ANNOUNCEMENTS

- Homework #10, due Wednesday, Oct. 31 before class

  Conceptual questions: Chapter 10, #2 and #10
  Problems: Chapter 10, #40, #46

- Study Chapter 10, sections 1 through 4 and 6 through 8

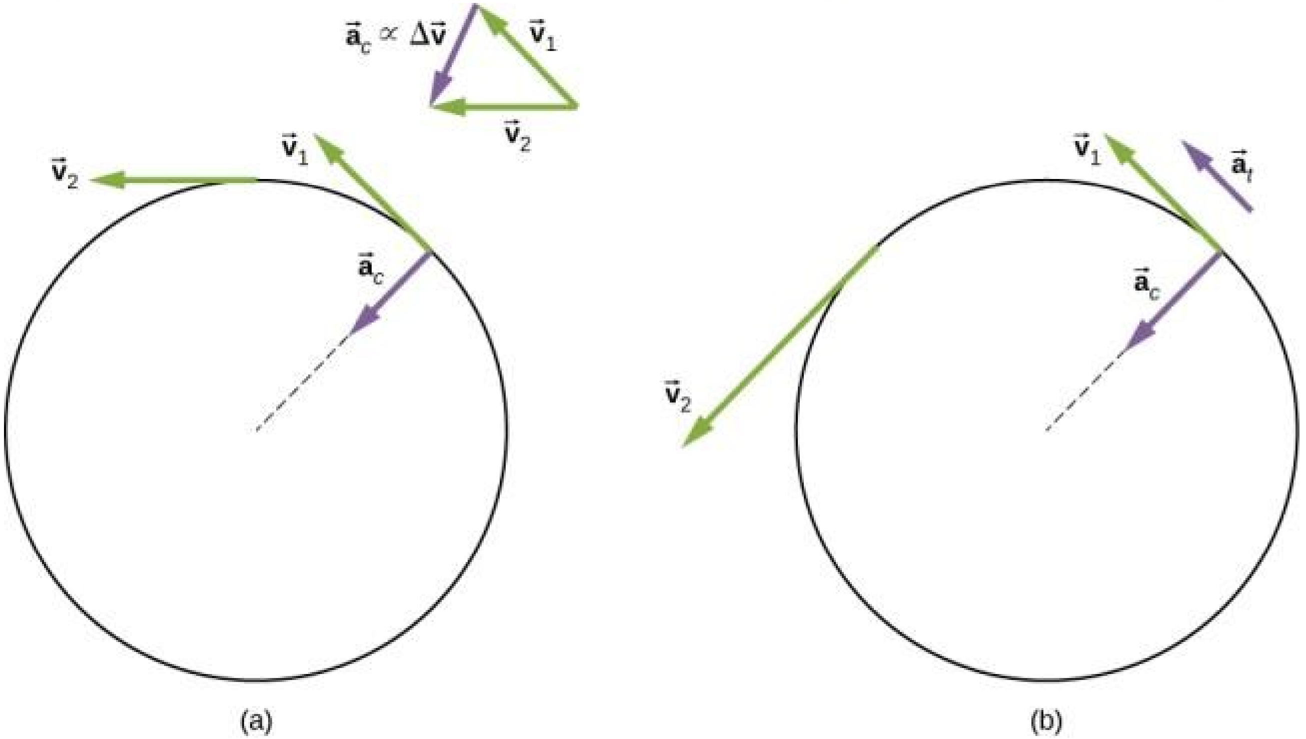
- Second in-class test: Friday, Nov. 2, Chapters 5-10

ANGULAR VS LINEAR VARIABLES AND EQUATIONS

	Linear	Rotational
Position	x	θ
Velocity	$v = \frac{dx}{dt}$	$\omega = \frac{d\theta}{dt}$
Acceleration	$a = \frac{dv}{dt}$	$\alpha = \frac{d\omega}{dt}$

$\theta_f = \theta_0 + \bar{\omega}t$	$x = x_0 + \bar{v}t$
$\omega_f = \omega_0 + \alpha t$	$v_f = v_0 + at$
$\theta_f = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	$x_f = x_0 + v_0 t + \frac{1}{2}at^2$
$\omega_f^2 = \omega_0^2 + 2\alpha(\Delta\theta)$	$v_f^2 = v_0^2 + 2a(\Delta x)$

θ	s	$\theta = \frac{s}{r}$
ω	v_t	$\omega = \frac{v_t}{r}$
α	a_t	$\alpha = \frac{a_t}{r}$
	a_c	$a_c = \frac{v_t^2}{r}$

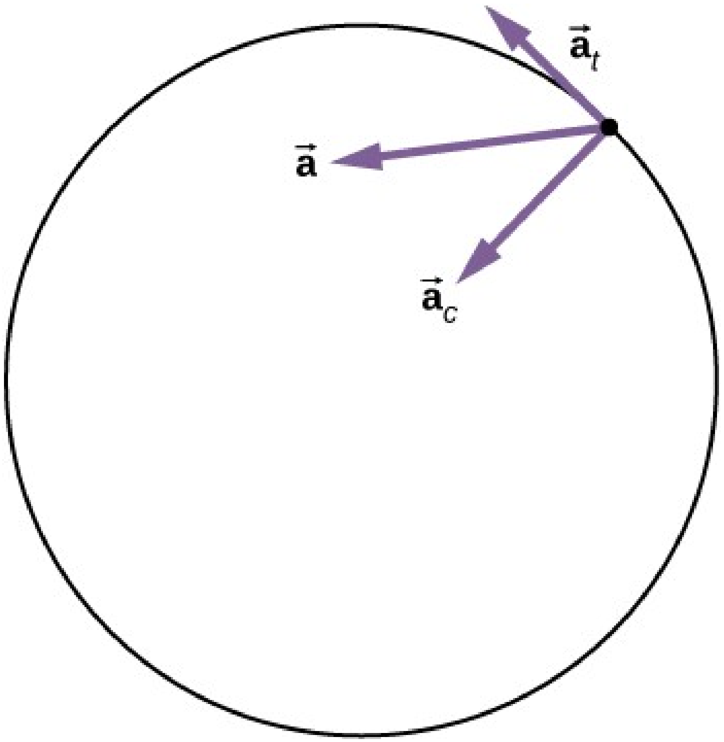


(a) Uniform circular motion: The centripetal acceleration a_c has its vector inward toward the axis of rotation. There is no tangential acceleration.

(b) Nonuniform circular motion: An angular acceleration produces an inward centripetal acceleration that is changing in magnitude, plus a tangential acceleration a_t .

$$\vec{a} = \vec{a}_c + \vec{a}_t.$$

$$|\vec{a}| = \sqrt{a_c^2 + a_t^2}.$$

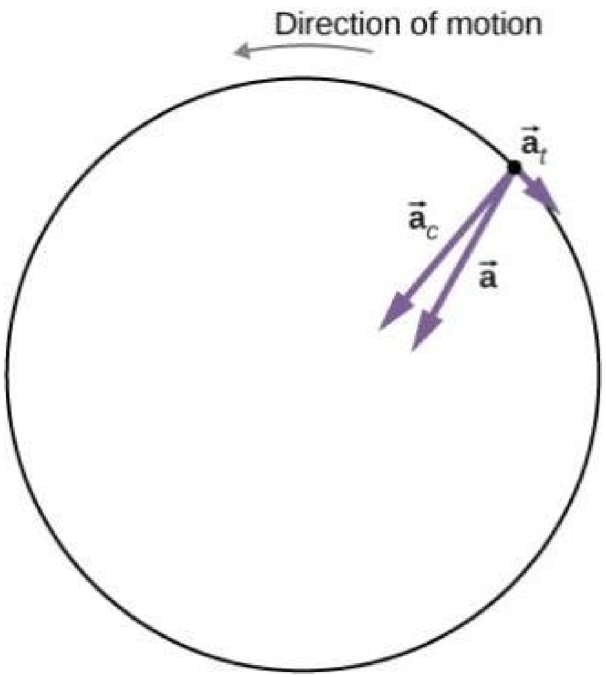


$$\vec{a} = \vec{a}_c + \vec{a}_t.$$

A particle is executing circular motion and has an angular acceleration. The total linear acceleration of the particle is the vector sum of the centripetal acceleration and tangential acceleration vectors. The total linear acceleration vector is at an angle in between the centripetal and tangential accelerations.

Linear Acceleration of a Centrifuge

A centrifuge has a radius of 20 cm and accelerates from a maximum rotation rate of 10,000 rpm to rest in 30 seconds under a constant angular acceleration. It is rotating counterclockwise. What is the magnitude of the total acceleration of a point at the tip of the centrifuge at $t = 29.0 s$? What is the direction of the total acceleration vector?



The centripetal, tangential, and total acceleration vectors. The centrifuge is slowing down, so the tangential acceleration is clockwise, opposite the direction of rotation (counterclockwise).

Linear Acceleration of a Centrifuge

A centrifuge has a radius of 20 cm and accelerates from a maximum rotation rate of 10,000 rpm to rest in 30 seconds under a constant angular acceleration. It is rotating counterclockwise. What is the magnitude of the total acceleration of a point at the tip of the centrifuge at $t = 29.0\text{s}$? What is the direction of the total acceleration vector?

Solution

The angular acceleration is

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{0 - (1.0 \times 10^4)2\pi/60.0\text{ s(rad/s)}}{30.0\text{ s}} = -34.9\text{ rad/s}^2.$$

Therefore, the tangential acceleration is

$$a_t = r\alpha = 0.2\text{ m}(-34.9\text{ rad/s}^2) = -7.0\text{ m/s}^2.$$

The angular velocity at $t = 29.0$ s is

$$\begin{aligned}\omega &= \omega_0 + \alpha t = 1.0 \times 10^4 \left(\frac{2\pi}{60.0\text{ s}}\right) + \left(-34.9\text{ rad/s}^2\right)(29.0\text{ s}) \\ &= 1047.2\text{ rad/s} - 1012.71 = 35.1\text{ rad/s}.\end{aligned}$$

Thus, the tangential speed at $t = 29.0$ s is

$$v_t = r\omega = 0.2\text{ m}(35.1\text{ rad/s}) = 7.0\text{ m/s}.$$

We can now calculate the centripetal acceleration at $t = 29.0$ s:

$$a_c = \frac{v^2}{r} = \frac{(7.0\text{ m/s})^2}{0.2\text{ m}} = 245.0\text{ m/s}^2.$$

Since the two acceleration vectors are perpendicular to each other, the magnitude of the total linear acceleration is

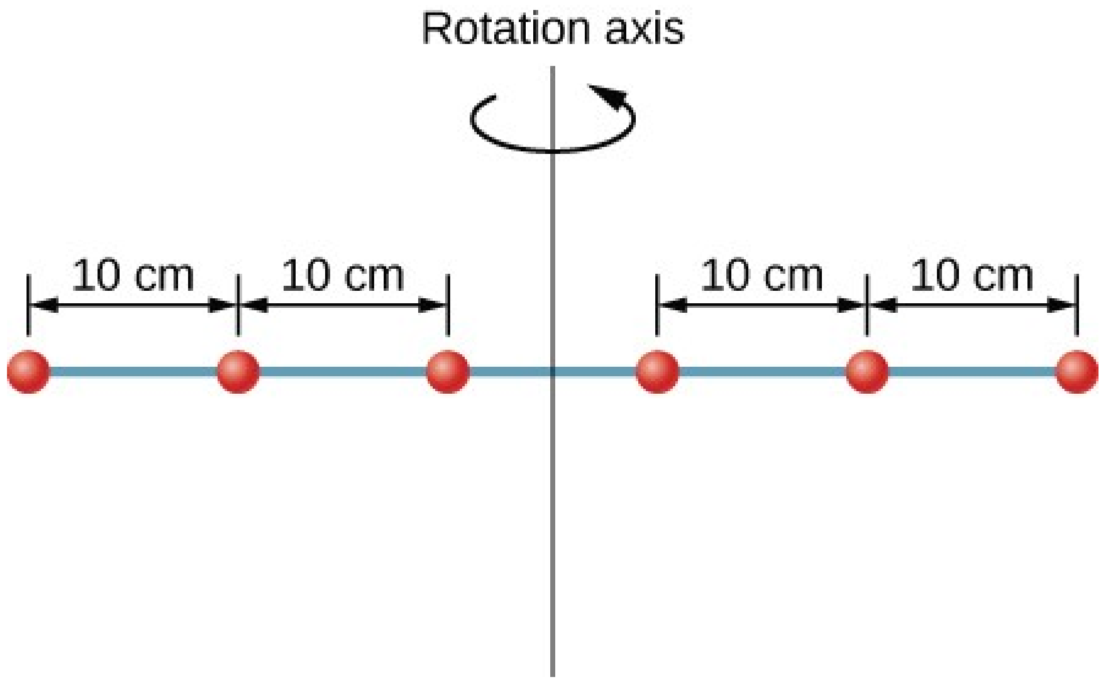
$$|\vec{\mathbf{a}}| = \sqrt{a_c^2 + a_t^2} = \sqrt{(245.0)^2 + (-7.0)^2} = 245.1\text{ m/s}^2.$$

Since the centrifuge has a negative angular acceleration, it is slowing down. The total acceleration vector is as shown in Figure 10.16. The angle with respect to the centripetal acceleration vector is

$$\theta = \tan^{-1}\frac{-7.0}{245.0} = -1.6°.$$

The negative sign means that the total acceleration vector is angled toward the clockwise direction.

ROTATIONAL ENERGY AND MOMENT OF INERTIA



Six washers are spaced 10 cm apart on a rod of negligible mass and rotating about a vertical axis.

$$K = \sum_j \frac{1}{2} m_j v_j^2 = \sum_j \frac{1}{2} m_j (r_j \omega_j)^2 \longrightarrow K = \frac{1}{2}\left(\sum_j m_j r_j^2\right)\omega^2.$$

$$I = \sum_j m_j r_j^2. \qquad K = \frac{1}{2} I \omega^2.$$

ANALOGY TO LINEAR QUANTITIES

Rotational	Translational
$I = \sum_j m_j r_j^2$	m
$K = \frac{1}{2} I \omega^2$	$K = \frac{1}{2} m v^2$

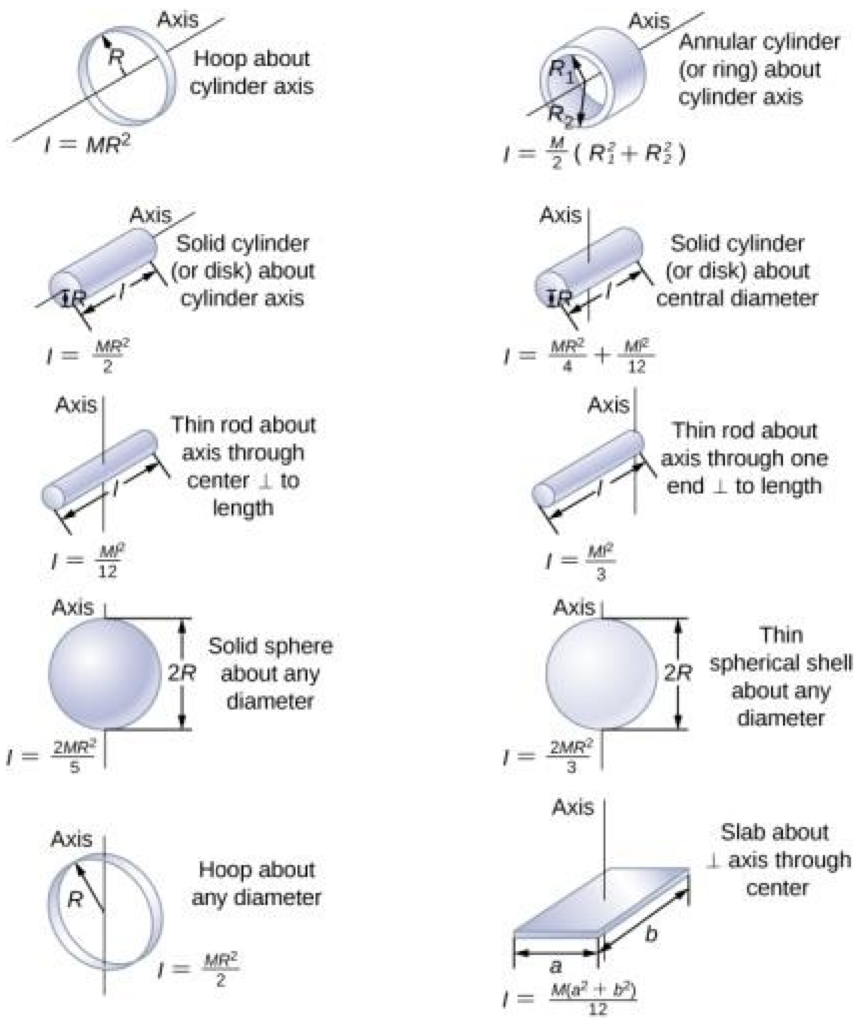
Table 10.4 Rotational and Translational Kinetic Energies and Inertia

EXAMPLE 10.8

Moment of Inertia of a System of Particles

Six small washers are spaced 10 cm apart on a rod of negligible mass and 0.5 m in length. The mass of each washer is 20 g. The rod rotates about an axis located at 25 cm, as shown in Figure 10.19. (a) What is the moment of inertia of the system? (b) If the two washers closest to the axis are removed, what is the moment of inertia of the remaining four washers? (c) If the system with six washers rotates at 5 rev/s, what is its rotational kinetic energy?

FIGURE 10.20

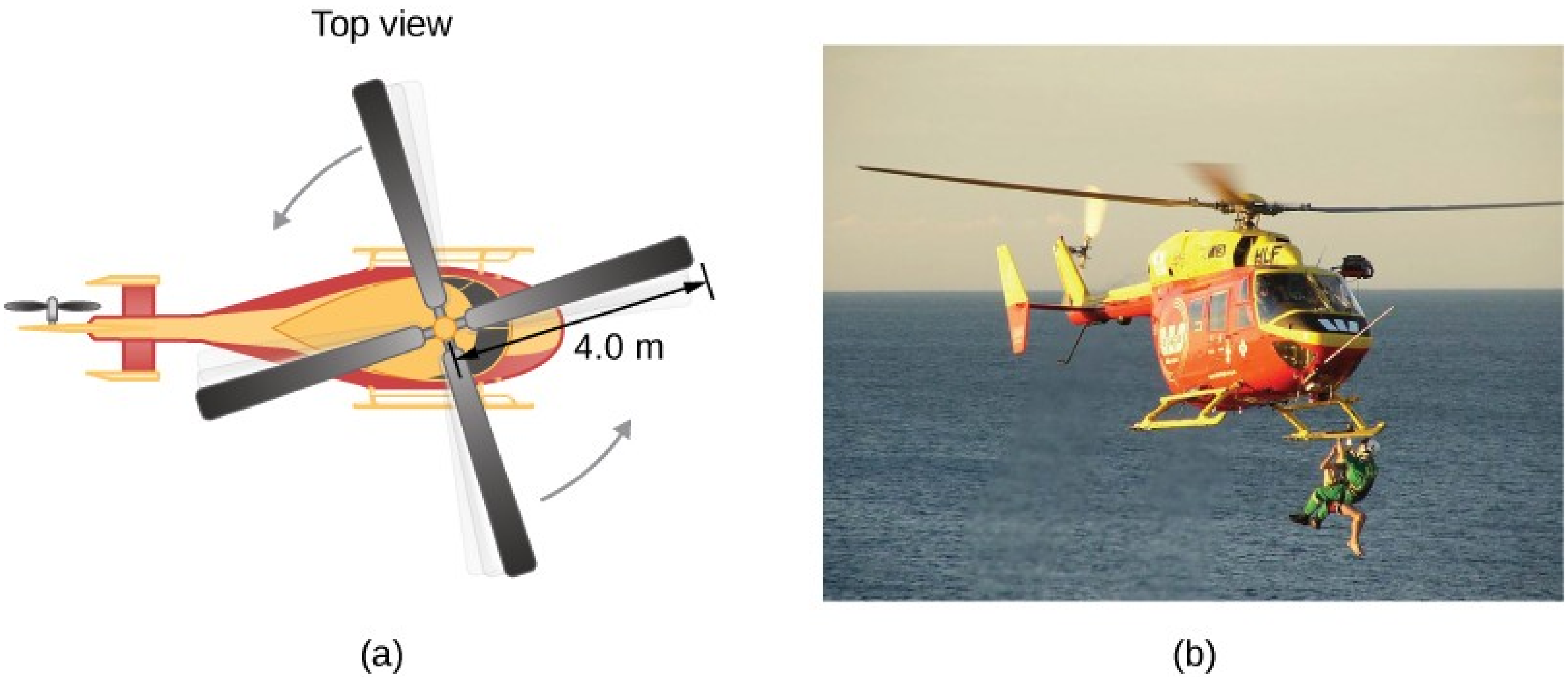


Values of rotational inertia for common shapes of objects.

EXAMPLE 10.9

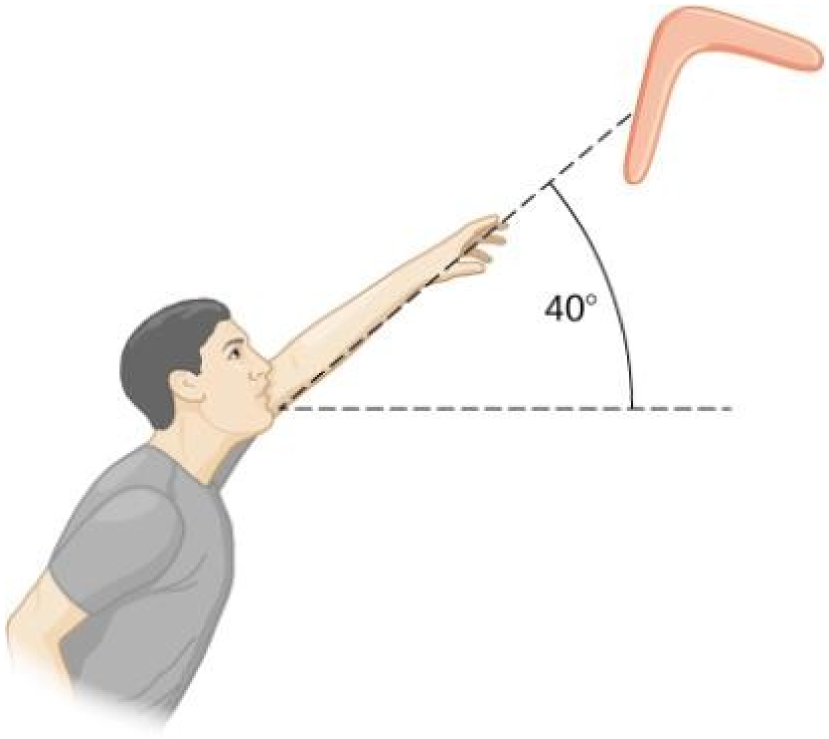
Calculating Helicopter Energies

A typical small rescue helicopter has four blades: Each is 4.00 m long and has a mass of 50.0 kg (Figure 10.21). The blades can be approximated as thin rods that rotate about one end of an axis perpendicular to their length. The helicopter has a total loaded mass of 1000 kg. (a) Calculate the rotational kinetic energy in the blades when they rotate at 300 rpm. (b) Calculate the translational kinetic energy of the helicopter when it flies at 20.0 m/s, and compare it with the rotational energy in the blades.



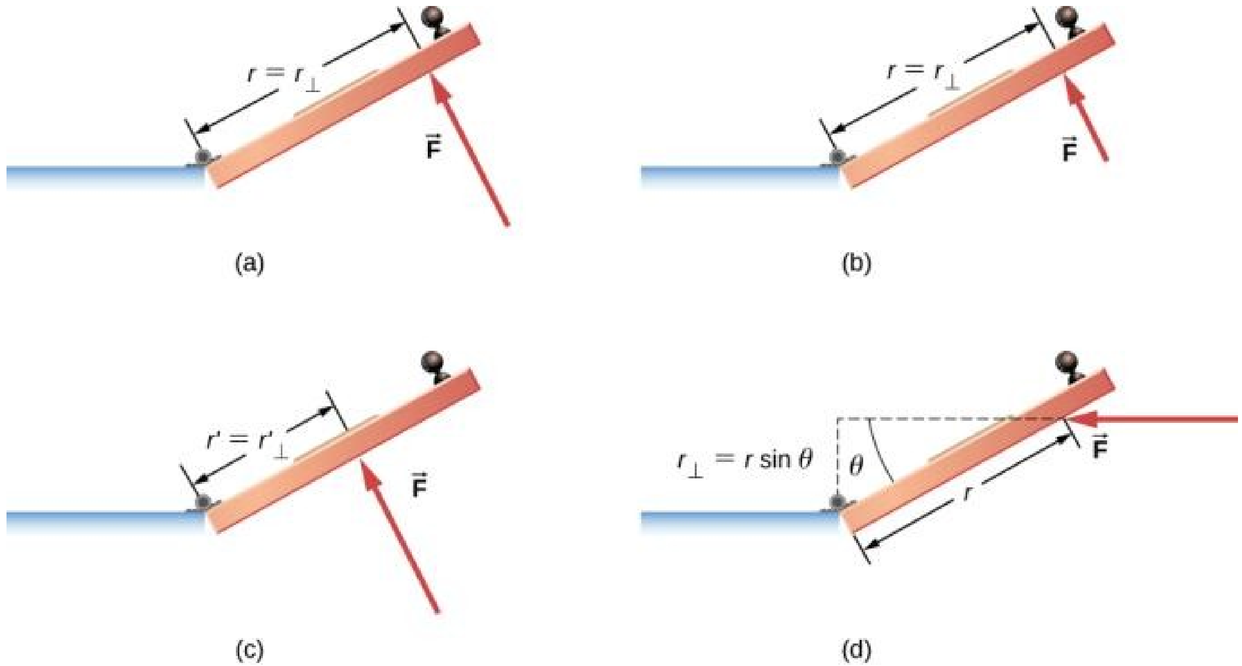
Energy in a Boomerang

A person hurls a boomerang into the air with a velocity of 30.0 m/s at an angle of $40.0°$ with respect to the horizontal (Figure 10.22). It has a mass of 1.0 kg and is rotating at 10.0 rev/s. The moment of inertia of the boomerang is given as $I = \frac{1}{12}mL^2$ where $L = 0.7\text{ m}$. (a) What is the total energy of the boomerang when it leaves the hand? (b) How high does the boomerang go from the elevation of the hand, neglecting air resistance?



Defining Torque

So far we have defined many variables that are rotational equivalents to their translational counterparts. Let's consider what the counterpart to force must be. Since forces change the translational motion of objects, the rotational counterpart must be related to changing the rotational motion of an object about an axis. We call this rotational counterpart **torque**.

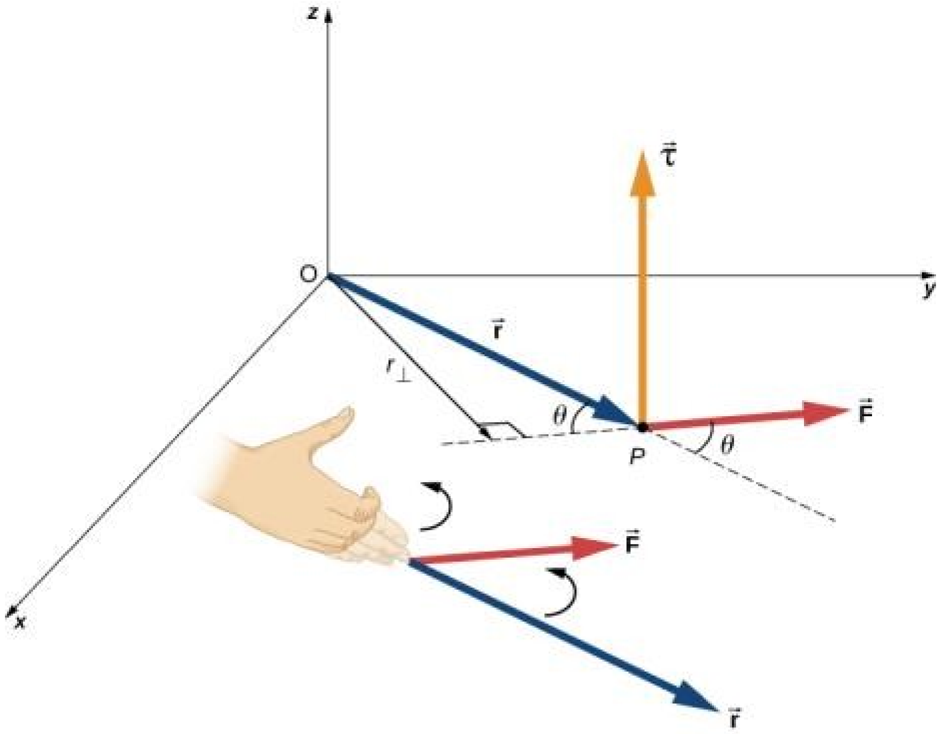


TORQUE

When a force $\vec{\mathbf{F}}$ is applied to a point *P* whose position is $\vec{\mathbf{r}}$ relative to *O* (Figure 10.32), the torque $\vec{\tau}$ around *O* is

$$\vec{\boldsymbol{\tau}} = \vec{\mathbf{r}} \times \vec{\mathbf{F}}. \tag{10.22}$$

TORQUE



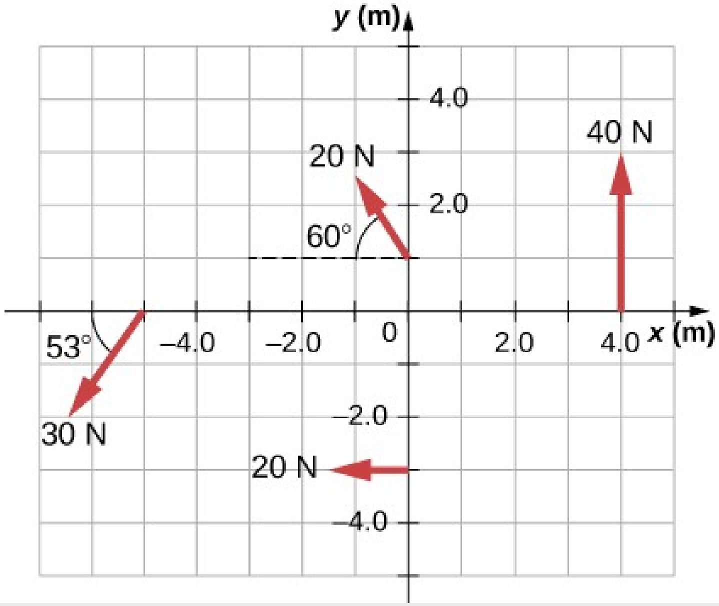
The torque is perpendicular to the plane defined by $\vec{\mathbf{r}}$ and $\vec{\mathbf{F}}$ and its direction is determined by the right-hand rule.

$$|\vec{\tau}| = |\vec{\mathbf{r}} \times \vec{\mathbf{F}}| = rF\sin\theta,$$

$$|\vec{\tau}| = r_\perp F.$$

TOTAL TORQUE

$$\vec{\tau}_{net} = \sum_i |\vec{\tau}_i| .$$



Solution

Use $|\vec{\tau}| = r_{\perp}F = rF\sin\theta$ to find the magnitude and $\vec{\tau} = \vec{r} \times \vec{F}$ to determine the sign of the torque.

The torque from force 40 N in the first quadrant is given by $(4)(40)\sin 90° = 160\ \text{N} \cdot \text{m}$.

The cross product of $\vec{r}$ and $\vec{F}$ is out of the page, positive.

The torque from force 20 N in the third quadrant is given by$-(3)(20)\sin 90° = -60\ \text{N} \cdot \text{m}$.

The cross product of $\vec{r}$ and $\vec{F}$ is into the page, so it is negative.

The torque from force 30 N in the third quadrant is given by $(5)(30)\sin 53° = 120\ \text{N} \cdot \text{m}$.

The cross product of $\vec{r}$ and $\vec{F}$ is out of the page, positive.

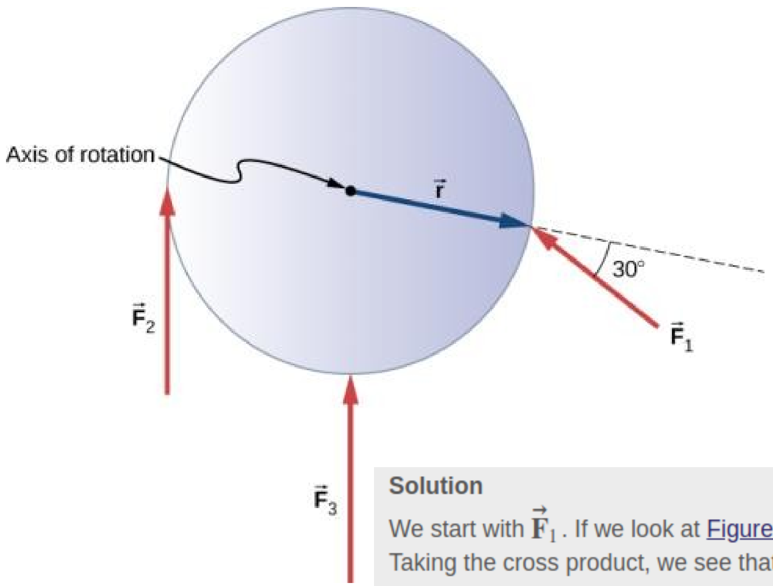
The torque from force 20 N in the second quadrant is given by $(1)(20)\sin 30° = 10\ \text{N} \cdot \text{m}$.

The cross product of $\vec{r}$ and $\vec{F}$ is out of the page.

The net torque is therefore $\tau_{net} = \sum_i |\tau_i| = 160 - 60 + 120 + 10 = 230\ \text{N} \cdot \text{m}$.

Calculating Torque on a rigid body

Figure 10.35 shows several forces acting at different locations and angles on a flywheel. We have $\left|\vec{\mathbf{F}}_1\right| = 20\ \text{N}$, $\left|\vec{\mathbf{F}}_2\right| = 30\ \text{N}$, $\left|\vec{\mathbf{F}}_3\right| = 30\ \text{N}$, and $r = 0.5\ \text{m}$. Find the net torque on the flywheel about an axis through the center.



Three forces acting on a flywheel.

Solution

We start with $\vec{\mathbf{F}}_1$. If we look at Figure 10.35, we see that $\vec{\mathbf{F}}_1$ makes an angle of $90° + 60°$ with the radius vector $\vec{\mathbf{r}}$. Taking the cross product, we see that it is out of the page and so is positive. We also see this from calculating its magnitude:

$$\left|\vec{\boldsymbol{\tau}}_1\right| = rF_1 \sin 150° = 0.5\ \text{m}(20\ \text{N})(0.5) = 5.0\ \text{N} \cdot \text{m}.$$

Next we look at $\vec{\mathbf{F}}_2$. The angle between $\vec{\mathbf{F}}_2$ and $\vec{\mathbf{r}}$ is $90°$ and the cross product is into the page so the torque is negative. Its value is

$$\left|\vec{\boldsymbol{\tau}}_2\right| = -rF_2 \sin 90° = -0.5\ \text{m}(30\ \text{N}) = -15.0\ \text{N} \cdot \text{m}.$$

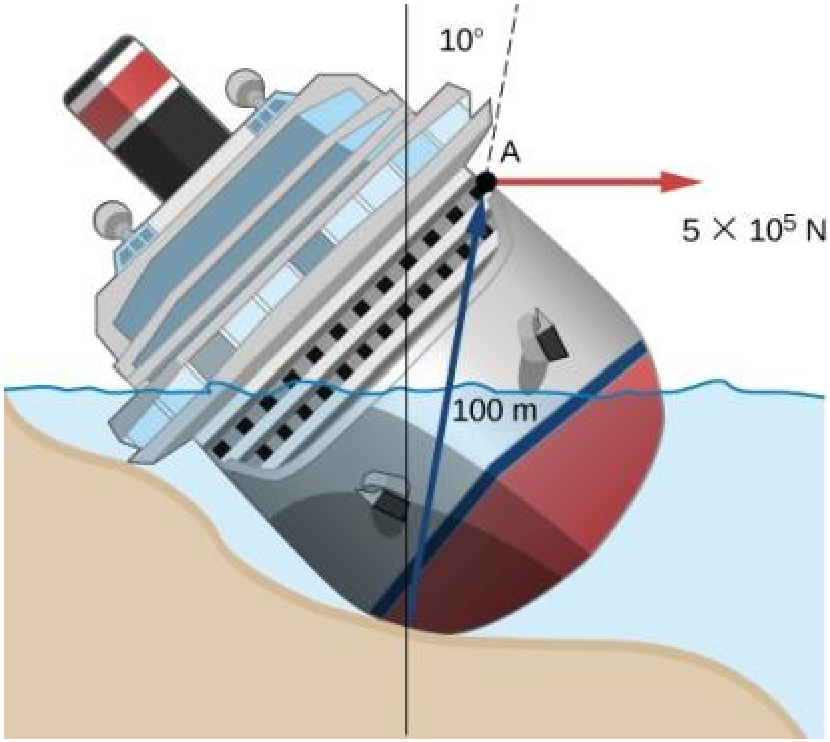
When we evaluate the torque due to $\vec{\mathbf{F}}_3$, we see that the angle it makes with $\vec{\mathbf{r}}$ is zero so $\vec{\mathbf{r}} \times \vec{\mathbf{F}}_3 = 0$. Therefore, $\vec{\mathbf{F}}_3$ does not produce any torque on the flywheel.

We evaluate the sum of the torques:

$$\tau_{\text{net}} = \sum_i \left|\boldsymbol{\tau}_i\right| = 5 - 15 = -10\ \text{N} \cdot \text{m}.$$

DIY

A large ocean-going ship runs aground near the coastline, similar to the fate of the *Costa Concordia*, and lies at an angle as shown below. Salvage crews must apply a torque to right the ship in order to float the vessel for transport. A force of 5.0×10^5 N acting at point *A* must be applied to right the ship. What is the torque about the point of contact of the ship with the ground (Figure 10.36)?

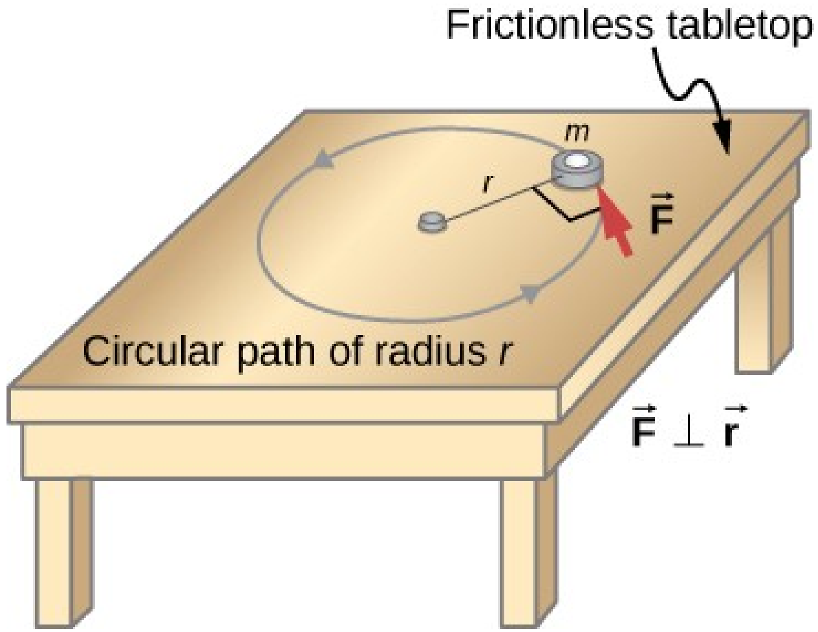


NEWTON'S SECOND LAW FOR ROTATION

If more than one torque acts on a rigid body about a fixed axis, then the sum of the torques equals the moment of inertia times the angular acceleration:

$$\sum_i \tau_i = I\alpha. \tag{10.25}$$

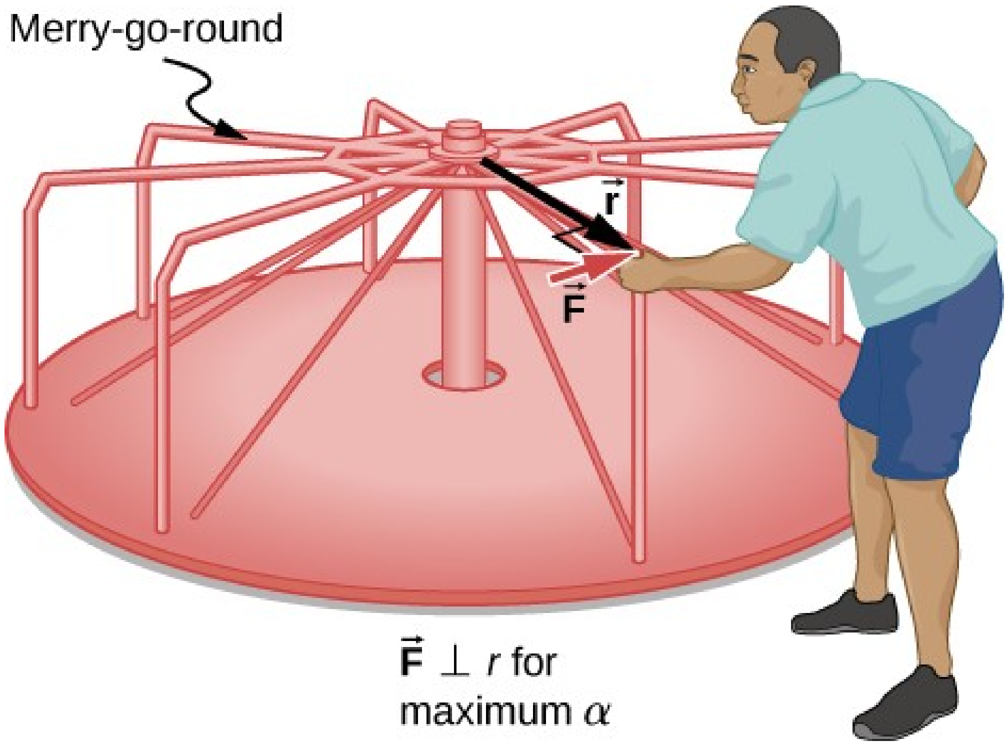
$$\Sigma\vec{\boldsymbol{\tau}} = I\vec{\boldsymbol{\alpha}}.$$



An object is supported by a horizontal frictionless table and is attached to a pivot point by a cord that supplies centripetal force. A force $\vec{\mathbf{F}}$ is applied to the object perpendicular to the radius *r*, causing it to accelerate about the pivot point. The force is perpendicular to *r*.

Calculating the Effect of Mass Distribution on a Merry-Go-Round

Consider the father pushing a playground merry-go-round in Figure 10.38. He exerts a force of 250 N at the edge of the 50.0-kg merry-go-round, which has a 1.50-m radius. Calculate the angular acceleration produced (a) when no one is on the merry-go-round and (b) when an 18.0-kg child sits 1.25 m away from the center. Consider the merry-go-round itself to be a uniform disk with negligible friction.



A father pushes a playground merry-go-round at its edge and perpendicular to its radius to achieve maximum torque.

Solution

a. The moment of inertia of a solid disk about this axis is given in Figure 10.20 to be

$$\frac{1}{2}MR^2.$$

We have $M = 50.0 \text{ kg}$ and $R = 1.50 \text{ m}$, so

$$I = (0.500)(50.0 \text{ kg})(1.50 \text{ m})^2 = 56.25 \text{ kg-m}^2.$$

To find the net torque, we note that the applied force is perpendicular to the radius and friction is negligible, so that

$$\tau = rF\sin\theta = (1.50 \text{ m})(250.0 \text{ N}) = 375.0 \text{ N-m}.$$

Now, after we substitute the known values, we find the angular acceleration to be

$$\alpha = \frac{\tau}{I} = \frac{375.0 \text{ N-m}}{56.25 \text{ kg-m}^2} = 6.67\frac{\text{rad}}{\text{s}^2}.$$

WORK-ENERGY THEOREM FOR ROTATION

The work-energy theorem for a rigid body rotating around a fixed axis is

$$W_{AB} = K_B - K_A \tag{10.29}$$

where

$$K = \frac{1}{2} I \omega^2$$

and the rotational work done by a net force rotating a body from point *A* to point *B* is

$$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta. \tag{10.30}$$

Power for Rotational Motion

Power always comes up in the discussion of applications in engineering and physics. Power for rotational motion is equally as important as power in linear motion and can be derived in a similar way as in linear motion when the force is a constant. The linear power when the force is a constant is $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$. If the net torque is constant over the angular displacement, Equation 10.25 simplifies and the net torque can be taken out of the integral. In the following discussion, we assume the net torque is constant. We can apply the definition of power derived in Power to rotational motion. From Work and Kinetic Energy, the instantaneous power (or just power) is defined as the rate of doing work,

$$P = \frac{dW}{dt}.$$

If we have a constant net torque, Equation 10.25 becomes $W = \tau\theta$ and the power is

$$P = \frac{dW}{dt} = \frac{d}{dt}(\tau\theta) = \tau\frac{d\theta}{dt}$$

or

$$P = \tau\omega. \tag{10.31}$$

SUMMARY

Rotational	Translational
$I = \sum_i m_i r_i^2$	m
$K = \frac{1}{2} I\omega^2$	$K = \frac{1}{2} mv^2$
$\sum_i \tau_i = I\alpha$	$\sum_i \vec{\mathbf{F}}_i = m\vec{\mathbf{a}}$
$W_{AB} = \int_{\theta_A}^{\theta_B} \left(\sum_i \tau_i \right) d\theta$	$W = \int \vec{\mathbf{F}} \cdot d\vec{\mathbf{s}}$
$P = \tau\omega$	$P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$