ANNOUNCEMENTS

Homework #1, due Monday, Aug. 27 before class

- Answer conceptual questions: Chapter 1, #4 and #8
- Solve problems: Chapter 1, #16 and #46

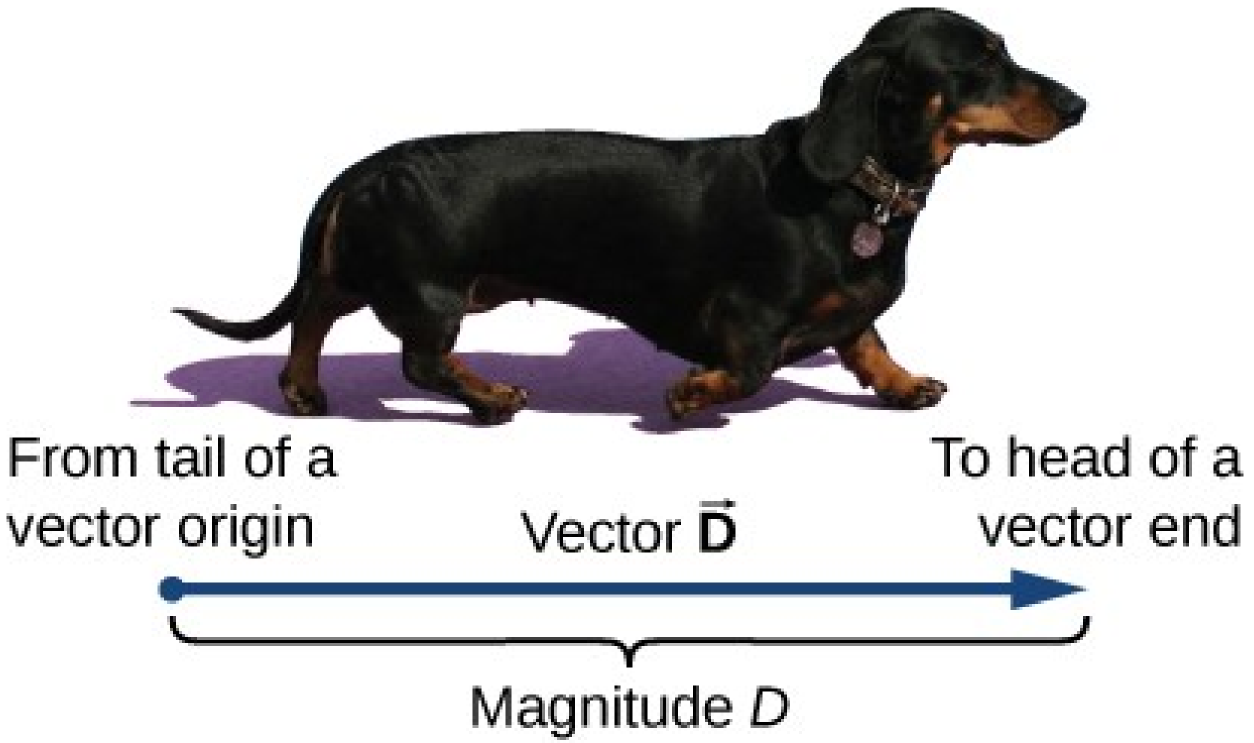
Also read sections 2.1 and 2.2 of the textbook

There will be a 5-minute quiz on Monday at beginning of class

FIGURE 2.1

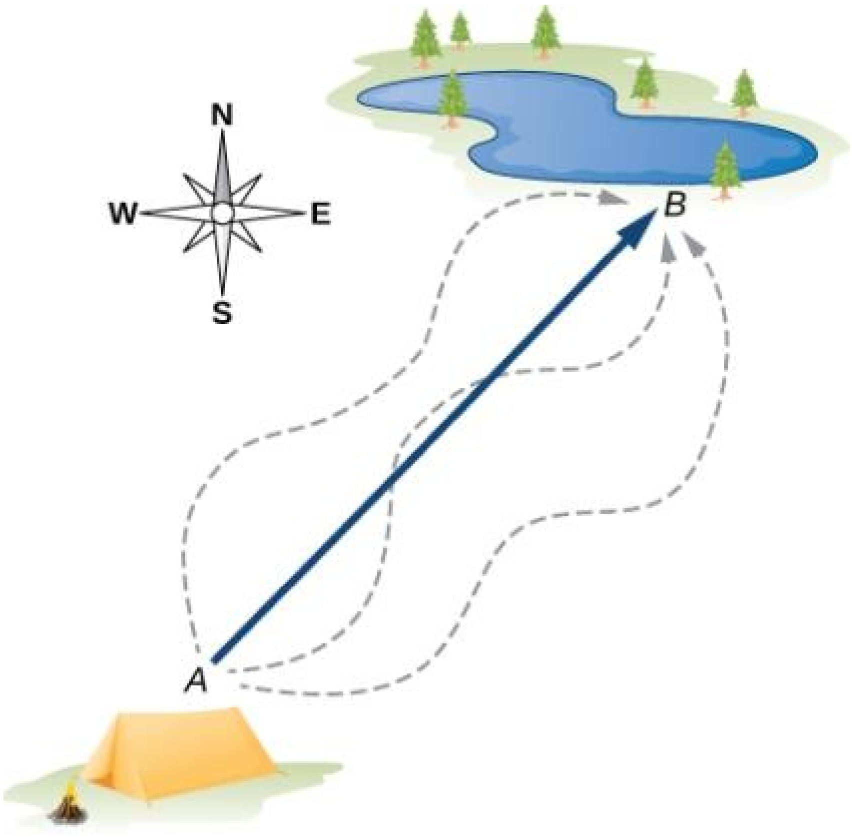
A signpost gives information about distances and directions to towns or to other locations relative to the location of the signpost. Distance is a scalar quantity. Knowing the distance alone is not enough to get to the town; we must also know the direction from the signpost to the town. The direction, together with the distance, is a vector quantity commonly called the displacement vector. A signpost, therefore, gives information about displacement vectors from the signpost to towns. (credit: modification of work by “studio tdes”/Flickr)

FIGURE 2.2



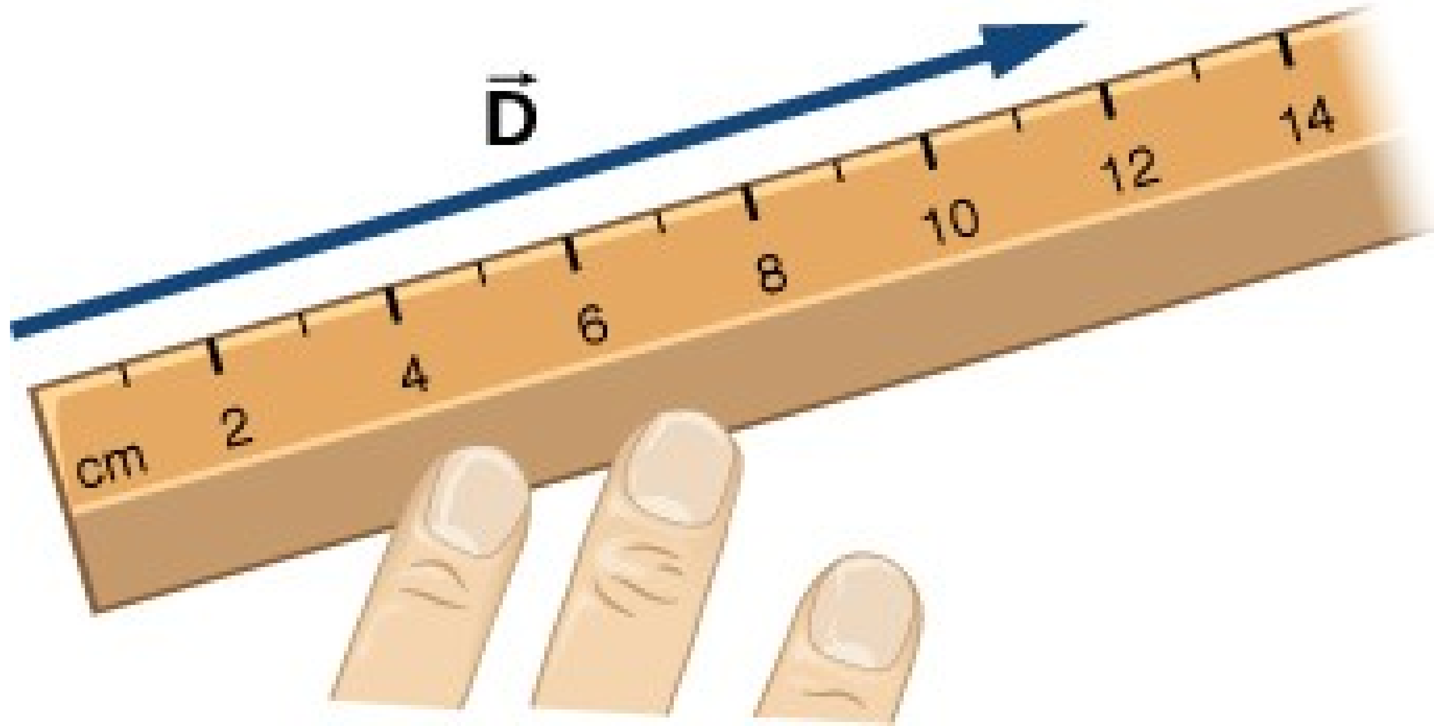
We draw a vector from the initial point or origin (called the “tail” of a vector) to the end or terminal point (called the “head” of a vector), marked by an arrowhead. Magnitude is the length of a vector and is always a positive scalar quantity. (credit: modification of work by Cate Sevilla)

FIGURE 2.3



The displacement vector from point *A* (the initial position at the campsite) to point *B* (the final position at the fishing hole) is indicated by an arrow with origin at point A and end at point *B*. The displacement is the same for any of the actual paths (dashed curves) that may be taken between points *A* and *B*.

FIGURE 2.4



A displacement of magnitude 6 km is drawn to scale as a vector of length 12 cm when the length of 2 cm represents 1 unit of displacement (which in this case is 1 km).

FIGURE 2.5

Various relations between two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.

(a) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because $A \neq B$.

(b) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they are not parallel and $A \neq B$.

(c) $|\vec{\mathbf{A}}| \neq |-\vec{\mathbf{A}}|$ because they have different directions (even though $|\vec{\mathbf{A}}| = |-\vec{\mathbf{A}}| = A$).

(d) $\vec{\mathbf{A}} = \vec{\mathbf{B}}$ because they are parallel and have identical magnitudes $A = B$.

(e) $\vec{\mathbf{A}} \neq \vec{\mathbf{B}}$ because they have different directions (are not parallel); here, their directions differ by 90°—meaning, they are orthogonal.

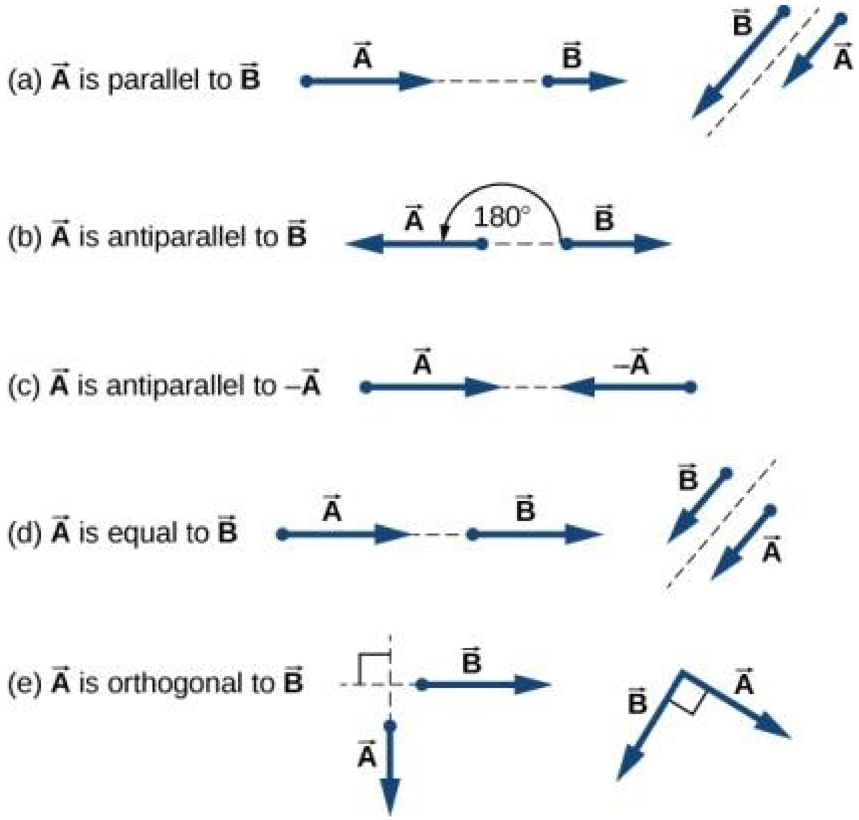
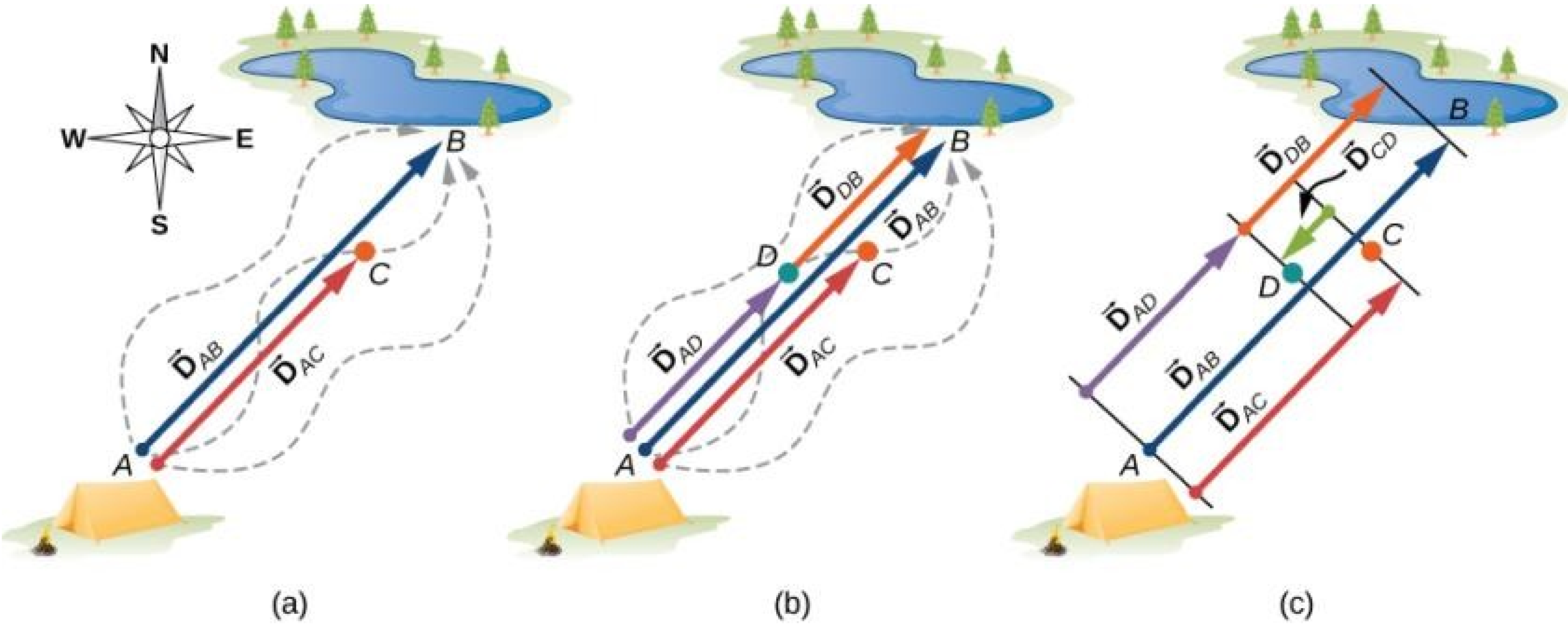


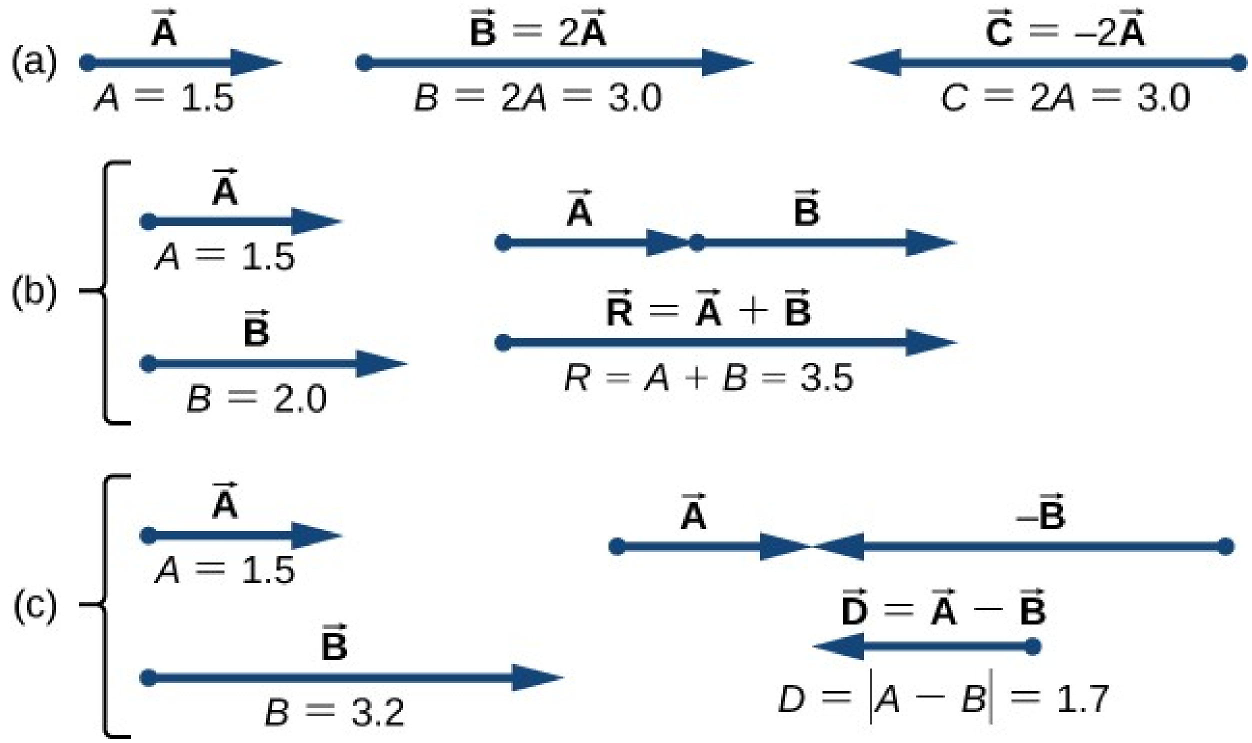
FIGURE 2.6



Displacement vectors for a fishing trip.

a) Stopping to rest at point *C* while walking from camp (point *A*) to the pond (point *B*).
b) Going back for the dropped tackle box (point *D*).
c) Finishing up at the fishing pond.

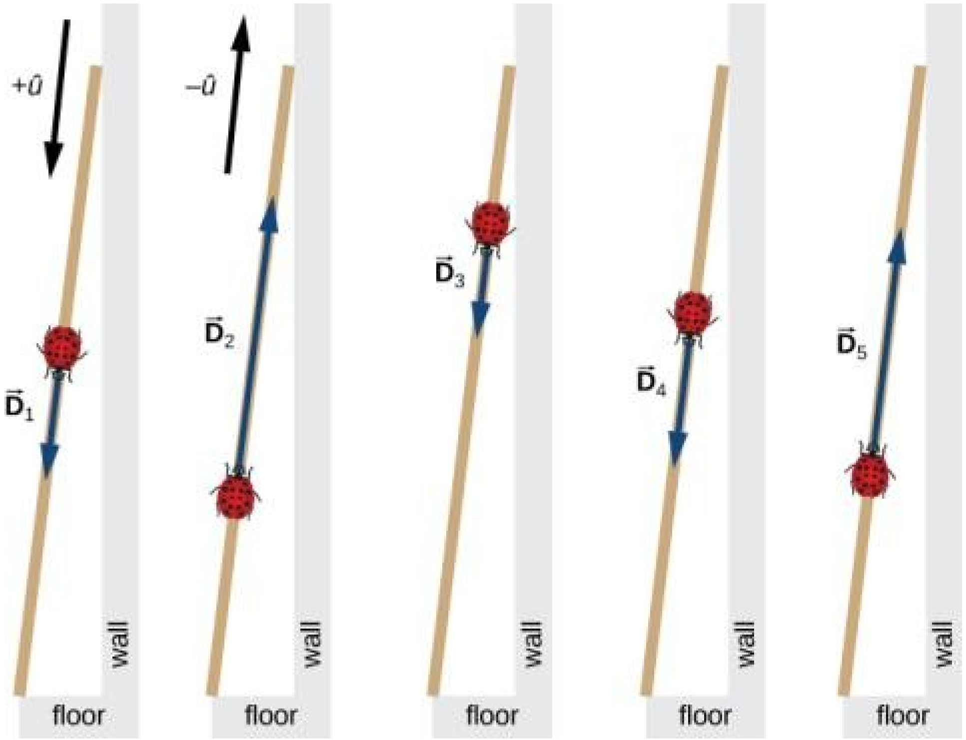
FIGURE 2.7



Algebra of vectors in one dimension.

a) Multiplication by a scalar.
b) Addition of two vectors.
c) Subtraction of two vectors.

FIGURE 2.8

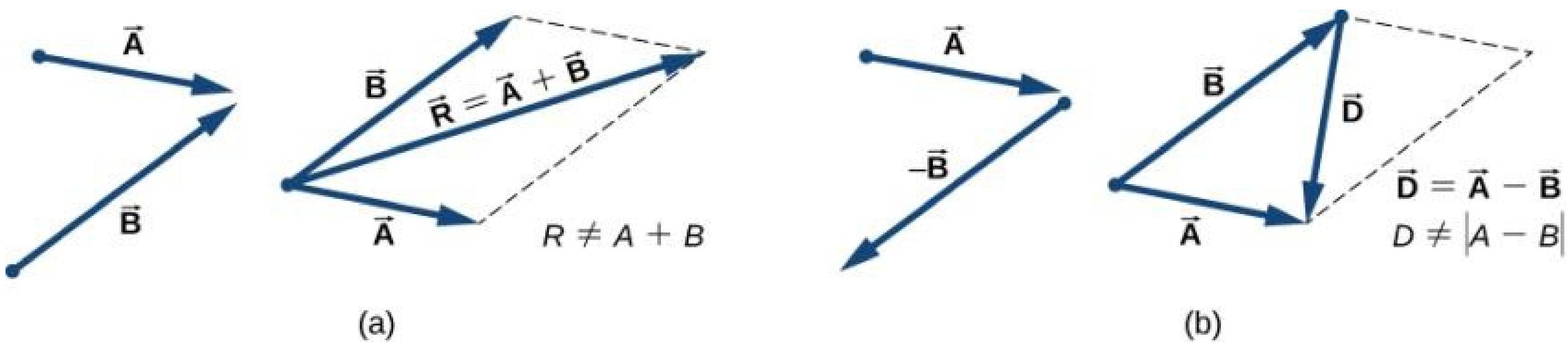


Five displacements of the ladybug. Note that in this schematic drawing, magnitudes of displacements are not drawn to scale. (credit: modification of work by “Persian Poet Gal”/Wikimedia Commons)

FIGURE 2.9

In navigation, the laws of geometry are used to draw resultant displacements on nautical maps.

FIGURE 2.10

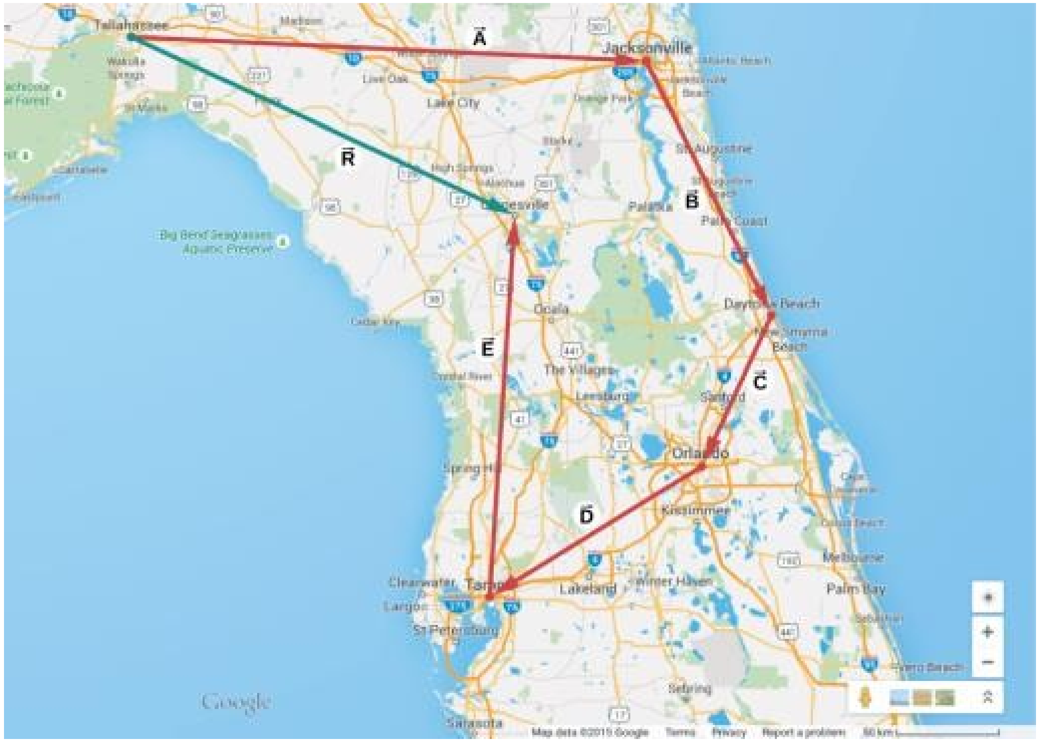


The parallelogram rule for the addition of two vectors. Make the parallel translation of each vector to a point where their origins (marked by the dot) coincide and construct a parallelogram with two sides on the vectors and the other two sides (indicated by dashed lines) parallel to the vectors.

(a) Draw the resultant vector $\vec{\mathbf{R}}$ along the diagonal of the parallelogram from the common point to the opposite corner. Length $\vec{\mathbf{R}}$ of the resultant vector is not equal to the sum of the magnitudes of the two vectors.

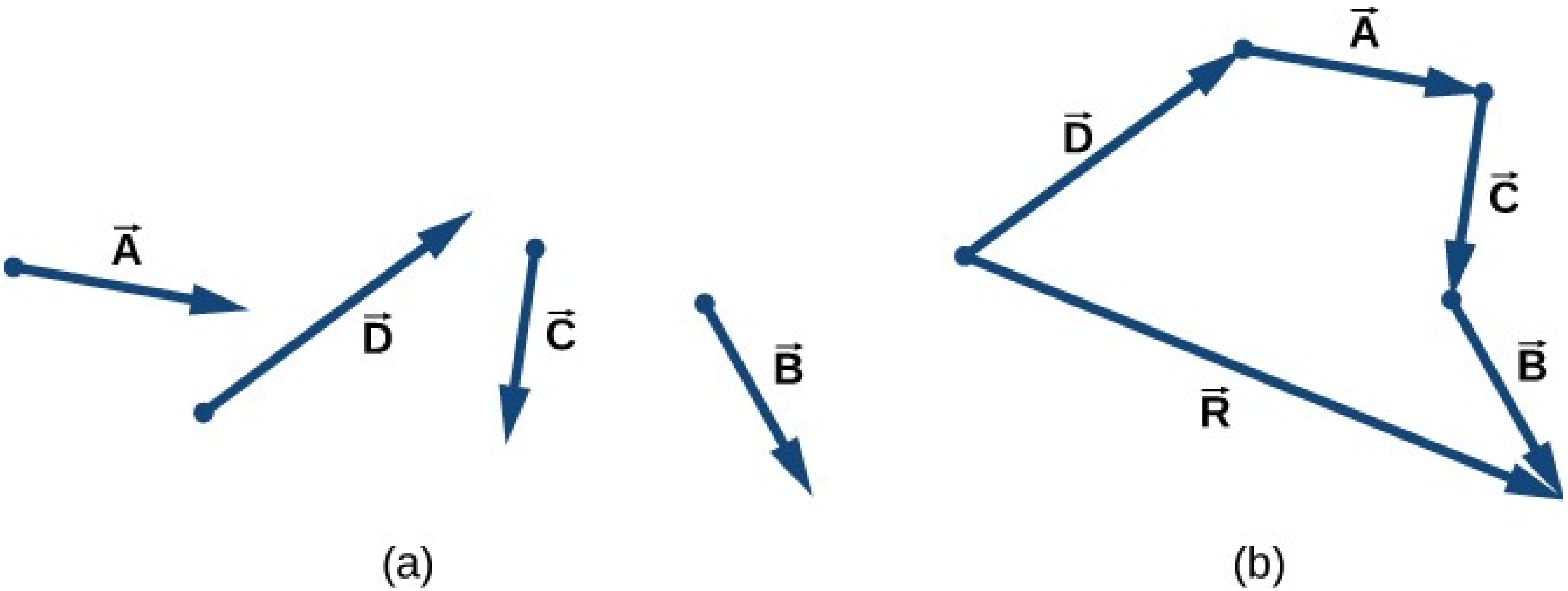
(b) Draw the difference vector $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$ along the diagonal connecting the ends of the vectors. Place the origin of vector $\vec{\mathbf{D}}$ at the end of vector $\vec{\mathbf{B}}$ and the end (arrowhead) of vector $\vec{\mathbf{D}}$ at the end of vector $\vec{\mathbf{A}}$. Length D of the difference vector is not equal to the difference of magnitudes of the two vectors.

FIGURE 2.11



When we use the parallelogram rule four times, we obtain the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}} + \vec{\mathbf{E}}$, which is the green vector connecting Tallahassee with Gainesville.

FIGURE 2.12

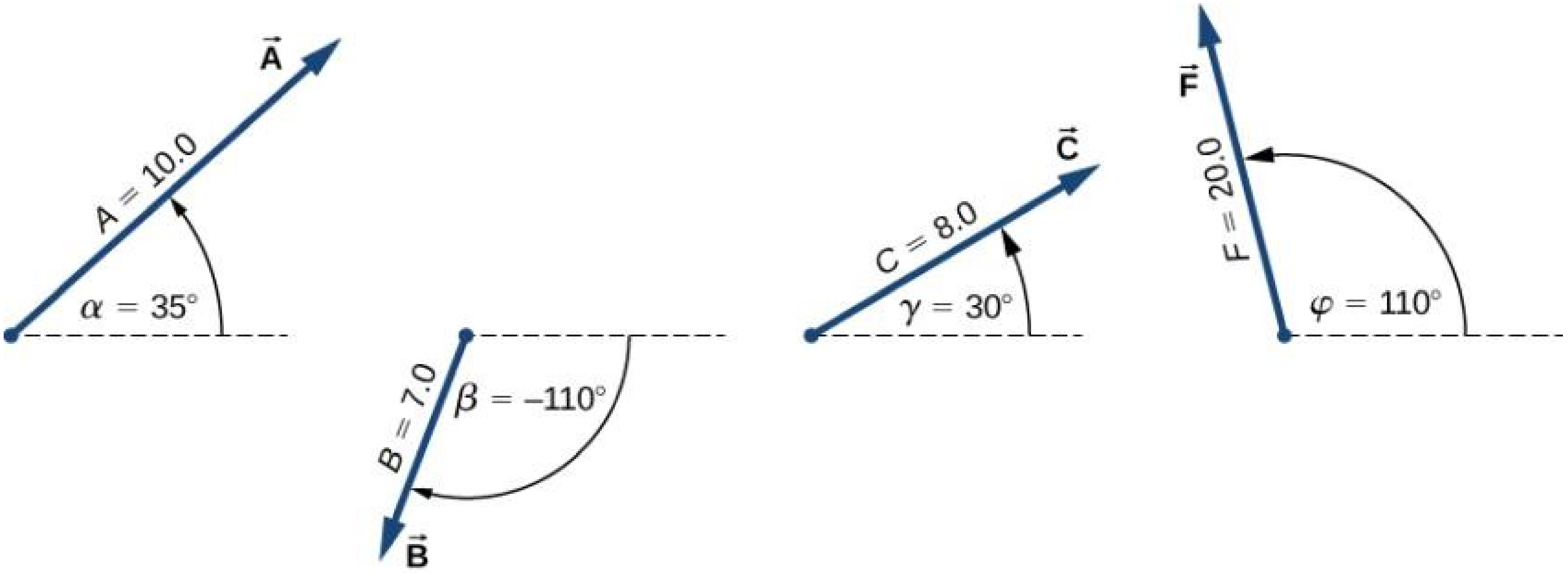


Tail-to-head method for drawing the resultant vector $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}} + \vec{\mathbf{D}}$.

(a) Four vectors of different magnitudes and directions.

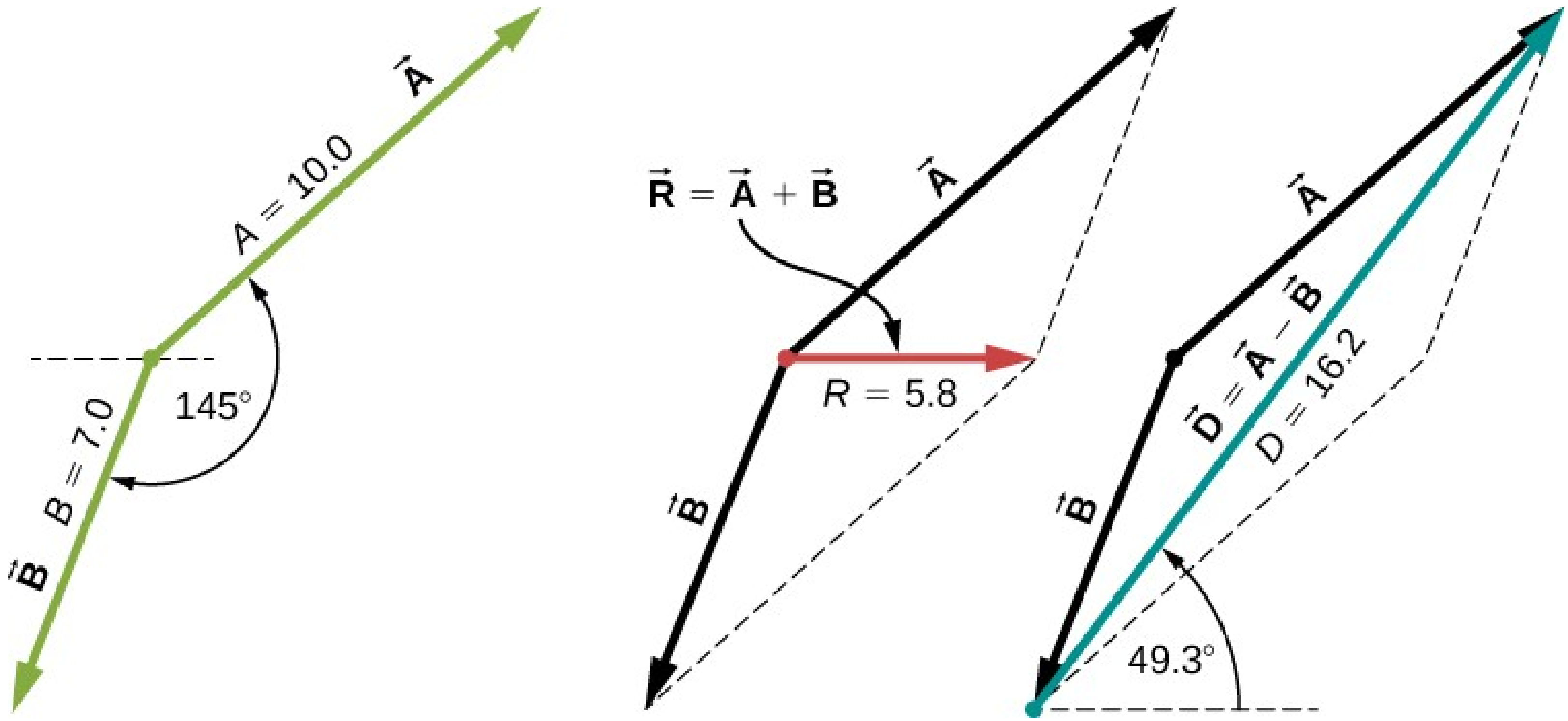
(b) Vectors in (a) are translated to new positions where the origin ("tail") of one vector is at the end ("head") of another vector. The resultant vector is drawn from the origin ("tail") of the first vector to the end ("head") of the last vector in this arrangement.

FIGURE 2.13



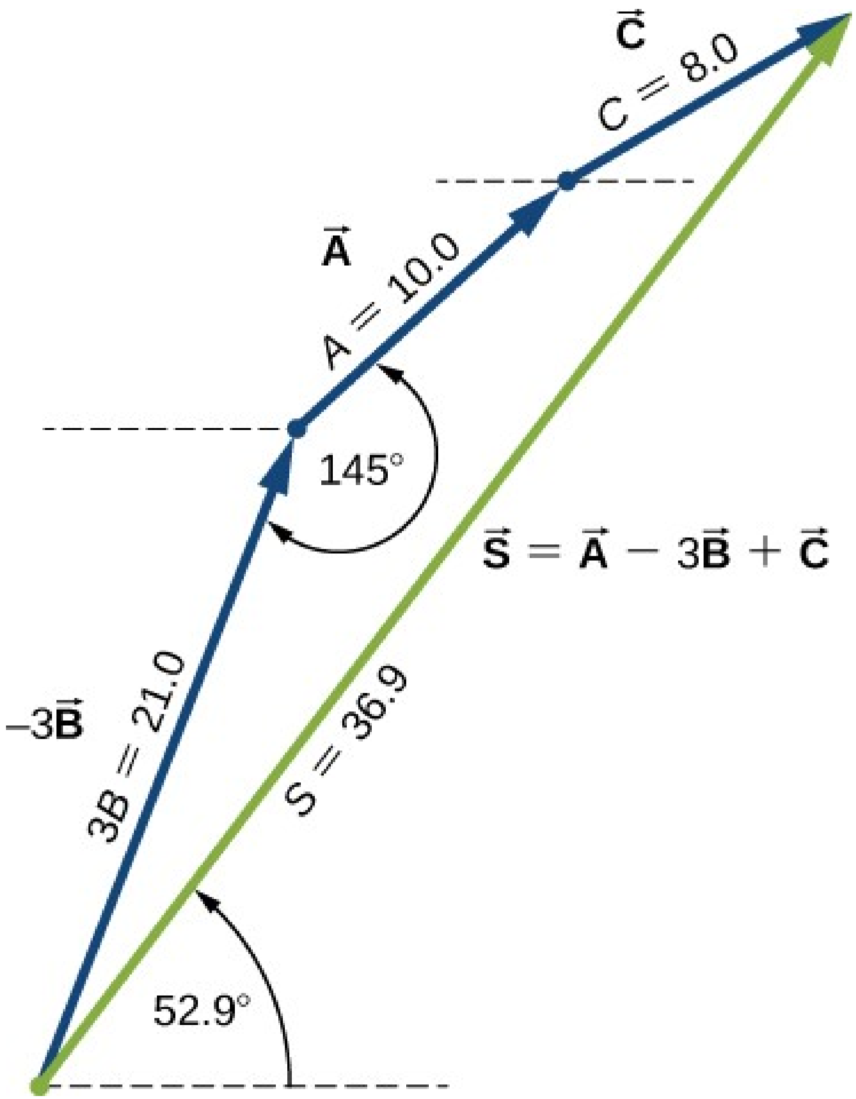
Vectors used in **Example 2.2** and in the Check Your Understanding feature that follows.

FIGURE 2.14



Using the parallelogram rule to solve (a) (finding the resultant, red) and (b) (finding the difference, blue).

FIGURE 2.15



Using the tail-to-head method to solve (c) (finding vector $\vec{S}$, green).

5-MINUTE QUIZ

Answer these three questions:

1) The volume of Earth is on the order of 10^{21} m^3. What is this in cubic kilometers (km^3)?
2)
3) The density of aluminum is 2.7 g/cm^3. What is the density in kilograms per cubic meter?
4)
5) Consider the physical quantities m and a with dimensions [m] = M and [a] = LT^{-2}, respectively. Assuming the equation F = ma to be dimensionally consistent, find the dimension of F.

ANNOUNCEMENTS

Homework #2, due Friday, Aug. 31 before class

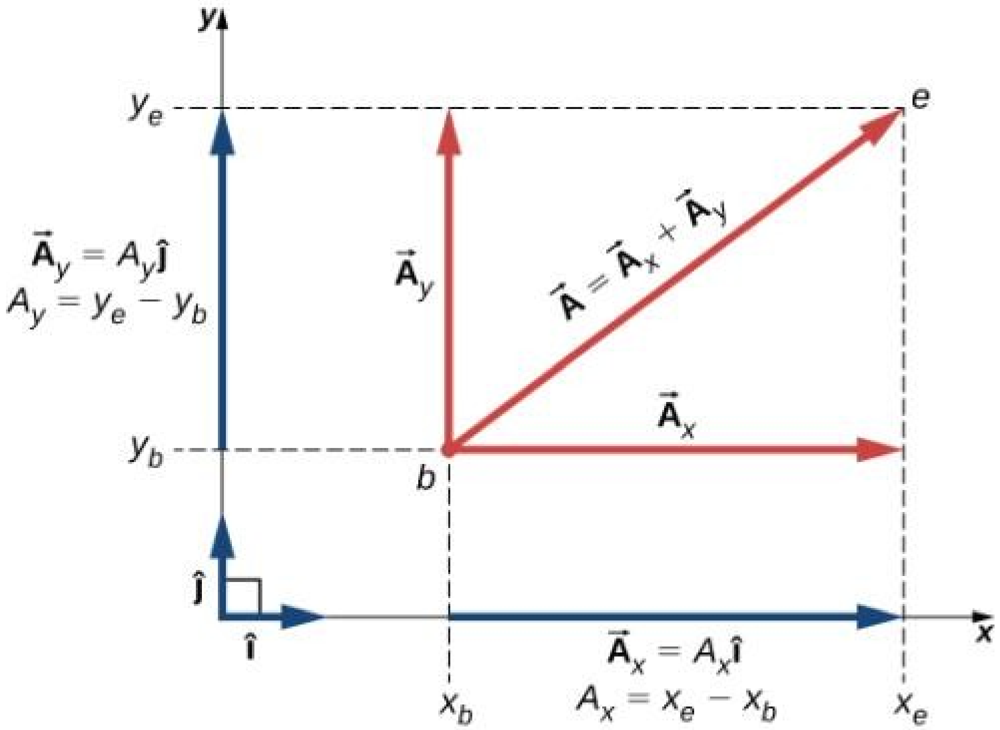
- Answer conceptual questions: Chapter 2, #8 and #20
- Solve problems: Chapter 2, #34 and #54

Finish reading chapter 2 by next class

There will be a 5-minute quiz on Wednesday September 5 at beginning of class Covering all chapter 2

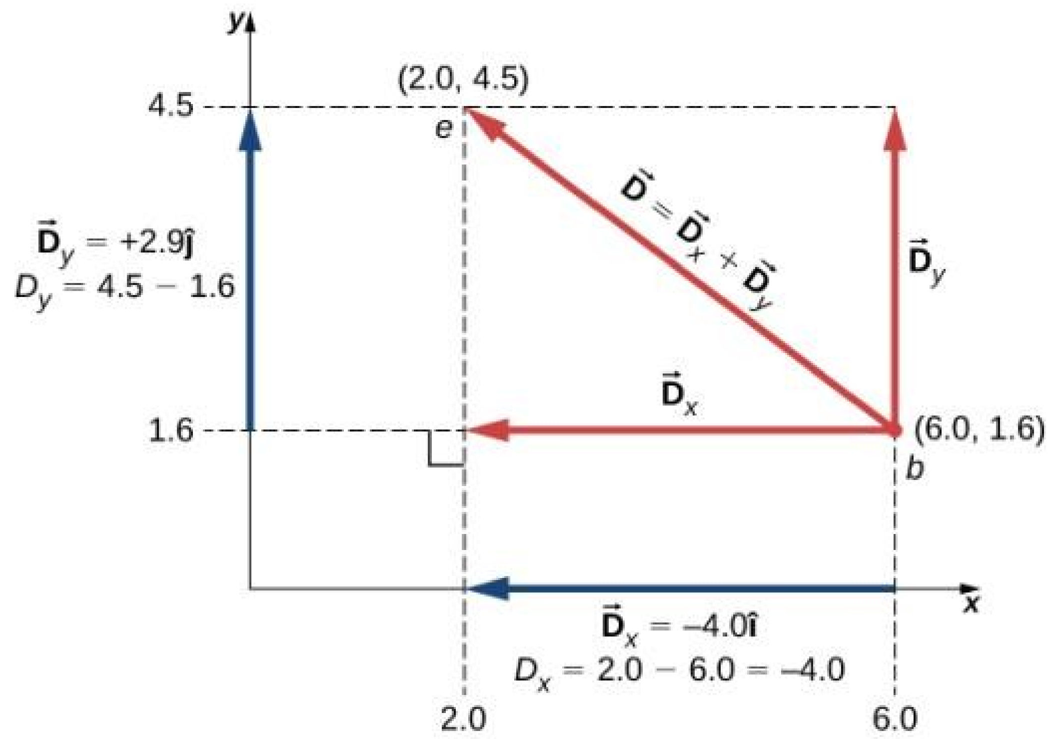
 start this week!

VECTOR COMPONENTS



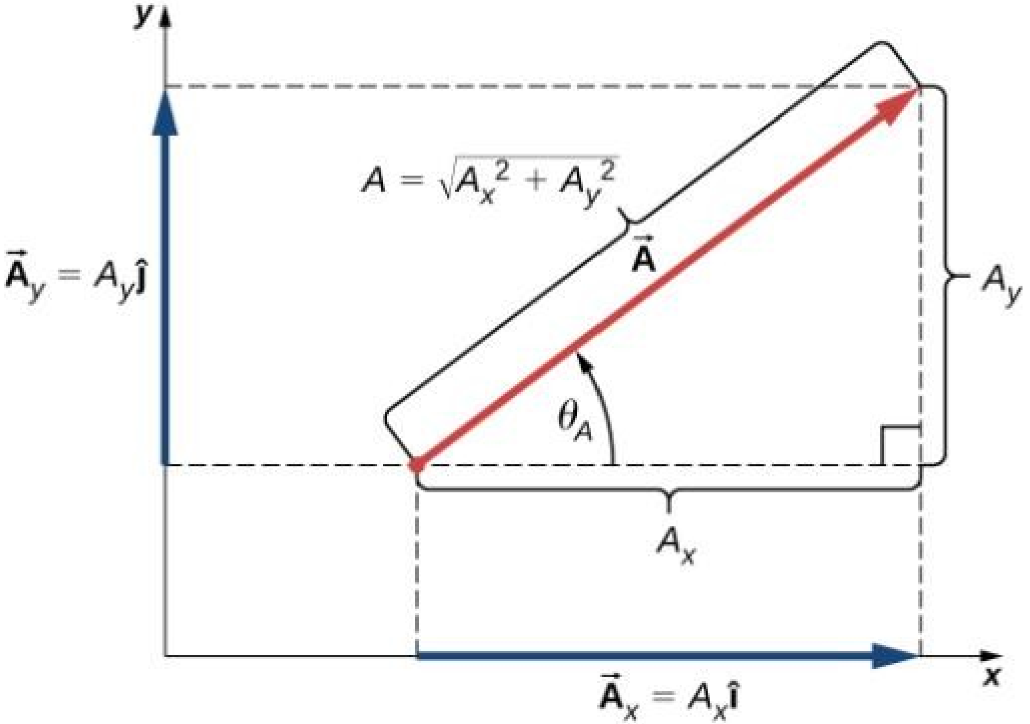
Vector $\vec{\mathbf{A}}$ in a plane in the Cartesian coordinate system is the vector sum of its vector *x*- and *y*-components. The *x*-vector component $\vec{\mathbf{A}}_x$ is the orthogonal projection of vector $\vec{\mathbf{A}}$ onto the *x*-axis. The *y*-vector component $\vec{\mathbf{A}}_y$ is the orthogonal projection of vector onto the *y*-axis. The numbers A_x and A_y that multiply the unit vectors are the scalar components of the vector.

FIGURE 2.17



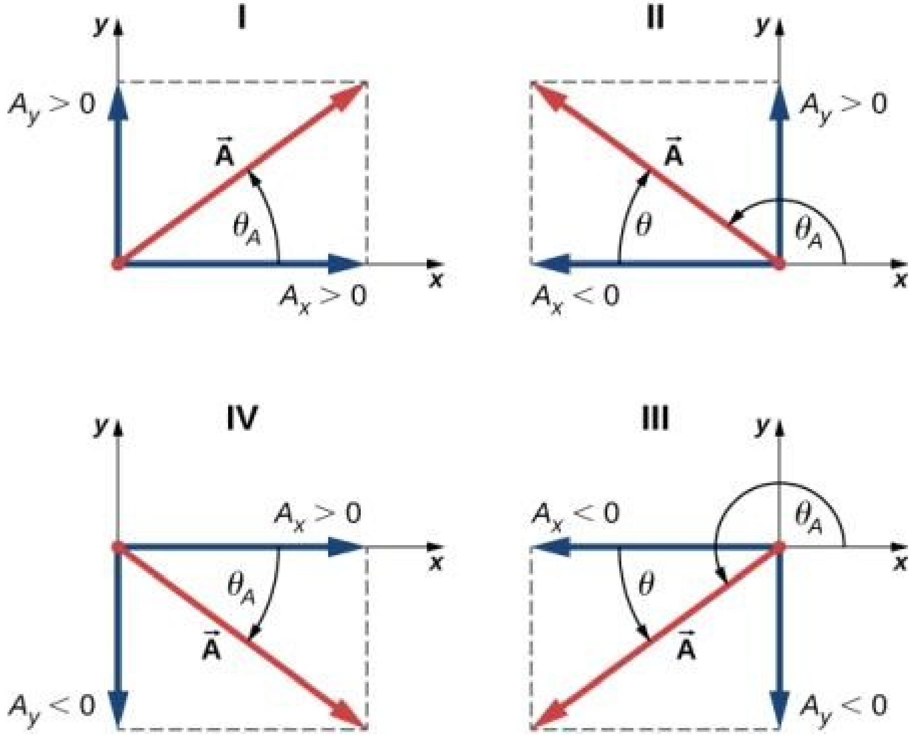
The graph of the displacement vector. The vector points from the origin point at *b* to the end point at *e*.

FIGURE 2.18



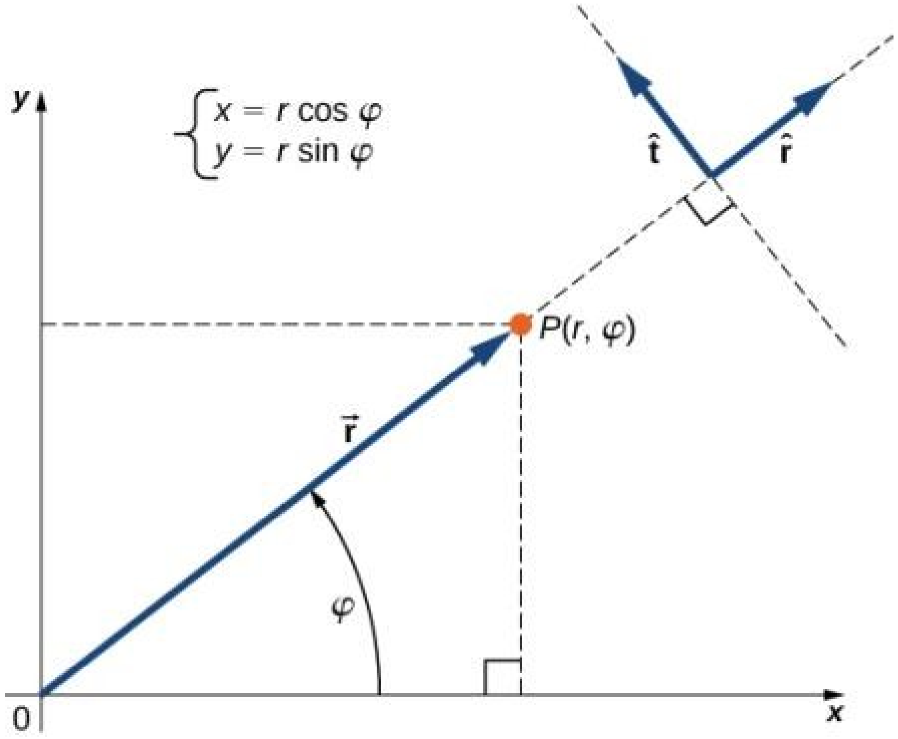
For vector $\vec{\mathbf{A}}$, its magnitude A and its direction angle θ_A are related to the magnitudes of its scalar components because A, A_x, and A_y form a right triangle.

FIGURE 2.19



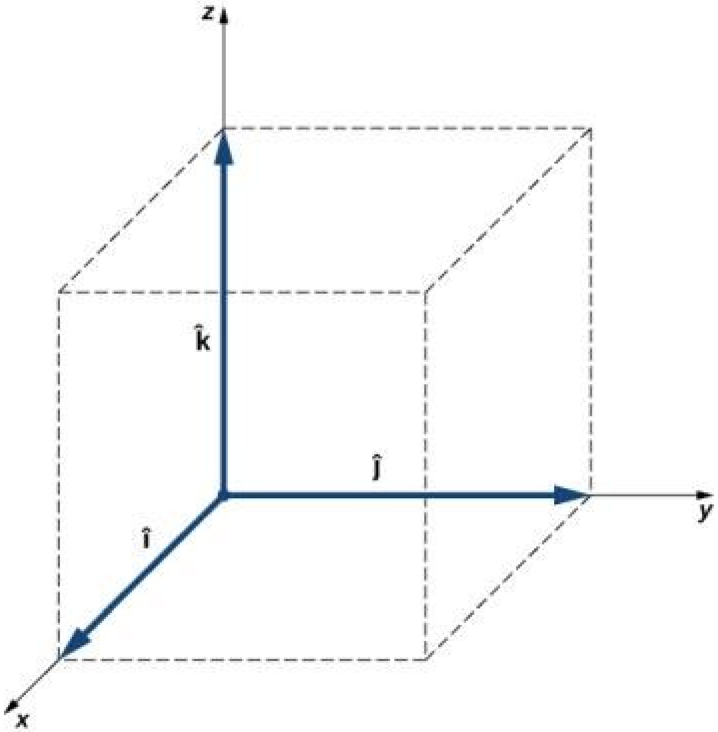
Scalar components of a vector may be positive or negative. Vectors in the first quadrant (I) have both scalar components positive and vectors in the third quadrant have both scalar components negative. For vectors in quadrants II and III, the direction angle of a vector is $\theta_A = \theta + 180°$.

FIGURE 2.20



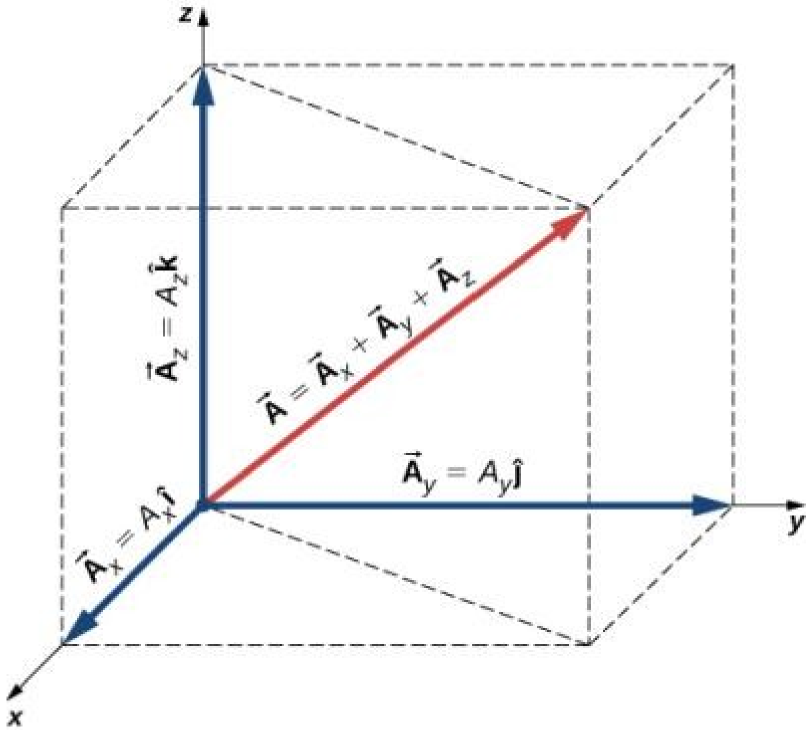
Using polar coordinates, the unit vector defines the positive direction along the radius *r* (radial direction) and, orthogonal to it, the unit vector defines the positive direction of rotation by the angle *φ*.

FIGURE 2.21



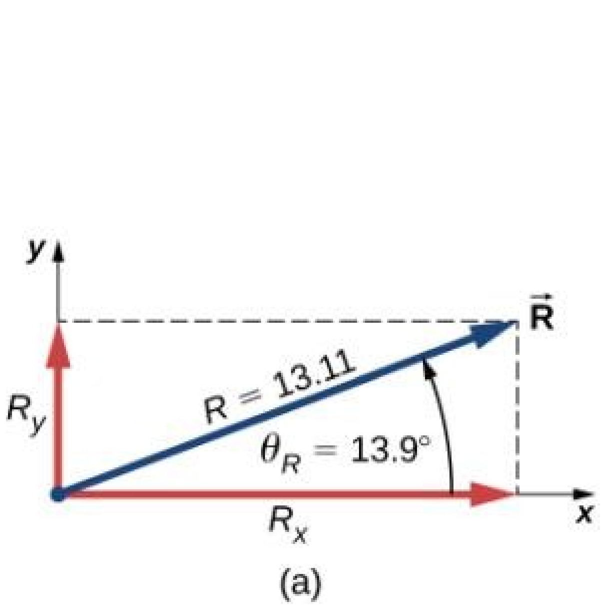
Three unit vectors define a Cartesian system in three-dimensional space. The order in which these unit vectors appear defines the orientation of the coordinate system. The order shown here defines the right-handed orientation.

FIGURE 2.22



A vector in three-dimensional space is the vector sum of its three vector components.

EXAMPLE 2.9



Graphical illustration of the solutions obtained analytically in **Example 2.9**.

ANNOUNCEMENTS

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- Solve problems: Chapter 2, #34 and #54

There will be a 5-minute quiz on Wednesday September 5 at beginning of class Covering all chapter 2

 start this week!

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EXAMPLE 2.9

Analytical Computation of a Resultant

Three displacement vectors $\vec{\mathbf{A}}$, $\vec{\mathbf{B}}$, and $\vec{\mathbf{C}}$ in a plane (Figure 2.13) are specified by their magnitudes *A* = 10.0, *B* = 7.0, and *C* = 8.0, respectively, and by their respective direction angles with the horizontal direction $\alpha = 35°$, $\beta = -110°$, and $\gamma = 30°$. The physical units of the magnitudes are centimeters. Resolve the vectors to their scalar components and find the following vector sums: (a) $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}} + \vec{\mathbf{C}}$, (b) $\vec{\mathbf{D}} = \vec{\mathbf{A}} - \vec{\mathbf{B}}$, and (c) $\vec{\mathbf{S}} = \vec{\mathbf{A}} - 3\vec{\mathbf{B}} + \vec{\mathbf{C}}$.

EXAMPLE 2.9

Analytical Computation of a Resultant

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Strategy

First, we use Equation 2.17 to find the scalar components of each vector and then we express each vector in its vector component form given by Equation 2.12. Then, we use analytical methods of vector algebra to find the resultants.

Solution

We resolve the given vectors to their scalar components:

$$\begin{cases} A_x = A\cos\alpha = (10.0\text{ cm})\cos 35° = 8.19\text{ cm} \\ A_y = A\sin\alpha = (10.0\text{ cm})\sin 35° = 5.73\text{ cm} \end{cases}$$
$$\begin{cases} B_x = B\cos\beta = (7.0\text{ cm})\cos(-110°) = -2.39\text{ cm} \\ B_y = B\sin\beta = (7.0\text{ cm})\sin(-110°) = -6.58\text{ cm} \end{cases}.$$
$$\begin{cases} C_x = C\cos\gamma = (8.0\text{ cm})\cos 30° = 6.93\text{ cm} \\ C_y = C\sin\gamma = (8.0\text{ cm})\sin 30° = 4.00\text{ cm} \end{cases}$$

For (a) we may substitute directly into Equation 2.25 to find the scalar components of the resultant:

$$\begin{cases} R_x = A_x + B_x + C_x = 8.19\text{ cm} - 2.39\text{ cm} + 6.93\text{ cm} = 12.73\text{ cm} \\ R_y = A_y + B_y + C_y = 5.73\text{ cm} - 6.58\text{ cm} + 4.00\text{ cm} = 3.15\text{ cm} \end{cases}.$$

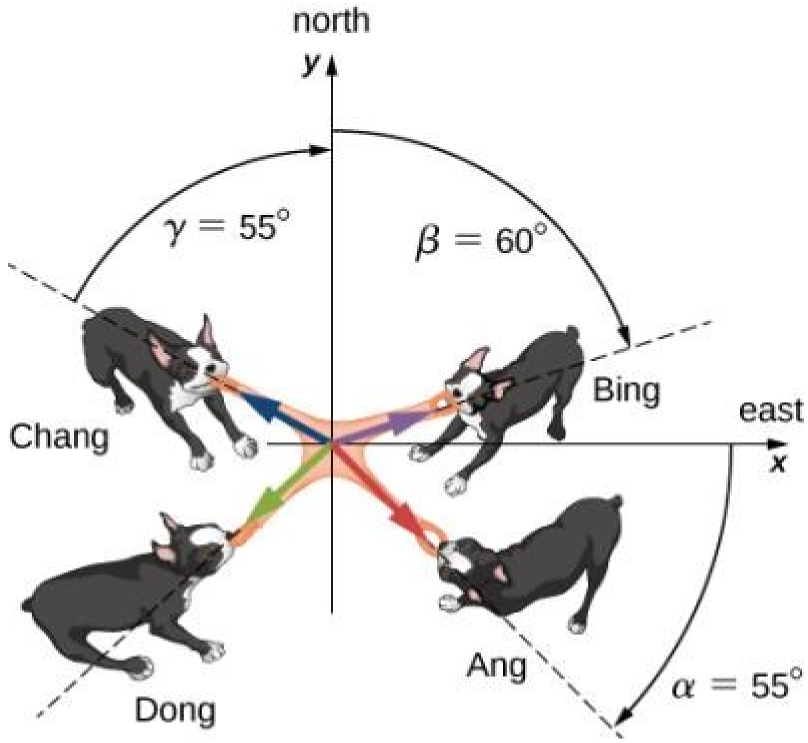
Therefore, the resultant vector is $\vec{\mathbf{R}} = R_x\hat{\mathbf{i}} + R_y\hat{\mathbf{j}} = (12.7\hat{\mathbf{i}} + 3.1\hat{\mathbf{j}})\text{cm}$.

EXAMPLE 2.10

EXAMPLE 2.10

The Tug-of-War Game

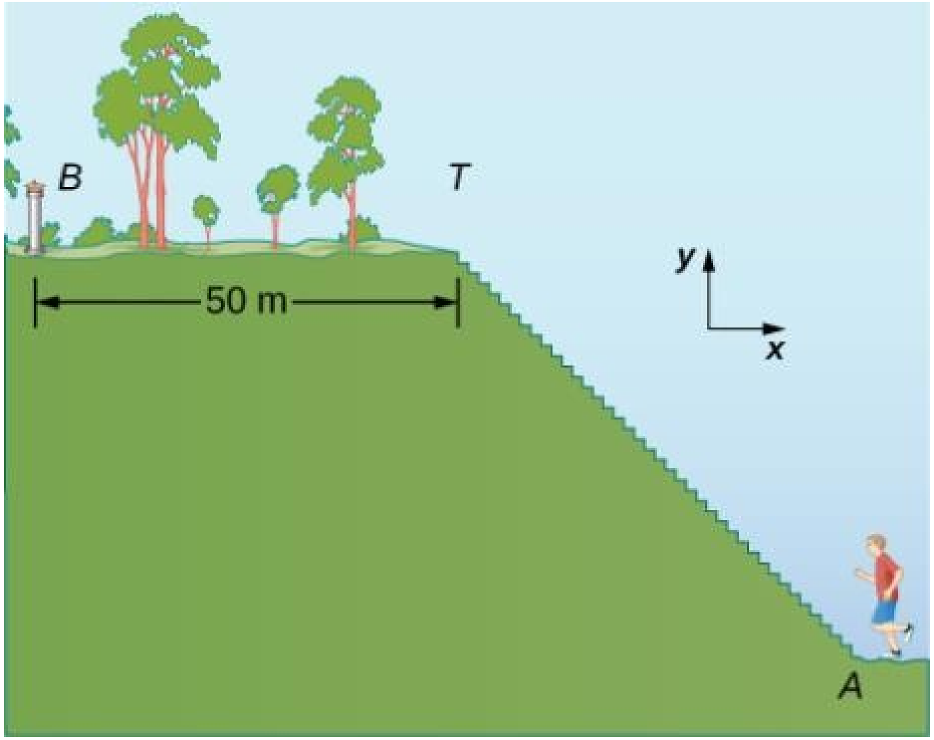
Four dogs named Astro, Balto, Clifford, and Dug play a tug-of-war game with a toy (Figure 2.25). Astro pulls on the toy in direction $\alpha = 55°$ south of east, Balto pulls in direction $\beta = 60°$ east of north, and Clifford pulls in direction $\gamma = 55°$ west of north. Astro pulls strongly with 160.0 units of force (N), which we abbreviate as A = 160.0 N. Balto pulls even stronger than Astro with a force of magnitude B = 200.0 N, and Clifford pulls with a force of magnitude C = 140.0 N. When Dug pulls on the toy in such a way that his force balances out the resultant of the other three forces, the toy does not move in any direction. With how big a force and in what direction must Dug pull on the toy for this to happen?

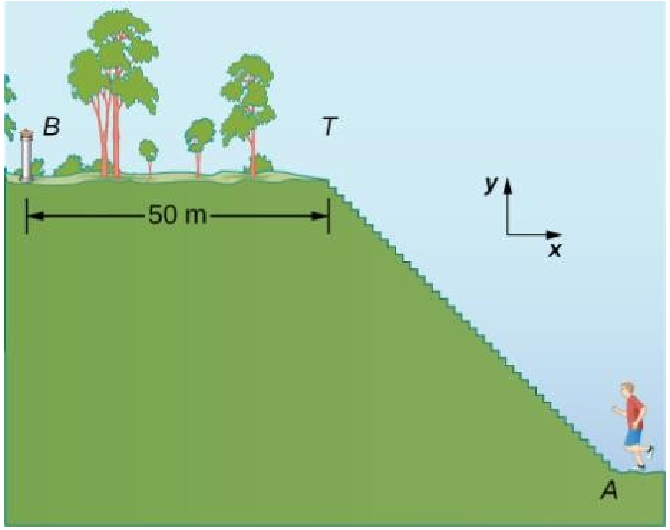


EXAMPLE 2.13

Displacement of a Jogger

A jogger runs up a flight of 200 identical steps to the top of a hill and then runs along the top of the hill 50.0 m before he stops at a drinking fountain (Figure 2.26). His displacement vector from point *A* at the bottom of the steps to point *B* at the fountain is $\vec{\mathbf{D}}_{AB} = (-90.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}})\text{m}$. What is the height and width of each step in the flight? What is the actual distance the jogger covers? If he makes a loop and returns to point *A*, what is his net displacement vector?





Solution
In the coordinate system indicated in Figure 2.26, the jogger's displacement vector on the top of the hill is $\vec{\mathbf{D}}_{TB} = (-50.0 \text{ m})\hat{\mathbf{i}}$. His net displacement vector is

$$\vec{\mathbf{D}}_{AB} = \vec{\mathbf{D}}_{AT} + \vec{\mathbf{D}}_{TB}.$$

Therefore, his displacement vector $\vec{\mathbf{D}}_{TB}$ along the stairs is

$$\begin{aligned} \vec{\mathbf{D}}_{AT} &= \vec{\mathbf{D}}_{AB} - \vec{\mathbf{D}}_{TB} = (-90.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}})\text{m} - (-50.0 \text{ m})\hat{\mathbf{i}} = [(-90.0 + 50.0)\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}})]\text{m} \\ &= (-40.0\hat{\mathbf{i}} + 30.0\hat{\mathbf{j}})\text{m}. \end{aligned}$$

Its scalar components are $D_{ATx} = -40.0 \text{ m}$ and $D_{ATy} = 30.0 \text{ m}$. Therefore, we must have

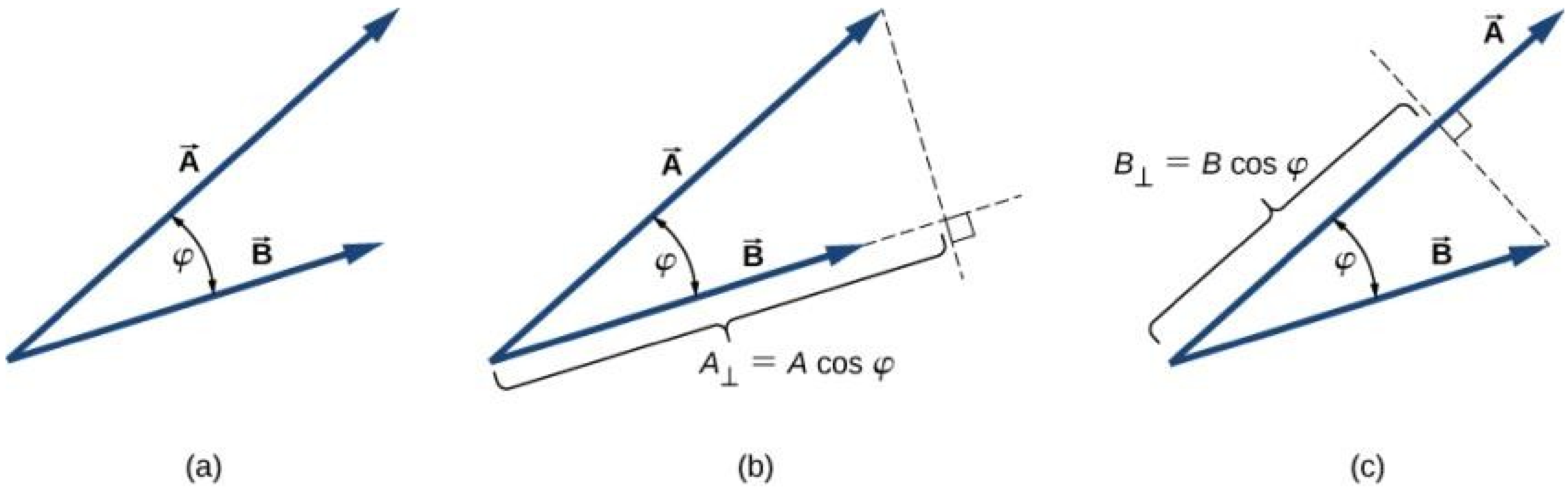
$$200w = |-40.0|\text{m and } 200h = 30.0 \text{ m}.$$

Hence, the step width is w = 40.0 m/200 = 0.2 m = 20 cm, and the step height is h = 30.0 m/200 = 0.15 m = 15 cm. The distance that the jogger covers along the stairs is

$$D_{AT} = \sqrt{D_{ATx}^2 + D_{ATy}^2} = \sqrt{(-40.0)^2 + (30.0)^2} \text{ m} = 50.0 \text{ m}.$$

Thus, the actual distance he runs is $D_{AT} + D_{TB} = 50.0 \text{ m} + 50.0 \text{ m} = 100.0 \text{ m}$. When he makes a loop and comes back from the fountain to his initial position at point *A*, the total distance he covers is twice this distance, or 200.0 m. However, his net displacement vector is zero, because when his final position is the same as his initial position, the scalar components of his net displacement vector are zero (Equation 2.13).

SCALAR PRODUCT



The scalar product of two vectors.

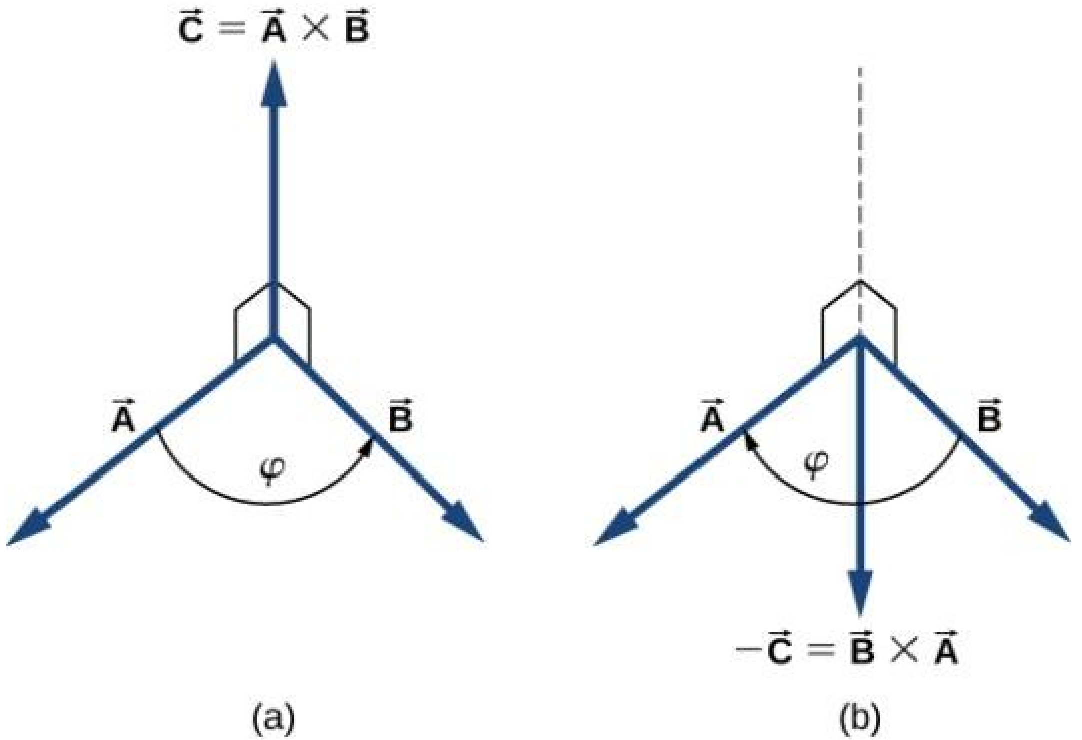
a) The angle between the two vectors.
b) The orthogonal projection $A_{\perp}$ of vector onto the direction of vector .
c) The orthogonal projection $B_{\perp}$ of vector onto the direction of vector .

EXAMPLE 2.16

Angle between Two Forces

Three dogs are pulling on a stick in different directions, as shown in Figure 2.28. The first dog pulls with force $\vec{\mathbf{F}}_1 = (10.0\hat{\mathbf{i}} - 20.4\hat{\mathbf{j}} + 2.0\hat{\mathbf{k}})\mathrm{N}$, the second dog pulls with force $\vec{\mathbf{F}}_2 = (-15.0\hat{\mathbf{i}} - 6.2\hat{\mathbf{k}})\mathrm{N}$, and the third dog pulls with force $\vec{\mathbf{F}}_3 = (5.0\hat{\mathbf{i}} + 12.5\hat{\mathbf{j}})\mathrm{N}$. What is the angle between forces $\vec{\mathbf{F}}_1$ and $\vec{\mathbf{F}}_2$?

VECTOR PRODUCT

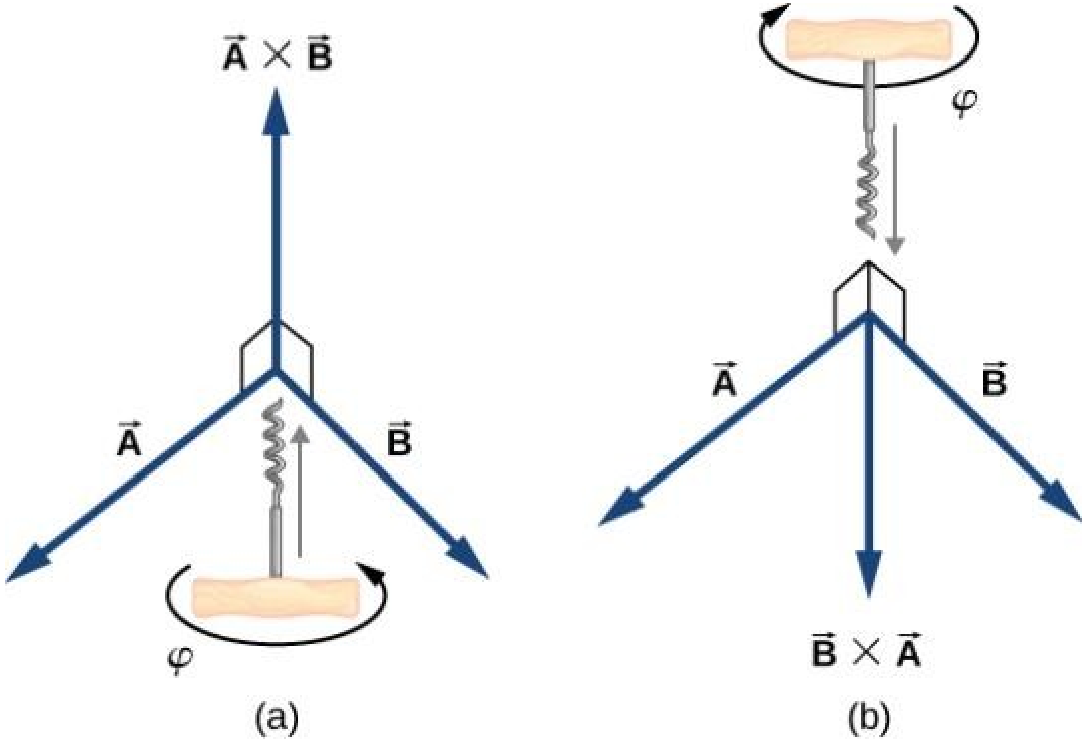


The vector product of two vectors is drawn in three-dimensional space.

(a) The vector product $\vec{A} \times \vec{B}$ is a vector perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$. Small squares drawn in perspective mark right angles between $\vec{A}$ and $\vec{C}$, and between $\vec{B}$ and $\vec{C}$ so that if $\vec{A}$ and $\vec{B}$ lie on the floor, vector $\vec{C}$ points vertically upward to the ceiling.

(b) The vector product $\vec{B} \times \vec{A}$ is a vector antiparallel to vector $\vec{A} \times \vec{B}$.

VECTOR PRODUCT



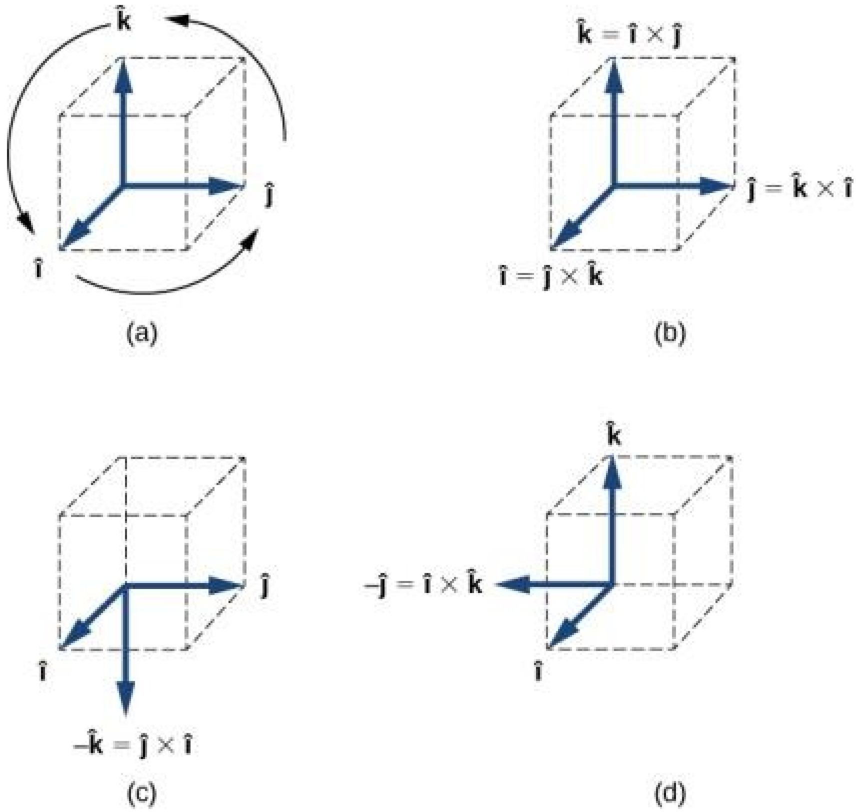
The corkscrew right-hand rule can be used to determine the direction of the cross product $\vec{A} \times \vec{B}$. Place a corkscrew in the direction perpendicular to the plane that contains vectors $\vec{A}$ and $\vec{B}$, and turn it in the direction from the first to the second vector in the product. The direction of the cross product is given by the progression of the corkscrew.

(a) Upward movement means the cross-product vector points up.

(b) Downward movement means the cross-product vector points downward.

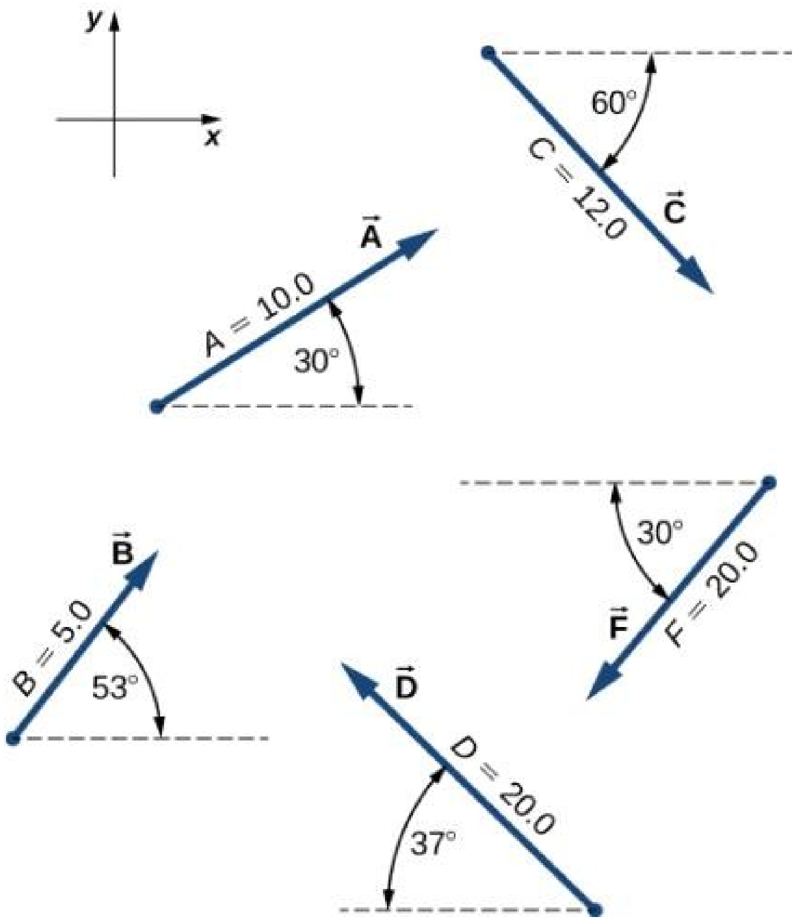
FIGURE 2.32

a) The diagram of the cyclic order of the unit vectors of the axes.
b) The only cross products where the unit vectors appear in the cyclic order. These products have the positive sign.
c) (d) Two examples of cross products where the unit vectors do not appear in the cyclic order. These products have the negative sign.



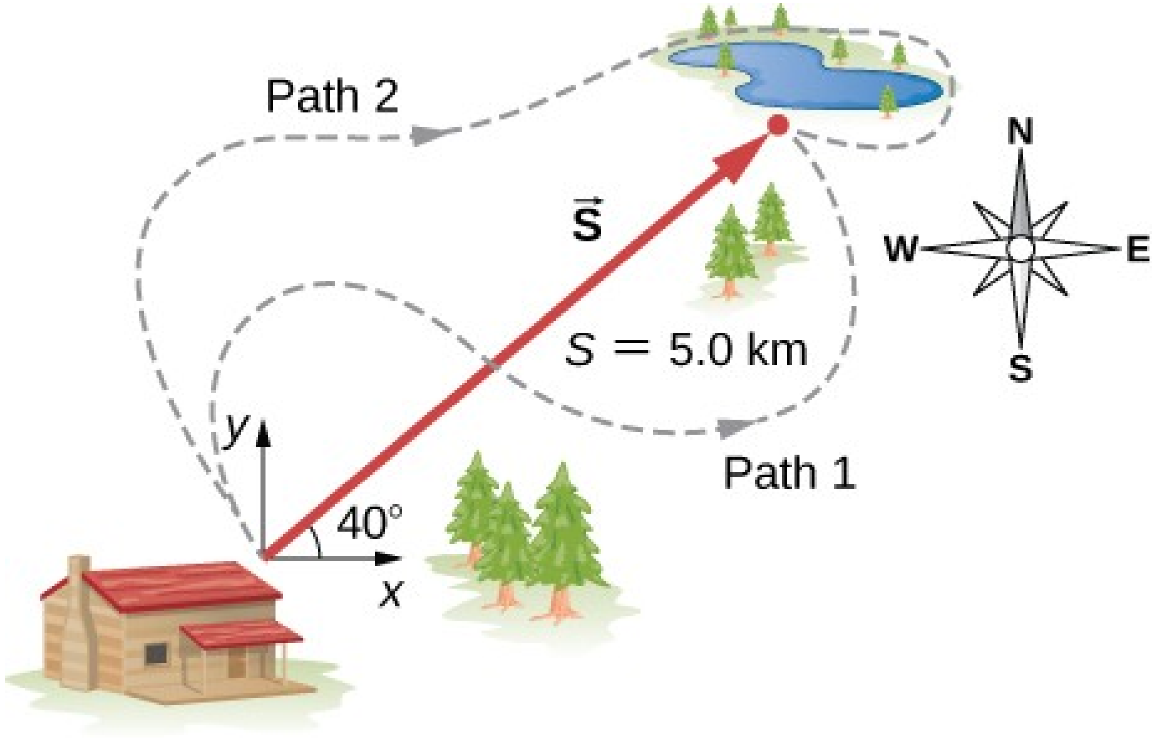
EXERCISE 28

28 . For the vectors given in the following figure, use a graphical method to find the following resultants: (a) $\vec{\mathbf{A}}+\vec{\mathbf{B}}$, (b) $\vec{\mathbf{C}}+\vec{\mathbf{B}}$, (c) $\vec{\mathbf{D}}+\vec{\mathbf{F}}$, (d) $\vec{\mathbf{A}}-\vec{\mathbf{B}}$, (e) $\vec{\mathbf{D}}-\vec{\mathbf{F}}$, (f) $\vec{\mathbf{A}}+2\vec{\mathbf{F}}$, (g)$\vec{\mathbf{C}}-2\vec{\mathbf{D}}+3\vec{\mathbf{F}}$; and (h) $\vec{\mathbf{A}}-4\vec{\mathbf{D}}+2\vec{\mathbf{F}}$.



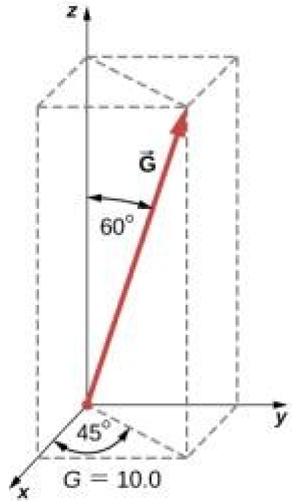
EXERCISE 33

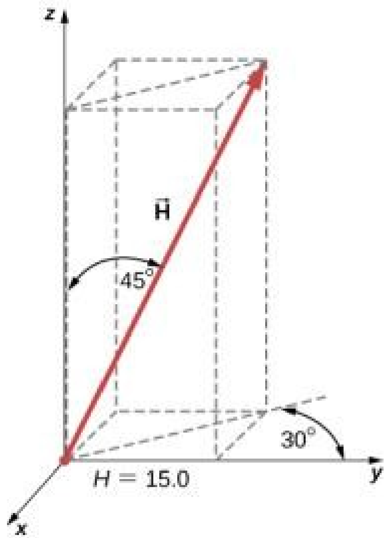
33 . A trapper walks a 5.0-km straight-line distance from his cabin to the lake, as shown in the following figure. Use a graphical method (the parallelogram rule) to determine the trapper's displacement directly to the east and displacement directly to the north that sum up to his resultant displacement vector. If the trapper walked only in directions east and north, zigzagging his way to the lake, how many kilometers would he have to walk to get to the lake?



EXERCISE 82

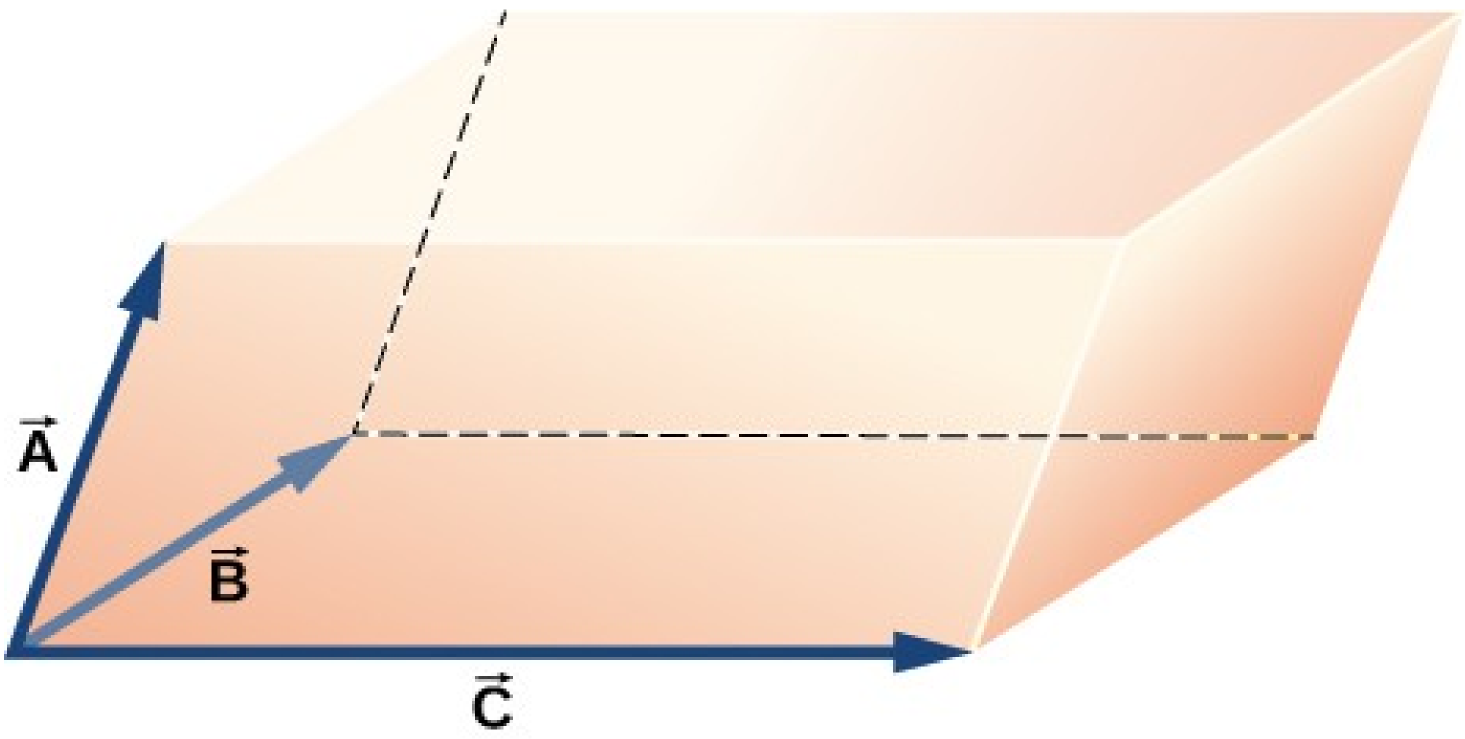
82 . Find the scalar components of three-dimensional vectors $\vec{\mathbf{G}}$ and $\vec{\mathbf{H}}$ in the following figure and write the vectors in vector component form in terms of the unit vectors of the axes.





EXERCISE 87

87 . Show that $(\vec{B} \times \vec{C}) \cdot \vec{A}$ is the volume of the parallelepiped, with edges formed by the three vectors in the following figure.



EXERCISE 91

91 . Distances between points in a plane do not change when a coordinate system is rotated. In other words, the magnitude of a vector is *invariant* under rotations of the coordinate system. Suppose a coordinate system S is rotated about its origin by angle φ to become a new coordinate system S', as shown in the following figure. A point in a plane has coordinates (*x*, *y*) in S and coordinates (x', y') in S'. (a) Show that, during the transformation of rotation, the coordinates in S' are expressed in terms of the coordinates in S by the following relations:

$$\begin{cases} x' = x\cos\varphi + y\sin\varphi \\ y' = -x\sin\varphi + y\cos\varphi \end{cases}.$$

(b) Show that the distance of point *P* to the origin is invariant under rotations of the coordinate system. Here, you have to show that

$$\sqrt{x^2 + y^2} = \sqrt{x'^2 + y'^2}.$$

(c) Show that the distance between points *P* and *Q* is invariant under rotations of the coordinate system. Here, you have to show that

$$\sqrt{(x_P - x_Q)^2 + (y_P - y_Q)^2} = \sqrt{(x'_P - x'_Q)^2 + (y'_P - y'_Q)^2}.$$