Appendix A: Math Review

A great deal of information can be obtained by first considering the type of equation being investigated.

- Is one variable squared? If yes $\rightarrow$ it is a quadratic equation.
- Are there trigonometric functions in the equation? If yes $\rightarrow$ it is a sinusoidal equation (or tangential, or ...).
- Is there an exponential (*i.e.*, the number "e," not an exponent) in the equation? If yes $\rightarrow$ it is an exponential equation.
- If none of the above, is this equation linear? If yes $\rightarrow$ use the slope-intercept equation of a line: $y = mx + b$.

Consider the *kinematic equations* below (from *Experiment* **??**: *Projectile Motion*).

$x = x_0 + v_{0_x} t + \frac{1}{2} a_x t^2$	Quadratic in Time
$y = y_0 + v_{0_y} t + \frac{1}{2} a_y t^2$	Quadratic in Time
$v_x = v_{0_x} + a_x t$	Linear
$v_y = v_{0_y} + a_y t$	Linear
$v_x^2 = v_{0_x}^2 + 2a_x(x - x_0)$	Quadratic in Velocity
$v_y^2 = v_{0_y}^2 + 2a_y(y - y_0)$	Quadratic in Velocity

We will use other types of equations this year as well:

$F_B = ILB \sin\theta$	Magnetic Force is Sinusoidal as the Angle Varies
$N - N_0 e^{\lambda t}$	Exponential Growth (Your calculator has the function e.)

General Physics requires mastery of college-level algebra and trigonometry.

If it has been some time since you used this level of math, it is important that you refresh your knowledge and skills. Your text for physics lecture may have a math appendix; if it does, please read through it to be certain you have the math knowledge required.

This appendix is a simple summary of math that is frequently used in General Physics Lab. Some experiments will require more advanced knowledge.

Solve for x means that the variable x is on one side of the equation, everything else in the equation is on the other side. If x is the only variable, the equation has a unique solution (has only one correct answer).

If unique solutions are required, the number of unknowns *must* equal the number of equations.

Consider the equation for the circumference of a circle: $C = \pi D$. There is 1 equation and 2 unknowns; there is no unique solution. Refer to Fig. A.1. *Any* point on the line is a correct solution. There are an infinite number of correct solutions.

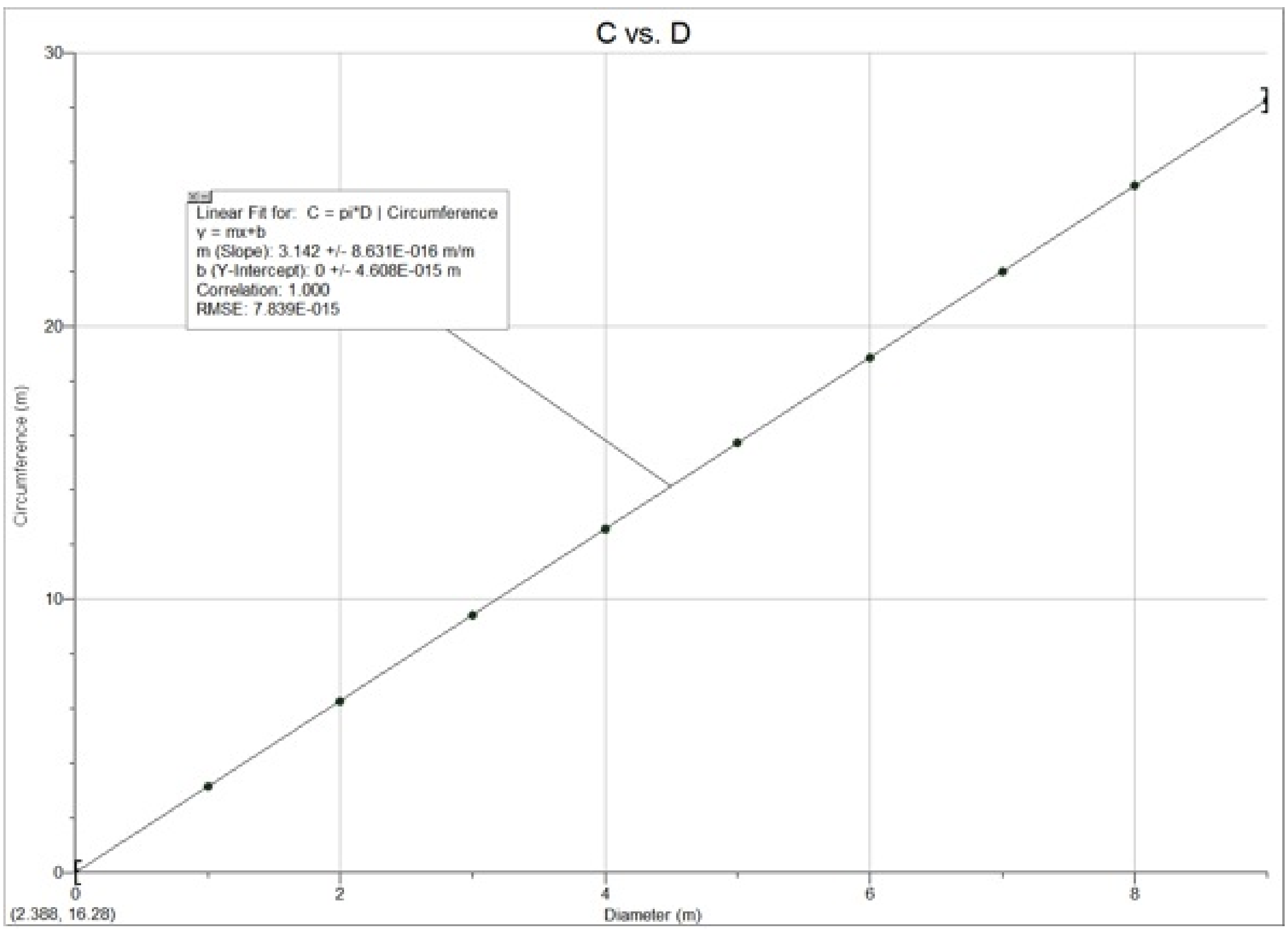


Figure A.1: Linear Plot

Solving 3 equations with 3 unknowns is required in lab. The variables are the unknowns.

There are various methods for solving a system of equations, many using advanced linear algebra of computing software, but the simplest and most straightforward is the *substitution method*, covered below:

Note that this method applies to any set of x equations with x unknowns, the procedure simply requires more or fewer repetitions, you will solve 2 equation 2 unknown type problems repeatedly, and abundantly in this course.

Choose one of the equations; if all 3 equations have all 3 unknowns, it does not matter which one you choose first. If this is not the case, remember you need to *eliminate* and unknown! Choose the equation that has all 3 unknowns. Designate this equation as Eq. 1. Solve Eq. 1 for one of the unknowns (it does not matter which one).

Substitute Eq. 1 into a second equation; designate this second equation as Eq. 2.
Note that the substitution *must* eliminate an unknown.
Designate the last equation as Eq. 3.
You now have 2 equations with 2 unknowns: Eq. 2 and Eq. 3.

Choose either Eq. 2 or Eq. 3, then solve it for an unknown;
Substitute the value of this unknown into the remaining equation.
Note that the substitution *must* eliminate an unknown.
You now have 1 equation and 1 unknown.

Solve the equation; substitute this value into the previous equation.
Solve the equation; substitute this value(s) into the first equation.

Solve the first equation. Done. Check your answer.

Example for "Solving 3 Equations with 3 Unknowns"

Given the following 3 equations, solve for x, y, and z:

$$x = y + z$$
$$1.5 - 3x - 4z = 0$$
$$-1.5 - 5y + 6x = 0$$

Choose the equations that has all 3 unknowns.

Designate this equation as Eq. 1. Solve Eq. 1 for one of the unknowns: z.

Eq 1: $x = y + z \to z = x - y$

Substitute Eq. 1 into a second equation; designate this second equation as Eq. 2.

Eq. 2: $1.5 - 3x - 4z = 0 \to 1.5 - 3x - 4(x - y) = 0 \to 1.5 - 7x + 4y = 0$

Choose Eq. 2 or Eq. 3, then solve it for an unknown: Eq. 2.

Eq. 2: $1.5 - 7x + 4y = 0 \to x = (1.5 + 4y)/7$

Substitute the value of this unknown into the remaining equation:

Eq. 3: $-1.5 - 5y + 6x = 0 \to -1.5 - 5y + 6(\frac{1.5+4y}{7}) = 0$

Solve the equation:

$-1.5 - 5y + \frac{9}{7} + \frac{24y}{7} = 0 \to \frac{24y}{7} - 5y = 1.5 - \frac{9}{7} \to \frac{-11y}{7} = \frac{3}{14} \to y = (\frac{-7}{11})(\frac{3}{14}) = -0.136$

Substitute this value into the previous equation and solve the equation:

Eq. 2: $x = \frac{1.5+4y}{7} \to x = \frac{1.5+4(-0.136)}{7} \to x = 0.137$

Substitute these value(s) into the first equation and solve the equation:

Eq. 1: $z = x - y \to z = 0.137 - (-0.136) \to z = 0.273$

Done. Check your answer!

Solving a quadratic equation is easily accomplished by the use of the *quadratic formula*.
A quadratic means that one variable is squared.

To use the quadratic formula, the equation must first be in the correct form: $ax^2 + bx + c = 0$.

The coefficients are a, b, and c; $a \neq 0$.

The coefficient of x^2 is a, the coefficient of x^1 is b, the coefficient of x^0 is c.

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$7x^2 - 3x = 5$ is a quadratic equation. $7x^2 - 3x - 5 = 0$ is the required form to use the quadratic formula.

The coefficients for this equation are $a = 7$, $b = -3$, and $c = -5$.

To use the quadratic formula, substitute the coefficients (*i.e.*, a, b, c) into the formula.

$$x = \frac{-(-3) \pm \sqrt{9 - (4)(7)(-5)}}{(2)(7)}$$

Solve the equation. *Note* that a quadratic equation will *always* have 2 solutions. An equation involving a variable raised to any power n has n solutions. For example: $y = x^7$, $n = 7$. There are 7 solutions.

Trigonometry

An angle is measured *from the +x-axis, in a counterclockwise direction* unless specified otherwise.

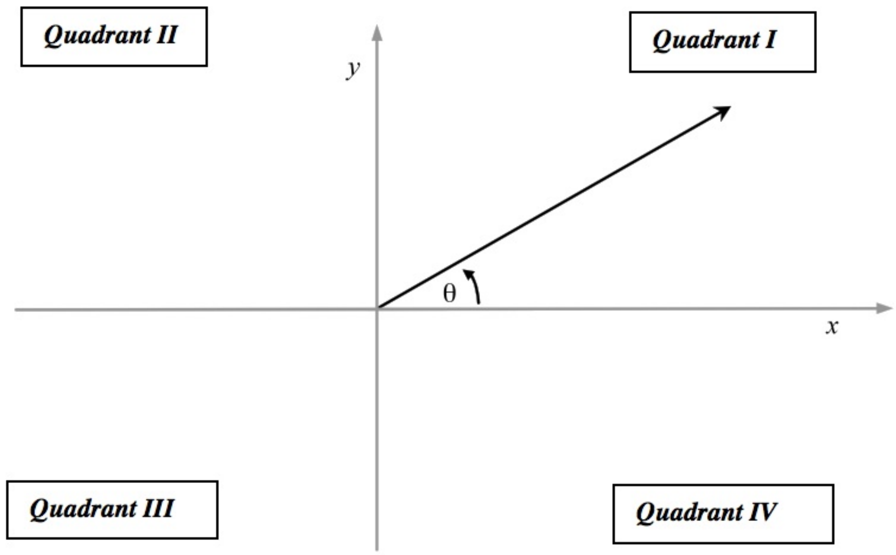
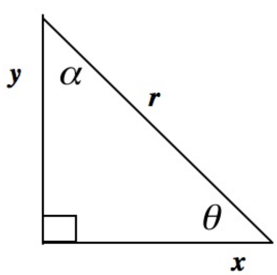


Figure A.2: Cartesian Graph

The sum of the angles of *any* triangle equals 180°. $180° = \pi$ (radians).

The *Pythagorean Theorem* states that $a^2 + b^2 = c^2$; we often use other variables: $x^2 + y^2 = r^2$ (refer to Fig. A.3).



$$\cos\theta = \frac{x}{r} \qquad \tan\theta = \frac{y}{x} \qquad \cos\alpha = \frac{y}{r} \qquad \tan\alpha = \frac{x}{y}$$

$$\sin\theta = \frac{y}{r} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right) \qquad \sin\alpha = \frac{x}{r} \qquad \alpha = \tan^{-1}\left(\frac{x}{y}\right)$$

Figure A.3: Right Triangle

Note that the $\cos\theta = \sin\alpha$, etc.

Note that to solve a trig function for an angle, you must take the inverse of the trig function.

When using a calculator to determine the inverse tangent of an angle, your calculator may give you an angle that is in the wrong quadrant [calculated angle will only be in quadrant I ($+\theta$) or quadrant IV ($-\theta$)]. To determine the correct quadrant, consider the ± sign of the argument: If x is negative, the angle *must* be in quadrant II or III; if y is negative, the angle *must* be in quadrant III or IV. If both x and y are negative, the angle *must* be in quadrant III (refer to Fig. A.2).

If the calculated angle is in the wrong quadrant, simply add 180° to the calculated angle.

If an angle is measured in radians, the arc length s equals the radius r times the angle θ (refer to Fig. A.4).

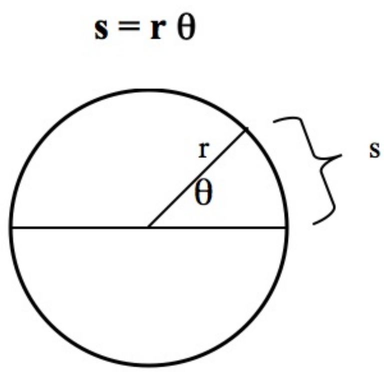


Figure A.4: Arc Length

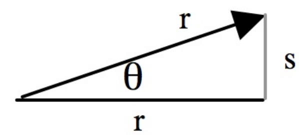


Figure A.5: Small Angle Approximation

If an angle is small and measured in radians: $\sin\theta \approx \tan\theta \approx \theta$ (refer to Fig. A.5).

Consider a circle of radius r (refer to Fig. A.6).
The circumference C of the circle is $C = 2\pi r$.
The area A of the circle is $A = \pi r^2$.

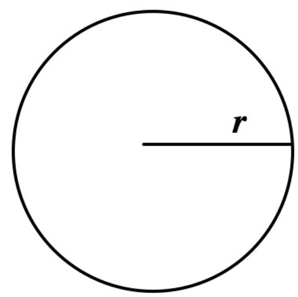


Figure A.6: Circle of radius r

Consider a right-circular cylinder of radius r and height h (refer to Fig. A.7).
The volume V of the cylinder is $V = \pi r^2 h$.

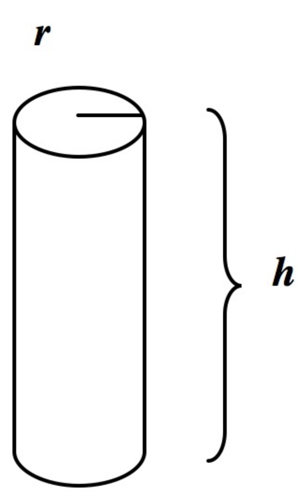


Figure A.7: Right-Circular Cylinder
radius r, height h