$$\frac{\frac{1}{8}I_0\omega_0^2}{\frac{1}{2}I_0\omega_0^2} = \frac{1}{4}.$$

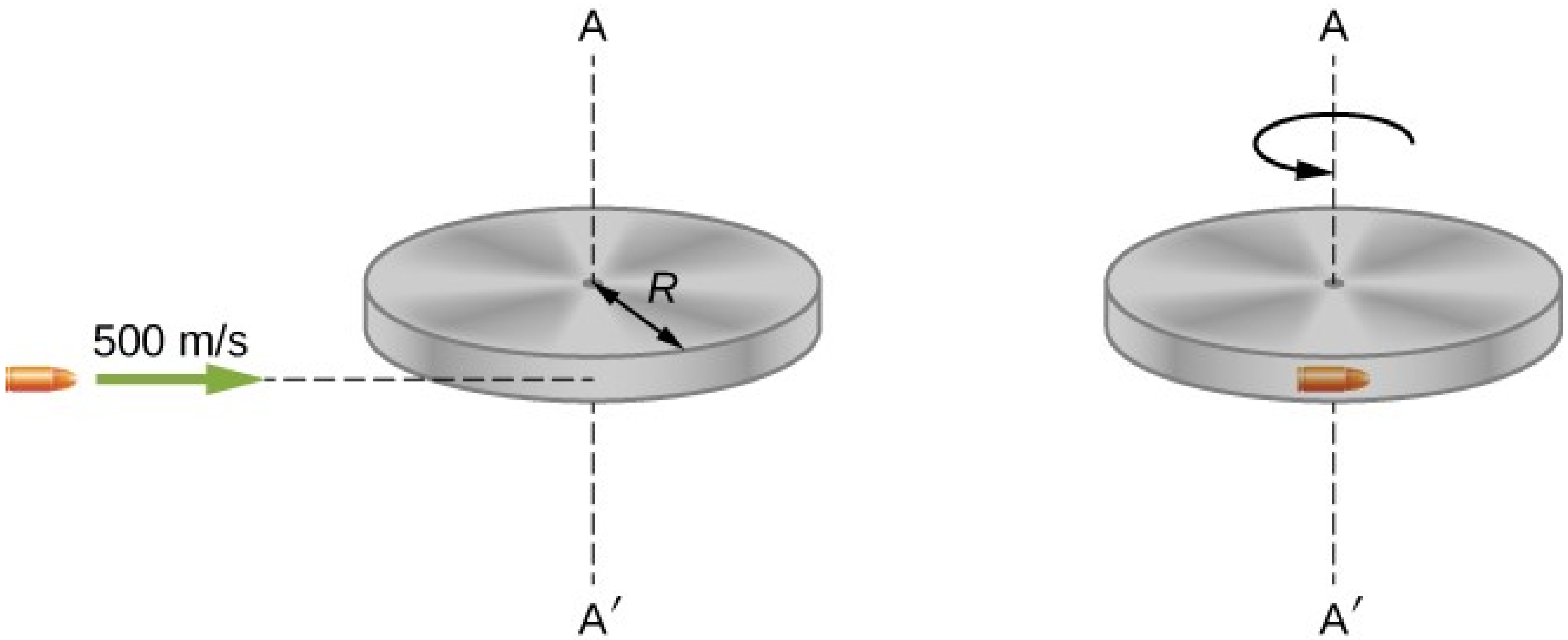
Thus, 3/4 of the initial kinetic energy is lost to the coupling of the two flywheels.

DIY



A gymnast dismounts from a high bar and executes a number of revolutions in the tucked position before landing upright.

DIY



A bullet is fired horizontally and becomes embedded in the edge of a disk that is free to rotate about its vertical axis.