ANNOUNCEMENTS

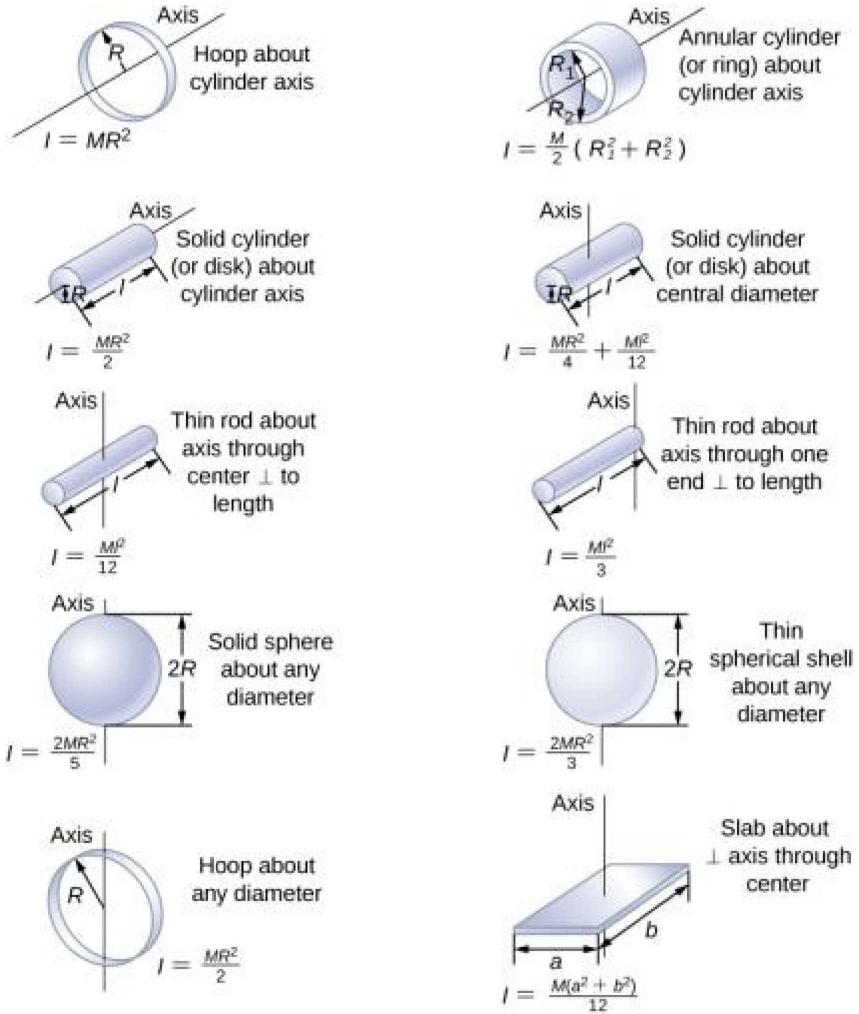
- Homework #10, due Wednesday, Oct. 31 before class

  Conceptual questions: Chapter 10, #2 and #10
  Problems: Chapter 10, #40, #46

- Study Chapter 10, sections 1 through 4 and 6 through 8

- Second in-class test: Friday, Nov. 2, Chapters 5-10

FIGURE 10.20



Values of rotational inertia for common shapes of objects.

ANALOGY TO LINEAR QUANTITIES

Rotational	Translational
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I\omega^2$	$K = \frac{1}{2} mv^2$

Table 10.4 Rotational and Translational Kinetic Energies and Inertia

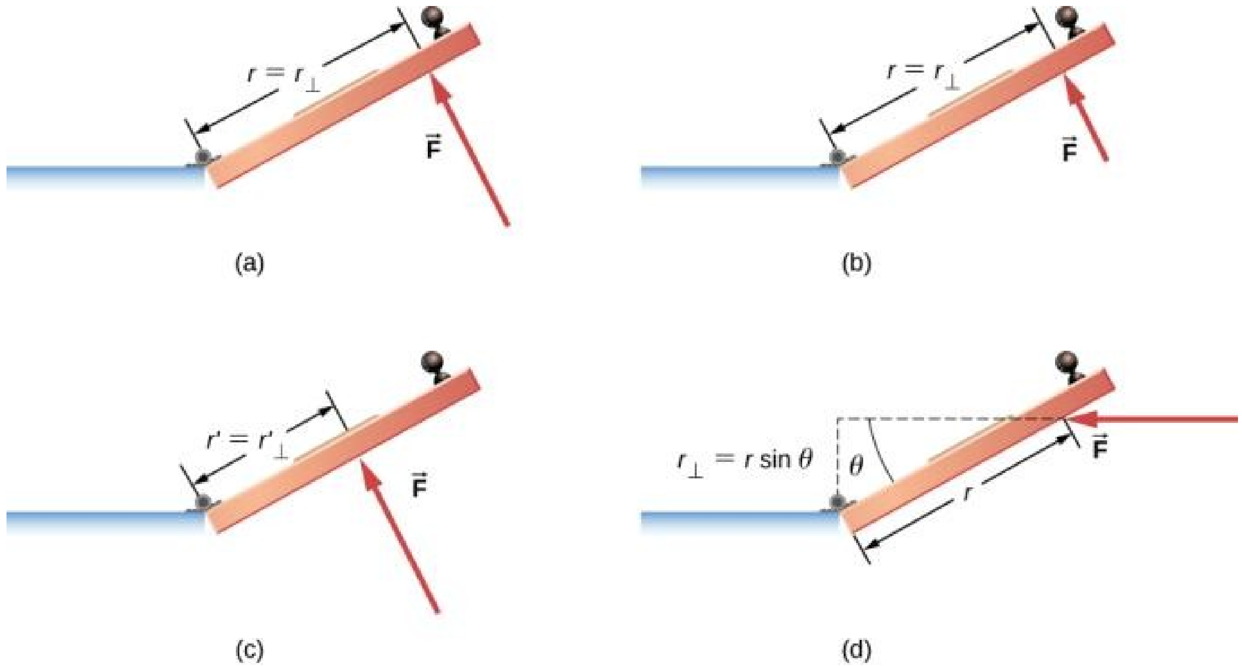
EXAMPLE 10.8

Moment of Inertia of a System of Particles

Six small washers are spaced 10 cm apart on a rod of negligible mass and 0.5 m in length. The mass of each washer is 20 g. The rod rotates about an axis located at 25 cm, as shown in Figure 10.19. (a) What is the moment of inertia of the system? (b) If the two washers closest to the axis are removed, what is the moment of inertia of the remaining four washers? (c) If the system with six washers rotates at 5 rev/s, what is its rotational kinetic energy?

Defining Torque

So far we have defined many variables that are rotational equivalents to their translational counterparts. Let's consider what the counterpart to force must be. Since forces change the translational motion of objects, the rotational counterpart must be related to changing the rotational motion of an object about an axis. We call this rotational counterpart **torque**.

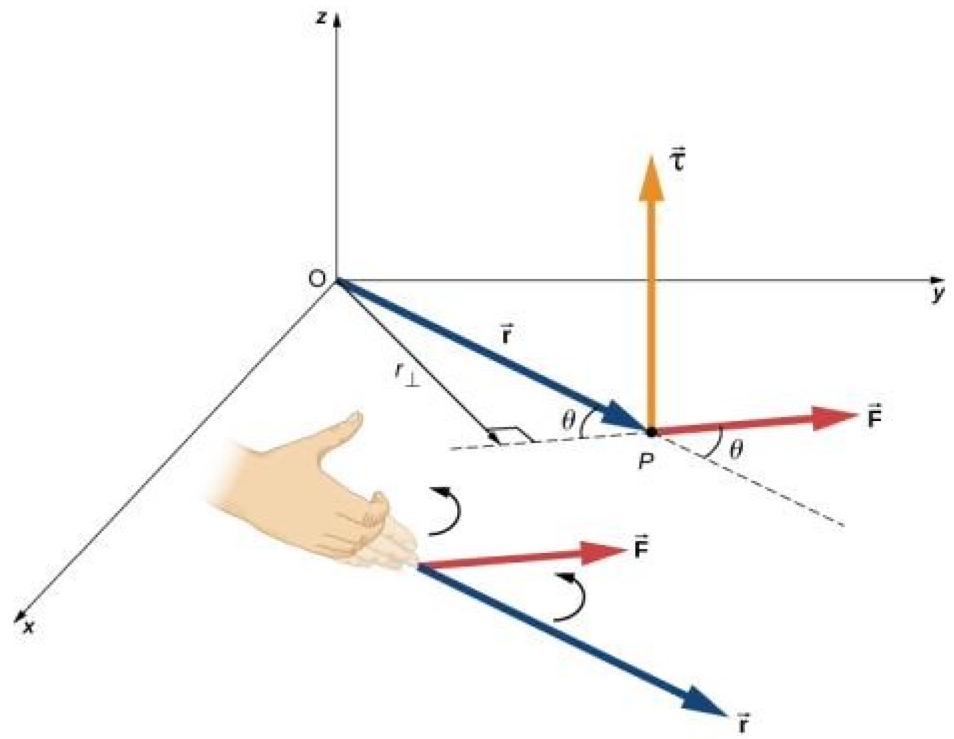


TORQUE

When a force $\vec{\mathbf{F}}$ is applied to a point *P* whose position is $\vec{\mathbf{r}}$ relative to *O* (Figure 10.32), the torque $\vec{\tau}$ around *O* is

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}. \tag{10.22}$$

TORQUE



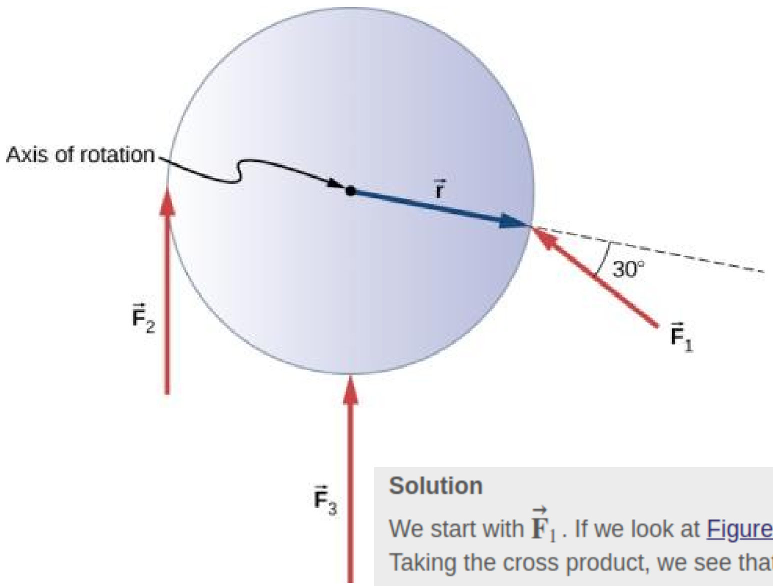
The torque is perpendicular to the plane defined by $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$ and its direction is determined by the right-hand rule.

$$\left|\vec{\tau}\right| = \left|\vec{\mathbf{r}} \times \vec{\mathbf{F}}\right| = rF\sin\theta,$$

$$\left|\vec{\tau}\right| = r_\perp F.$$

Calculating Torque on a rigid body

Figure 10.35 shows several forces acting at different locations and angles on a flywheel. We have $\left|\vec{\mathbf{F}}_1\right| = 20\text{ N}$, $\left|\vec{\mathbf{F}}_2\right| = 30\text{ N}$, $\left|\vec{\mathbf{F}}_3\right| = 30\text{ N}$, and $r = 0.5\text{ m}$. Find the net torque on the flywheel about an axis through the center.



Three forces acting on a flywheel.

Solution

We start with $\vec{\mathbf{F}}_1$. If we look at Figure 10.35, we see that $\vec{\mathbf{F}}_1$ makes an angle of $90° + 60°$ with the radius vector $\vec{\mathbf{r}}$. Taking the cross product, we see that it is out of the page and so is positive. We also see this from calculating its magnitude:

$$\left|\vec{\boldsymbol{\tau}}_1\right| = rF_1 \sin 150° = 0.5\text{ m}(20\text{ N})(0.5) = 5.0\text{ N}\cdot\text{m}.$$

Next we look at $\vec{\mathbf{F}}_2$. The angle between $\vec{\mathbf{F}}_2$ and $\vec{\mathbf{r}}$ is $90°$ and the cross product is into the page so the torque is negative. Its value is

$$\left|\vec{\boldsymbol{\tau}}_2\right| = -rF_2 \sin 90° = -0.5\text{ m}(30\text{ N}) = -15.0\text{ N}\cdot\text{m}.$$

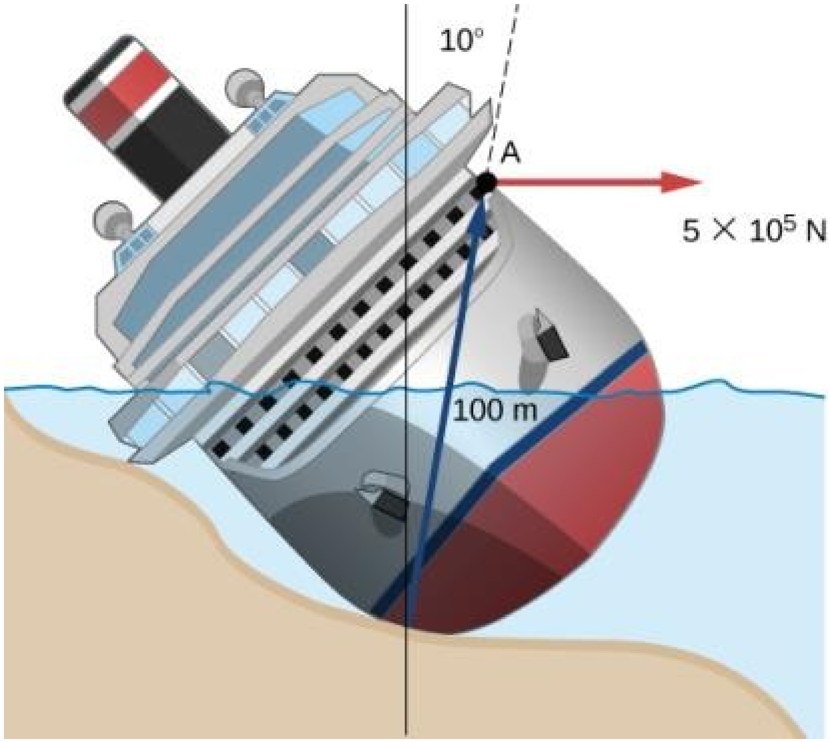
When we evaluate the torque due to $\vec{\mathbf{F}}_3$, we see that the angle it makes with $\vec{\mathbf{r}}$ is zero so $\vec{\mathbf{r}} \times \vec{\mathbf{F}}_3 = 0$. Therefore, $\vec{\mathbf{F}}_3$ does not produce any torque on the flywheel.

We evaluate the sum of the torques:

$$\tau_{\text{net}} = \sum_i |\boldsymbol{\tau}_i| = 5 - 15 = -10\text{ N}\cdot\text{m}.$$

DIY

A large ocean-going ship runs aground near the coastline, similar to the fate of the *Costa Concordia*, and lies at an angle as shown below. Salvage crews must apply a torque to right the ship in order to float the vessel for transport. A force of 5.0×10^5 N acting at point *A* must be applied to right the ship. What is the torque about the point of contact of the ship with the ground (Figure 10.36)?

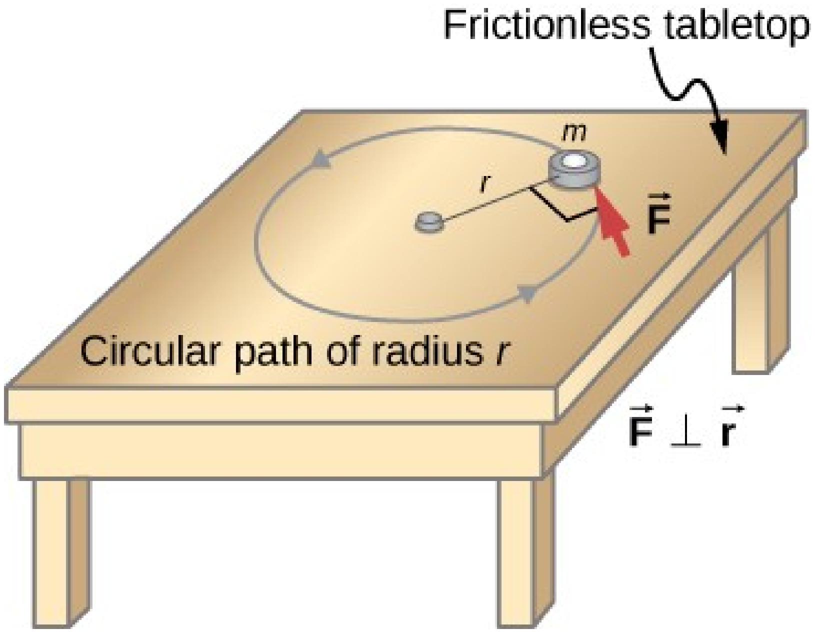


NEWTON'S SECOND LAW FOR ROTATION

If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration:

$$\sum_i \tau_i = I\alpha. \tag{10.25}$$

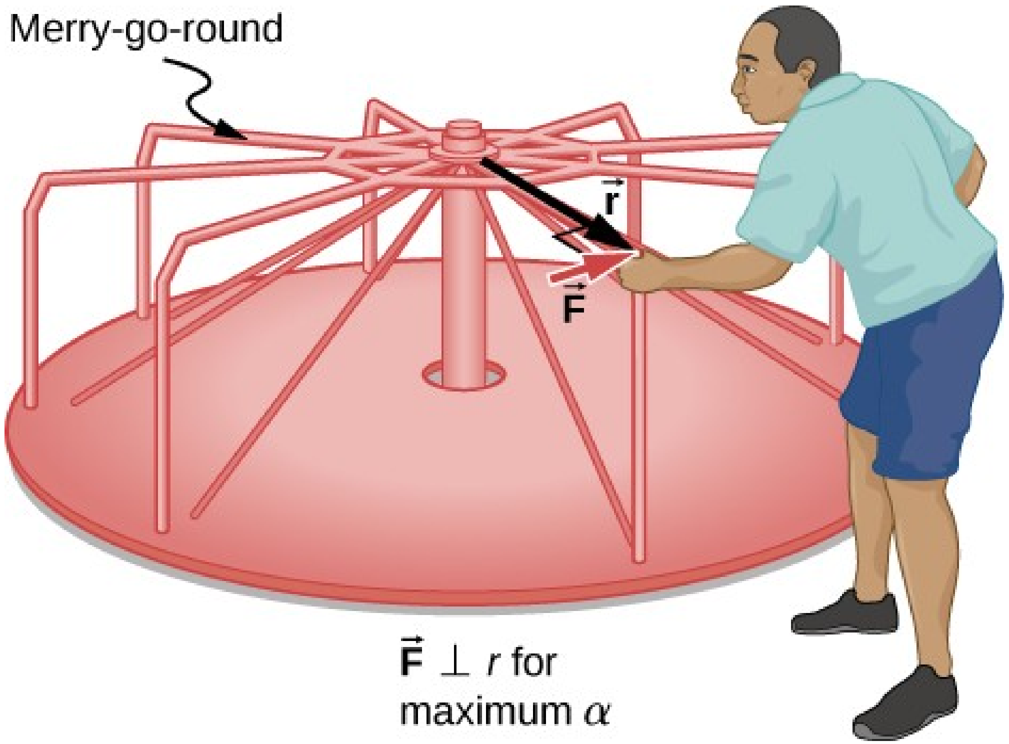
$$\Sigma\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}}.$$



An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force $\vec{\mathbf{F}}$ is applied to the object perpendicular to the radius *r*, causing it to accelerate about the pivot point. The force is perpendicular to *r*.

Calculating the Effect of Mass Distribution on a Merry-Go-Round

Consider the father pushing a playground merry-go-round in Figure 10.38. He exerts a force of 250 N at the edge of the 50.0-kg merry-go-round, which has a 1.50-m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible friction.



A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Solution

a. The moment of inertia of a solid disk about this axis is given in Figure 10.20 to be

$$\frac{1}{2}MR^2.$$

We have $M = 50.0\ \text{kg}$ and $R = 1.50\ \text{m}$, so

$$I = (0.500)(50.0\ \text{kg})(1.50\ \text{m})^2 = 56.25\ \text{kg-m}^2.$$

To find the net torque, we note that the applied force is perpendicular to the radius and friction is negligible, so that

$$\tau = rF\sin\theta = (1.50\ \text{m})(250.0\ \text{N}) = 375.0\ \text{N-m}.$$

Now, after we substitute the known values, we find the angular acceleration to be

$$\alpha = \frac{\tau}{I} = \frac{375.0\ \text{N-m}}{56.25\ \text{kg-m}^2} = 6.67\frac{\text{rad}}{\text{s}^2}.$$

WORK-ENERGY THEOREM FOR ROTATION

The work-energy theorem for a rigid body rotating around a fixed axis is

$$W_{AB} = K_B - K_A \tag{10.29}$$

where

$$K = \frac{1}{2} I \omega^2$$

and the rotational work done by a net force rotating a body from point *A* to point *B* is

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta. \tag{10.30}$$

Power for Rotational Motion

Power always comes up in the discussion of applications in engineering and physics. Power for rotational motion is equally as important as power in linear motion and can be derived in a similar way as in linear motion when the force is a constant. The linear power when the force is a constant is $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$. If the net torque is constant over the angular displacement, Equation 10.25 simplifies and the net torque can be taken out of the integral. In the following discussion, we assume the net torque is constant. We can apply the definition of power derived in Power to rotational motion. From Work and Kinetic Energy, the instantaneous power (or just power) is defined as the rate of doing work,

$$P = \frac{dW}{dt}.$$

If we have a constant net torque, Equation 10.25 becomes $W = \tau\theta$ and the power is

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\frac{d\theta}{dt}$$

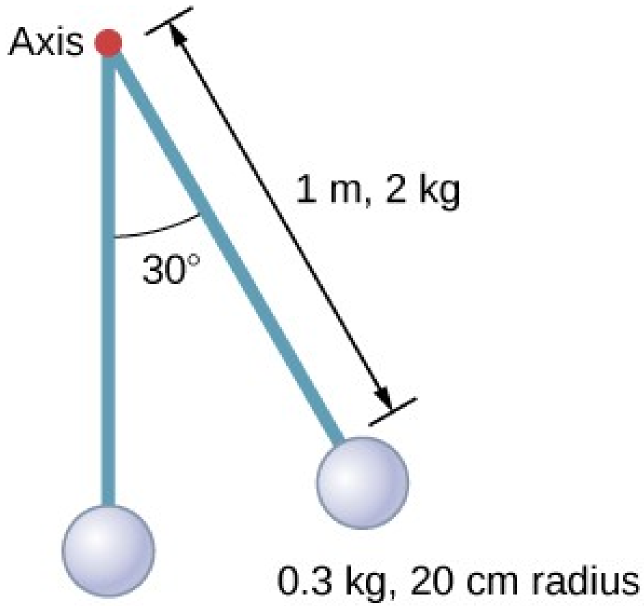
or

$$P = \tau\omega. \tag{10.31}$$

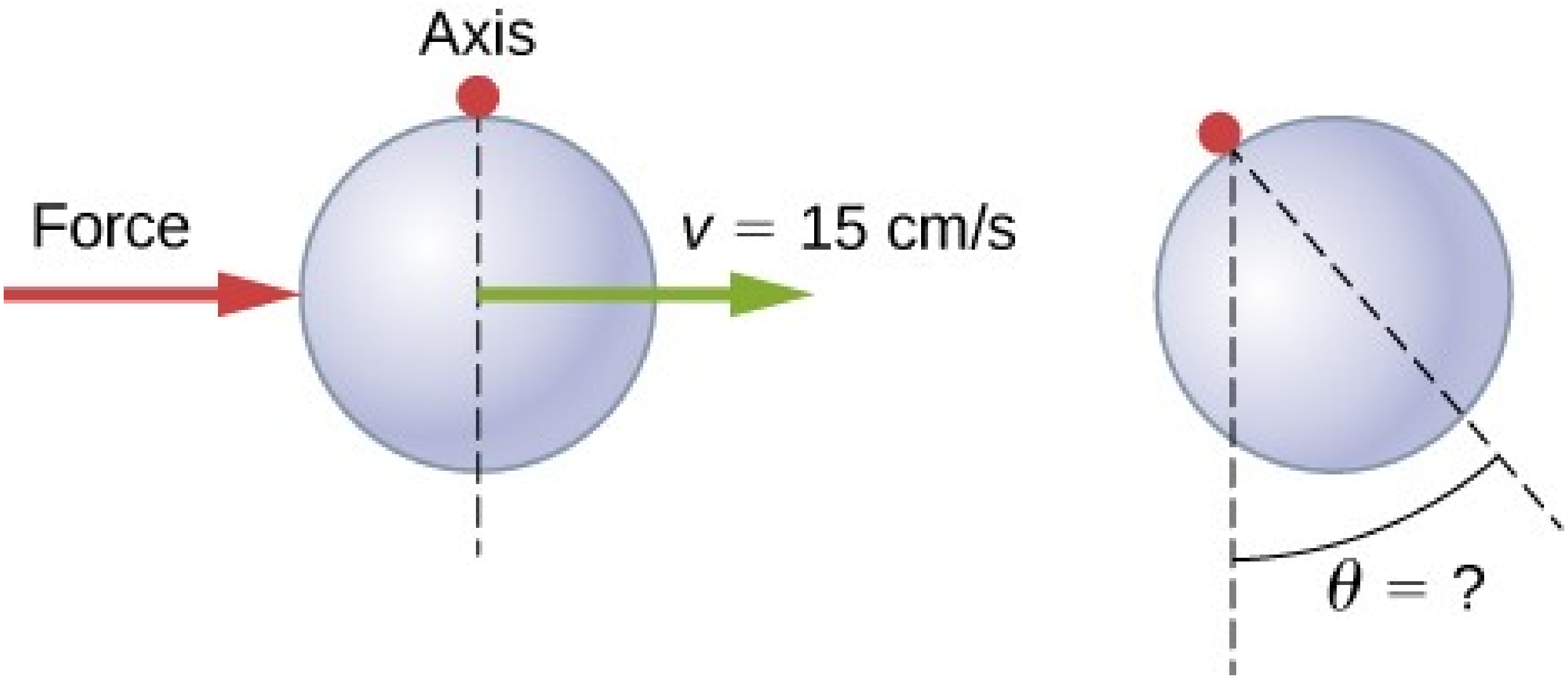
SUMMARY

Rotational	Translational
$I = \sum_i m_i r_i^2$	m
$K = \frac{1}{2} I\omega^2$	$K = \frac{1}{2} mv^2$
$\sum_i \tau_i = I\alpha$	$\sum_i \vec{\mathbf{F}}_i = m\vec{\mathbf{a}}$
$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta$	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$
$P = \tau\omega$	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$

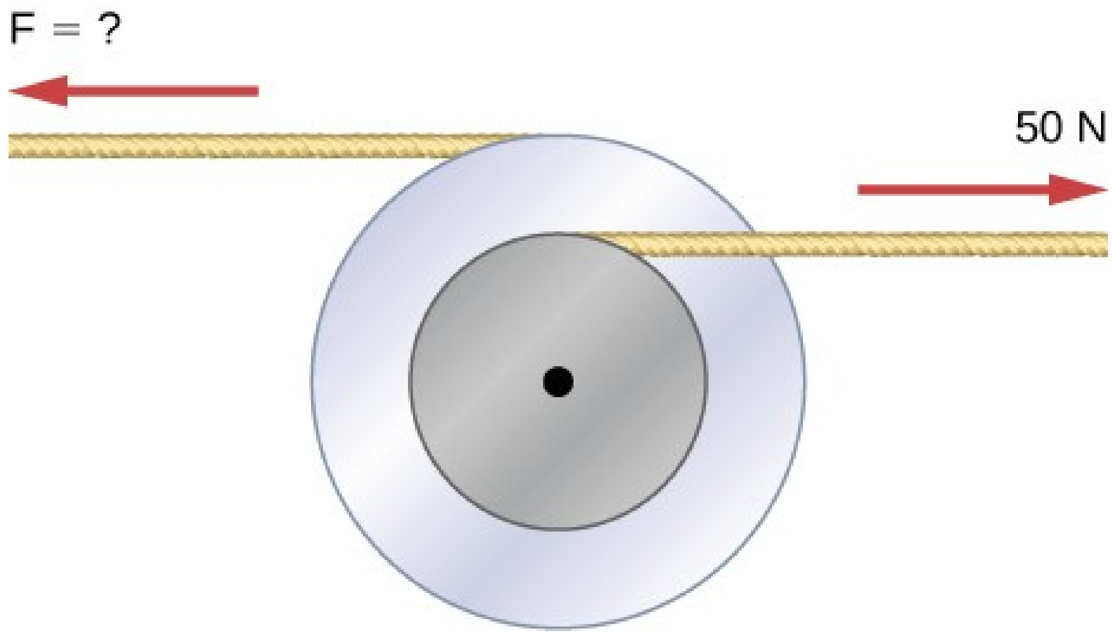
A pendulum consists of a rod of mass 2 kg and length 1 m with a solid sphere at one end with mass 0.3 kg and radius 20 cm (see the following figure). If the pendulum is released from rest at an angle of $30°$, what is the angular velocity at the lowest point?



A solid sphere of radius 10 cm is allowed to rotate freely about an axis. The sphere is given a sharp blow so that its center of mass starts from the position shown in the following figure with speed 15 cm/s. What is the maximum angle that the diameter makes with the vertical?



Two flywheels of negligible mass and different radii are bonded together and rotate about a common axis (see below). The smaller flywheel of radius 30 cm has a cord that has a pulling force of 50 N on it. What pulling force needs to be applied to the cord connecting the larger flywheel of radius 50 cm such that the combination does not rotate?



A seesaw has length 10.0 m and uniform mass 10.0 kg and is resting at an angle of $30°$ with respect to the ground (see the following figure). The pivot is located at 6.0 m. What magnitude of force needs to be applied perpendicular to the seesaw at the raised end so as to allow the seesaw to barely start to rotate?