# Contents

1 Measurement & Analysis .................................................. 5  
2 Motion: Uniform and Non-Uniform ...................................... 11  
3 Vector Addition ............................................................. 15  
4 Projectile Motion ............................................................ 21  
5 Newton’s Second Law ....................................................... 27  
6 Coefficients of Friction .................................................... 33  
7 Conservation of Energy and Linear Momentum .......................... 39  
8 Torques and Rotational Motion ........................................... 45  
9 Moments of Inertia ........................................................... 49  
10 Archimedes’ Principle ...................................................... 53  
11 Simple Harmonic Motion .................................................. 57  
12 Speed of Sound in Air ...................................................... 61  
13 Electrostatics ................................................................. 65  
14 Electric Fields and Potentials .......................................... 71  
15 Ohm’s Law .................................................................... 77  
16 Series and Parallel Circuits .............................................. 83  
17 Kirchhoff’s Laws for Circuits .......................................... 89  
18 Earth’s Magnetic Field ..................................................... 95  
19 The Current Balance ....................................................... 101  
20 Exponentials and Oscilloscopes ....................................... 107  
21 Geometric Optics ........................................................... 117  
22 Thin Lenses ................................................................. 125  
23 Wave Optics ................................................................. 133  
A Measurement and Uncertainty ......................................... 139  
B Computers and Software ................................................. 149  
C Equipment ................................................................. 155  
D Math Review ............................................................... 167
# List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>Densities of Selected Materials</td>
<td>9</td>
</tr>
<tr>
<td>13.1</td>
<td>Electrostatic Series</td>
<td>67</td>
</tr>
<tr>
<td>14.1</td>
<td>Electric Quantities and Units</td>
<td>75</td>
</tr>
<tr>
<td>15.1</td>
<td>Resistor Color Code Values</td>
<td>78</td>
</tr>
<tr>
<td>15.2</td>
<td>Effect of Electric Current on the Body</td>
<td>81</td>
</tr>
<tr>
<td>19.1</td>
<td>Wire Loops Effective Lengths</td>
<td>104</td>
</tr>
</tbody>
</table>
Experiment 1: Measurement & Analysis

Figure 1.1: Measurement Materials

**EQUIPMENT**

1-Meter Stick  
Digital Balance  
500 g, 1 kg, 2 kg Stamped Masses  
Metal Cylinder  
(2) Vernier Calipers  
(10) Cylinders (Plastic, Wood)  
String
**Advance Reading**

*Text:* SI units for fundamental quantities, prefixes, significant figures, converting units, dimensional analysis.

**Objective**

Enhance measurement and graphing skills; introduce error analysis.

**Theory**

The fundamental quantities of mechanics are length, mass, and time. The SI units for these quantities are the meter (m), kilogram (kg), and second (s), respectively. All other mechanical quantities can be stated in terms of these quantities. For example, the unit of force in the SI system is the newton (N):

\[ 1 \text{ N} = 1 \text{ kg} \cdot \text{m/s}^2. \]

In terms of fundamental quantities, a newton is \([\text{mass} \cdot \text{length}]/[\text{time}^2]\).

In this experiment, you will use a variety of instruments to measure the fundamental quantities of length, mass, and time (see Fig. 1.1). You will then use these measurements to calculate other quantities and perform error analysis (refer to Appendix A).

---

**Experiment 1: Measurement & Analysis**

All measurements have uncertainty; they can only be so precise. The precision of the measuring instrument used determines the uncertainty of a measurement. The size of the smallest scale division (the resolution) of an instrument determines the uncertainty of measurements obtained with that instrument. Uncertainty, \(\delta\) (Greek letter delta), is either equal to the resolution or equal to \(\frac{1}{2}\) the resolution of the measuring instrument, depending on how the instrument is designed.

- **Digital Instrument:** \(\delta = \text{the resolution}\)
- **Analog Instrument:** \(\delta = \frac{1}{2}\) the resolution

Analog instruments, such as rulers, are read by eye and can be estimated within ± half of a “tick” mark. Digital instruments can be as imprecise as a full digit in either direction. A digital reading of “5.2 g” might be obtained for masses ranging between (5.100...1) g and (5.2999...) g, so their uncertainty is fully equal to the resolution of the instrument.

Measurements of time and measurements made with a vernier caliper are considered digital, due to the manner in which they are read.
Name: ________________________________

1. What are the fundamental quantities of mechanics, their symbols, their respective units, and their symbols? (25 pts)

2. State the symbol and value for the following prefixes: (5 pts ea.)

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>mega</td>
<td></td>
<td></td>
</tr>
<tr>
<td>kilo</td>
<td></td>
<td></td>
</tr>
<tr>
<td>centi</td>
<td></td>
<td></td>
</tr>
<tr>
<td>milli</td>
<td></td>
<td></td>
</tr>
<tr>
<td>micro</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. When comparing experimental values, what determines the correct equation to use? (Hint: Always read the Advance Reading material before you attempt the Prelab; this information is located in Appendix A.) (20 pts)

4. Convert the following: (10 pts ea.)

- 1.00 in. = __________ cm
- 6.10 in. = __________ cm
- 4 cm² = __________ m²
**PROCEDURE**

**PART 1: Estimating Length**

1. Close your eyes and hold your hands about 50 cm apart on the table. Have your partner measure the actual distance between your hands using a ruler.

2. Record this value in the table provided. What is the resolution of the ruler? Use this to determine the uncertainty in your measurement.

3. Repeat for 100 cm.

4. Have your lab partner estimate 50 cm and 100 cm in the same way.

5. Calculate your percent error for each estimate. How much did your measured value differ from what you expected? How large is that difference compared to the length you expected? (50 cm, 100 cm) Percent error can be calculated as follows: (Eq. A.1)

\[
\% \text{ error} = \frac{\text{measured} - \text{expected}}{\text{expected}} \times 100
\]

**PART 2: Estimating Mass**

6. Pick up several of the stamped masses on the lab table to get a sense of their weight.

7. Close your eyes and have your partner hand you one of the stamped masses. Estimate its weight in grams and record it in the table provided.

8. Estimate again as your partner hands you a second mass.

9. Give your partner two different masses to estimate in the same way.

**PART 3: Measuring Density**


11. Measure the mass of the metal cylinder provided. What is the resolution of the balance? Use this to determine the measurement uncertainty.

12. Measure the height and diameter of the metal cylinder using the vernier caliper.

13. Calculate the volume of the cylinder. This is equal to the area of the base times the height. Show your work.

14. What is the density, \( \rho \), of the metal cylinder?

15. Identify the metal in the cylinder using the density chart (Table 1.1, Page 9). Compare your value of density to the accepted value. What is your percent error?

**PART 4: Graphing**

16. Measure diameter and circumference of the 10 plastic and/or wood cylinders provided. Use a piece of string for measuring circumference.

17. Graph \( C \) vs. \( D \) on the computer. Open the Graphical Analysis software provided and enter your data in the columns on the left.

18. Fit the data with a linear regression line. Select from the drop-down menu bar: Analyze \( \Rightarrow \) Linear Fit

19. Determine the slope of the best-fit line from the information box provided on-screen.
Experiment 1: Measurement & Analysis

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g/cm³)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solids</strong></td>
<td></td>
</tr>
<tr>
<td><strong>Metal:</strong></td>
<td></td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.70</td>
</tr>
<tr>
<td>Stainless Steel</td>
<td>7.8</td>
</tr>
<tr>
<td>Brass</td>
<td>8.44 - 8.75</td>
</tr>
<tr>
<td>Bronze</td>
<td>8.74 - 8.89</td>
</tr>
<tr>
<td>Copper</td>
<td>8.96</td>
</tr>
<tr>
<td>Lead</td>
<td>11.3</td>
</tr>
<tr>
<td>Mercury</td>
<td>13.5336</td>
</tr>
<tr>
<td><strong>Rock:</strong></td>
<td></td>
</tr>
<tr>
<td>Granite</td>
<td>2.64 - 2.76</td>
</tr>
<tr>
<td>Slate</td>
<td>2.6 - 3.3</td>
</tr>
<tr>
<td>Diamond</td>
<td>3.51</td>
</tr>
<tr>
<td>Garnet</td>
<td>3.15 - 4.3</td>
</tr>
<tr>
<td>Corundum</td>
<td>3.9 - 4.0</td>
</tr>
<tr>
<td><strong>Wood:</strong></td>
<td></td>
</tr>
<tr>
<td>Pine (Yellow)</td>
<td>0.37 - 0.60</td>
</tr>
<tr>
<td>Oak</td>
<td>0.60 - 0.90</td>
</tr>
<tr>
<td>Ebony</td>
<td>1.11 - 1.33</td>
</tr>
<tr>
<td><strong>Misc.:</strong></td>
<td></td>
</tr>
<tr>
<td>Ice</td>
<td>0.917</td>
</tr>
<tr>
<td>Bone</td>
<td>1.7 - 2.0</td>
</tr>
<tr>
<td>Chalk</td>
<td>1.9 - 2.8</td>
</tr>
<tr>
<td>Glass (Lead)</td>
<td>3 - 4</td>
</tr>
<tr>
<td><strong>Fluids</strong></td>
<td></td>
</tr>
<tr>
<td>Atmosphere (STP)</td>
<td>0.001225</td>
</tr>
<tr>
<td>Water (20°C)</td>
<td>0.99821</td>
</tr>
<tr>
<td>Water (0°C)</td>
<td>0.99984</td>
</tr>
<tr>
<td>Mercury (20°C)</td>
<td>13.546</td>
</tr>
</tbody>
</table>

Table 1.1: Density of Selected Materials
Experiment 1: Measurement & Analysis
Experiment 2: Motion: Uniform and Non-Uniform

Figure 2.1: Experimental Setup

EQUIPMENT
Lab Pro Interface and USB Chord
Motion Detector
Power Supply
Basketball
**Advance Reading**

*Text:* Uniform motion, non-uniform motion, graphical representation of motion, position, displacement, distance, velocity, speed, acceleration, kinematic equations, slope of a line.

*Lab Manual:* Appendix B: COMPUTERS and SOFTWARE

**Objective**

To qualitatively analyze the kinematic equations by graphing a student, then objects, in motion; to enhance graph analysis and graphing skills.

**Theory**

*Kinematics* is the description of how objects move. An effective method of describing motion is to plot graphs of distance, velocity, and acceleration vs. time. Using a motion detector and computer software, your motion as you move around the room will be graphed. Qualitative analysis of these graphs will help you develop a better understanding of the concepts of kinematics and enhance your graph analysis skills.

Definition of terms:

**Distance** is the length of the path traveled.

**Displacement** is the change in position.

*Example:*
Your friend lives on the north side of Memphis, 100 miles from Ole Miss. You visit your friend and return to Ole Miss. Let’s assume towards Memphis is the positive direction of our coordinate system.

Your traveled a distance of 200 miles.
Your displacement from Ole Miss to Memphis was +100 miles.
Your displacement from Memphis to Ole Miss was −100 miles.
Your total displacement was 0 miles.

**Speed** is the distance traveled/total time traveled

**Velocity** is displacement/time

*Example:*
You drove for 2 hours to visit your friend and 2 hours on the return trip to Ole Miss.
Your average speed was 50 mph.
Your average velocity from Ole Miss to Memphis was +50 mph.
Your average velocity from Memphis to Ole Miss was −50 mph.
Your average velocity for the entire trip was 0 mph.

**Acceleration** is the change in velocity/change in time. It can be either positive or negative.

*Note* that linear graphs (straight lines) are analyzed with the *slope-intercept* equation of a line:

\[ y = mx + b \] (2.1)
Sketch Cartesian coordinate systems on the back of this sheet as needed. The origin should be (0,0); no scale is necessary. Sketch means you don’t need a ruler, etc., but a curve should not look like a line, or vice versa. All lines/curves should end with an arrowhead.

1. Sketch a \( d \ vs. \ t \) (distance vs. time) graph for each of the following situations: (25 pts)
   (a) An object at rest
   (b) An object moving in a positive direction with a constant speed
   (c) An object moving in a negative direction with a constant speed
   (d) An object that is accelerating in a positive direction, starting from rest

2. Sketch the \( v \ vs. \ t \) (velocity vs. time) graph for each of the situations described above. (25 pts)

3. What is the shape of a velocity vs. time graph for any object that has a constant acceleration? (20 pts)

4. Consider a ball thrown straight up. It goes up, then falls down. (Start by defining the coordinate system. Is up or down the positive \( y \) direction?) What is the acceleration of the ball on the way up? What is the acceleration when it reaches its highest point? Is the acceleration positive or negative? What is the acceleration of the ball on the way down? Is the acceleration positive or negative? (25 pts)

5. What are the objectives of this experiment? (5 pts)
**PROCEDURE**

**PART 1: Position vs. Time**

1. Turn on your computer; connect the Lab Pro and motion detector device.

2. Open the Logger Pro software.

3. Find and open the file title “01a Graph matching.”
   \[\text{Applications} \Rightarrow \text{Logger Pro 3} \Rightarrow \text{Experiments} \Rightarrow \text{Physics with Vernier} \Rightarrow \text{01a Graph Matching}\]

4. Stand in front of the motion detector, with plenty of room to walk in front of it. Push “Collect” to begin collecting data and move towards/away from the detector however you like. Watch your measured distance as it is recorded onscreen; try to adjust it with your motion. Remain at least 0.5 meters from the motion detector at all times.

5. Print this graph with your name on it; select File ⇒ Printing Options and enter your own name.

6. Have your partner create his or her own graph and print it.

**PART 2: Position vs. Time Graph Matching**

7. Open 01b and choose one partner to reproduce it. Begin collecting data and match the position graph as closely as possible with your own motion.

8. Print the graph with your name on it once you have achieved a satisfactory graph.

9. Have your lab partner match 01c similarly and print the result.

**PART 3: Velocity vs. Time Graph Matching**

10. Open the file 01d. Push “Collect” and match the graph with your own velocity as closely as possible. Print the resulting graph.

11. Similarly, have your lab partner attempt to match 01e and print it.

**PART 4: Bouncing Ball**

12. Open the file titled “02 Ball.”

13. Hold the motion detector at shoulder height, pointing down to capture the motion of the ball.

14. Drop the ball beneath it and record its motion through four full bounces.

15. Adjust the scale to show exactly four bounces on the screen.

---

**Figure 2.2:** Position and velocity graphs of a bouncing ball. Note that the shape of the bounce appears to be inverted.

16. On the velocity graph, click and drag to select a section that appears to be a straight line with a positive slope (i.e. the ball is in the air). Use the **Linear Fit** button to obtain a linear fit for this segment of the graph. Repeat for each bounce, for a total of four segments. Print a copy for each lab partner, labeled with both of your names.

17. Use the **Examine** and **Tangent** buttons to determine position, slope, and time on both the position and velocity graphs. Take these measurements at four locations: traveling upwards, near the top, traveling downwards, and near the bottom (indicated in Fig. 2.2).
Experiment 3: Vector Addition

![Figure 3.1: Force Table](image)

**EQUIPMENT**

- Force Table
- (4) Pulleys
- (4) Mass Hangers
- Masses
- Level (TA’s Table)
- (2) Protractors
- (2) Rulers
- (4) Colored Pencils (bold colors)
Advance Reading

Text: Motion in one and two dimensions, vectors and vector addition.

Objective

The objective of this lab is to add vectors using both the tail-to-head method and the component method and to verify the results using a force table.

Theory

A scalar quantity is a number that has only a magnitude. When scalar quantities are added together (e.g., prices), the result is a sum.

Vectors are quantities that have both magnitude and direction; specific methods of addition are required. When vector quantities are added, the result is a resultant.

For example, if you walk 1 mile north, then 1 mile east, you will walk a total distance of 2 miles (distance is a scalar quantity). Displacement, a vector, involves both distance and direction. So the same 2 mile walk results in a displacement of \( \sqrt{2} \) miles northeast of where you began (\( \approx 1.41 \) miles, northeast of your starting position).

A negative vector has the same length as the corresponding positive vector, but with the opposite direction. Making a vector negative can be accomplished either by changing the sign of the magnitude or by simply adjusting the direction by 180°.

\[ \vec{V} = 5 \text{ N} 100^\circ \]
\[ -\vec{V} = -5 \text{ N} 100^\circ \]
\[ \text{or} \]
\[ -\vec{V} = 5 \text{ N} 280^\circ \]

Tail-to-Head Method

Vectors can be added together graphically by drawing them end-to-end. A vector can be moved to any location; so long as its magnitude and orientation are not changed, it remains the same vector. When adding vectors, the order in which the vectors are added does not change the resultant.

- Draw each vector on a coordinate system; begin each from the origin.
- Choose any vector drawn to be the first vector.
- Choose a second vector and redraw it, beginning from the end of the first.
- Repeat, adding as many vectors as are desired to the end of the “train” of vectors.
- The resultant is a vector that begins at the origin and ends at the tip of the last vector drawn. It is the shortest distance between the beginning and the end of the path created.

The tail-to-head method is often useful when working problems. A quick sketch, rather than measurements, can help verify your solutions.
Experiment 3: Vector Addition

Component Method

To add vectors by components, calculate how far each vector extends in each dimension. The lengths of the $x$- and $y$-components of a vector depend on the length of the vector and the sine or cosine of its direction, $\theta$:

$$\sin \theta = \frac{F_{iy}}{F_1} \quad \cos \theta = \frac{F_{ix}}{F_1}$$

Use algebra to solve for each component, $F_{ix}$ and $F_{iy}$, from these equations.

$$F_{ix} = |\vec{F}_1| \cos \theta \quad (3.1)$$

$$F_{iy} = |\vec{F}_1| \sin \theta \quad (3.2)$$

$$\theta = \tan^{-1}\left(\frac{F_{iy}}{F_{ix}}\right) \quad (3.3)$$

![Figure 3.3](image.png)

The components of $\vec{R}$ can be converted back into polar form $(R, \theta)$ using the Pythagorean theorem (Eq. 3.6) and the tangent function (Eq. 3.3):

$$|\vec{R}| = R = \sqrt{R_x^2 + R_y^2} \quad (3.6)$$

$$\theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) \quad (3.7)$$

Note: Verify the quadrant! A calculator will return only one of two possible angles (Eq. 3.7). To verify the quadrant, determine if $R_x$, $R_y$ are positive or negative. If your calculation puts the resultant in quadrant I, but $R_x$ and $R_y$ are both negative, it must be in quadrant III; simply add 180° to the angle.

Force Table Verification

We will use a force table to verify our results of vector addition and gain a hands-on perspective. The force table is a circular steel disc with angles 0° to 360° inscribed on the edge (refer to Figure 3.1).

As noted above, when adding vectors, a resultant vector is determined. To balance the force table, however, a force that is equal in magnitude but opposite in direction must be used. This force is the equilibrant, $\vec{E}$. $\vec{E} = -\vec{R}$.

For example, when a 10.0 N force at 0° and a 10.0 N force at 90° are added, the resultant force has a magnitude of 14.1 N at 45°. The equilibrant force has the same magnitude, but the direction is $180° + 45° = 225°$. The equilibrant must be used to balance the two 10.0 N forces.

When each vector is broken into components, add the $x$-components of each vector:

$$\sum_{i=1}^{n} F_{ix} = R_x \quad (3.4)$$

Then add all of the $y$-components:

$$\sum_{i=1}^{n} F_{iy} = R_y \quad (3.5)$$

The sums are the $x$- and $y$-components of the resultant vector, $\vec{R}$. 

![Image](image.png)
Prelab 3: Vector Addition

Name: ____________________________________________

1. What is a vector? (10 pts)

2. Name a vector quantity and its magnitude. (10 pts)

3. What is a scalar quantity? Give two examples. (10 pts)

4. What is the *equilibrant*? (10 pts)

For Questions 5, 6, and 7, use the following values:

\[ \vec{A} = 5.0 \text{ N at } 135.0^\circ \quad \vec{B} = 6.0 \text{ N at } 270.0^\circ \]

If you get the same answer for Questions 5 and 6, ask for help!

5. Using the component method, add vectors \( \vec{A} \) and \( \vec{B} \) \((i.e., \vec{R} = \vec{A} + \vec{B})\). (20 pts)

6. Using the component method, add vectors \( \vec{A} \) and \(-\vec{B} \) \((i.e., \vec{R} = \vec{A} - \vec{B})\). (20 pts)

7. Using the Tail-to-Head method, add vectors \( \vec{A} \) and \( \vec{B} \) \((\vec{R} = \vec{A} + \vec{B})\), on the back of this sheet; rulers and protractors will be provided in the physics building. Let 2.0 cm = 1.0 N. (20 pts)
**Experiment 3: Vector Addition**

**PROCEDURE**

**PART 1: Tail-to-Head Method**

1. Your TA will provide you with a set of three force vectors, record them in the table to the right. Let 1.00 N = 2.00 cm on graph paper.

2. Using a ruler and a protractor, draw the three vectors on the graph paper provided, starting each one from the origin. You should place the origin near the center of the page. Use different colored pencils for each vector.

3. Label the three force vectors.

4. Add together vectors $\vec{A}$ and $\vec{B}$ graphically: Draw vector $\vec{B}$ again, beginning from the tip (or “head”) of vector $\vec{A}$.

5. Next, add the third vector, $\vec{C}$, to the first two: Redraw it, beginning from the tip of $\vec{B}$.

6. Draw the resultant, $\vec{R}$, from the origin to the tip of the last vector drawn, $\vec{C}$. When the three forces $\vec{A}$, $\vec{B}$, and $\vec{C}$ act together, they behave as though they were only one force, $\vec{R}$.

7. Measure and record $\vec{R}$. Include uncertainty in your measurement of $|\vec{R}|$ and $\theta$.

8. Record the equilibrant that balances the three forces. Note that this has the same magnitude as $\vec{R}$.

**PART 2: Force Table**

9. Use the level to level the force table.

10. Set three pulleys on the force table in the magnitude and direction of $\vec{A}$, $\vec{B}$, and $\vec{C}$. Note: the mass hanger has its own mass. Let 1.00 N = 100 g on the force table.

11. Add a fourth vector to equalize the forces. This equilibrant force can have any magnitude and direction; you may use your equilibrant from Step 8 as a guide.

12. Have your TA pull the pin from your force table to see if $\Sigma F = 0$.

13. If the forces are unbalanced, adjust the magnitude and direction of the equilibrant. When they are balanced, the fourth vector is your experimental equilibrant.

14. Calculate your accuracy against the theoretical equilibrant. Show your work.

**PART 3: Vector Subtraction**

15. Vectors $\vec{D}$ and $\vec{E}$ are given by your TA. Record them in the table provided.

16. Calculate the resultant, $\vec{D} - \vec{E}$. It may help you to draw a sketch.

17. Find the equilibrant.

**QUESTIONS**

1. Compare the two methods for $\vec{R}$ ($R$ and $\theta$) for each set of vectors.

2. Consider six vectors that are added tail-to-head, ending up where they started from. What is the magnitude of $\vec{R}$?
Experiment 4: Projectile Motion

**EQUIPMENT**

- Pasco Ballistic Pendulum (Spring Gun)
- 2-Meter Stick
- Meter Stick
- Ruler
- Plumb Bob
- Carbon Paper
- Target Paper
- Launch Platform & C-clamps
- Wall Guards

Figure 4.1: Ballistic Pendulum (Spring Gun)
Advance Reading

Text: Motion in two dimensions (2-D), projectile motion, kinematic equations.

Lab Manual: Appendix A, Appendix D.

Objective

To measure the initial velocity of a projectile when fired from a spring gun and to predict the landing point when the projectile is fired at a non-zero angle of elevation.

Theory

Projectile motion is an example of motion with constant acceleration when air resistance is ignored. An object becomes a projectile at the very instant it is released (fired, kicked) and is influenced only by gravity.

The \(x\)- and \(y\)-components of a projectile’s motion are independent, connected only by time of flight, \(t\). Consider two objects at the same initial elevation. One object is launched at an angle \(\theta = 0^\circ\) at the same moment the second object is dropped. The two objects will land at the same time. This allows the two dimensions to be considered separately.

To predict where a projectile will land, one must know the object’s starting position, \(\vec{r}_0\), initial velocity, \(\vec{v}_0\), and the acceleration it experiences, \(\vec{a}\). Position as a function of time is then described as:

\[
\vec{r}(t) = \vec{r}_0 + \vec{v}_0 t + \frac{1}{2} \vec{a} t^2
\]

(4.1)

Eq. 4.1 is a vector equation; it can be resolved into \(x\)- and \(y\)-components:

\[
x = x_0 + v_{0x} t + \frac{1}{2} a_x t^2
\]

(4.2)

\[
y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2
\]

(4.3)

Velocity changes constantly in projectile motion. While horizontal acceleration is zero for the purposes of this experiment, the vertical component of a projectile’s velocity can be described as follows, with \(a_y = g\):

\[
v_y = v_{0y} + a_y t
\]

(4.4)

These are the kinematic equations for constant acceleration. Taken together, they describe the motion of projectiles and other constant-acceleration systems.

Part 2 of this experiment involves finding the components (\(v_{0x}\) and \(v_{0y}\)) of the velocity of a projectile fired at above the horizontal (\(\theta > 0^\circ\)). To determine the components of an initial velocity vector, refer to Eq. 3.1 and Eq. 3.2 of Experiment 3: Vector Addition.

When measuring the change in position for the projectile in this experiment, measure from the bottom of the ball to the floor for \(\Delta y\), and measure the end of the rod from which it is fired to where it lands for \(\Delta x\).

The ballistic pendulum (spring gun) should be held firmly in place when fired to prevent loss of momentum.
1. What is projectile motion? (15 pts)

2. Find the initial velocity, \( v_0 \), of a ball rolling off the table in the figure below. The launch position is the origin of the coordinate system, positive directions as specified. (25 pts)

![Diagram showing projectile motion](image)

3. For a ball shot with an initial speed of 8.0 m/s at \( \theta_0 = 30^\circ \), find \( v_{0x} \) and \( v_{0y} \). Always write the algebraic equations first, then write the equations with values inserted. (20 pts)

4. Given the information in Question 3, \( y = -0.8 \) m, use the quadratic formula to solve for \( t_1 \) and \( t_2 \). Note: It is unlikely that you will finish the experiment if you are not able to solve this type of quadratic equation. (20 pts)

\[
y = y_0 + v_{0y} t + \frac{1}{2} a_y t^2
\]

(Continued on back page)
On May 5, 1961, only 23 days after Yuri Gagarin of the then-Soviet Union became the first person in space, NASA astronaut Alan Shepard launched at 9:34 a.m. EDT aboard his Freedom 7 capsule, powered by a Redstone booster, to become the first American in space. His historic flight lasted 15 minutes, 28 seconds. Image Credit: NASA

6. Calculate the initial velocity in m/s, ignoring air resistance. Show work. (20 pts)
Experiment 4: Projectile Motion

PROCEDURE

PART 1: Horizontal Launch ($\theta_0 = 0^\circ$)

1. Align the back edge of the spring gun with the back edge of the table. The initial position of the projectile is defined to be the origin of your coordinate system, as shown above in Fig. 4.3. Measure $\Delta y$.

2. Calculate the time of flight, $t$.

3. Read this step and the next step before proceeding. When the spring gun is fired for the first time, you will need to note where the ball lands. This is the location for your target. The target is a sheet of white paper taped to the floor with a sheet of carbon paper placed on top. Do not tape the carbon paper.

4. Place the wastebasket against the wall where you expect the ball to hit. Do not allow the ball to hit the wall. When the flight path is secure, cock the spring gun to the third detent, then fire to determine the target location. Place the target.

5. Place the wastebasket against the wall, in the line of fire; do not allow the ball to hit the bare wall. Fire the spring gun once and measure $x$.

6. Determine $v_0$.

7. Record your value for $v_0$ in the table.

8. Fire the spring gun 5 more times. Measure and record $x_i$ for each trial. Remember that $x_i$ means $x_1$, $x_2$, $x_3$, etc.

9. Calculate the average displacement, $x_{avg}$, for all 6 trials.

10. Use $x_{avg}$ to calculate $v_{0-avg}$.

11. Record your value for $v_{0-avg}$ in the table.
PART 2: Inclined Launch ($\theta_0 > 0^\circ$)

12. Elevate the front of your spring gun by removing the launcher and reattaching it to the vertical mount slots at an angle. Be certain that the back edge of the spring gun is aligned with the back edge of the table. Adjust the wastebasket at the wall if necessary.

13. Measure the angle by holding the string of the plumb bob at the corner of the quarter sphere on the launcher and reading the angle where the string crosses the outer edge. Remeasure $y_0$ to the lowest point of the ball outline at the launch position diagram on the launcher. Record these values in your table.

14. Use the kinematic equations and the quadratic formula to calculate the displacement of the ball when fired from this angle and height. Note that $v_0_y$ is not equal to 0.0 m/s.

15. Place your target at the predicted location. Fire the spring gun a total of six times.

16. Determine $x_{avg}$.

17. Compare the theoretical and experimental values of $x$ for Part 2. If the values are substantially different, check your calculations or measurements.

QUESTIONS

1. If a ball has twice the mass but the same initial velocity, what effect would this have on its displacement (neglect air resistance)?

2. Consider the following statement: When a rifle is fired horizontally, the bullet leaves the barrel and doesn’t drop at all for the first 75 meters of flight. Is this statement true?

3. What is the acceleration of a projectile fired vertically upwards? Is it positive or negative? Sketch your coordinate system.

4. What is the acceleration of a projectile fired vertically downwards? Is it positive or negative? Use the same coordinate system you used for Question 3.

5. Determine the standard deviation, $\sigma$, for $x_i$ for Part 1. (Refer to Appendix A)

6. Determine $\sigma$ for $x_i$ for Part 2.
Experiment 5: Newton’s Second Law

**EQUIPMENT**

- Low-Friction Cart
- Pulley and String
- Triple-Beam Balance
- Digital Balance
- Stopwatch
- Meter Stick
- Mass Hanger
- (1) 10 g Mass
- (2) 20 g Masses
- (1) 50 g Mass
- Paper Clips (small masses)
- Plumb Bob
- Wastebasket

---

Figure 5.1: Modified Atwood’s Machine Setup

Figure 5.2: Note: String from cart to pulley must be horizontal and aligned with the pulley.
**Experiment 5: Newton’s Second Law**

**FREE-BODY DIAGRAM SOLUTION METHOD: INSTRUCTIONS**

**Step 1:** Sketch the problem/situation and specify the coordinate system for each object in your system.

**Step 2:** Draw all forces (arrows that represent these vectors) acting on each object in the system you are investigating. All forces should extend away from the object in the direction of the force. Remember that the length of the arrow is an indication of the magnitude of the force.

![Diagram](image)

**Step 3:** Write Newton’s 2nd Law ($\sum \vec{F} = m\vec{a}$) in component form ($\Sigma F_x = ma_x$ and $\Sigma F_y = ma_y$) for each object in the system. For this example, $m_A$ and $m_B$:

$m_A$: $\Sigma F_{Ax} = m_Aa_{Ax}$

$\Sigma F_{Ay} = m_Aa_{Ay}$

$m_B$: $\Sigma F_{Bx} = m_Ba_{Bx}$

$\Sigma F_{By} = m_Ba_{By}$

**Step 4:**

Refer to your diagram to sum the forces for each object as instructed by Sir Isaac Newton. For this example:

$\Sigma F_{Ax} = m_Aa_{Ax} = T - F_f$

$\Sigma F_{Ay} = m_Aa_{Ay} = F_N - m_Ag = 0.0 \text{ N}$

($a_{Ay}$: Constant velocity means this acceleration equals 0.0 m/s$^2$)

$\Sigma F_{Bx} = m_Ba_{Bx} = 0.0 \text{ N}$

($m_B$: No forces acting in the horizontal direction.)

$\Sigma F_{By} = m_Ba_{By} = T - mBg$

(Note that since $m_A$ and $m_B$ are connected and moving together, $T$ is the same for each object, and $a_{Ax} = -a_{By}$.)

**Step 5:**

Write the known quantities. Write the question.
Always write equations algebraically first, then insert the known values.
Now do the math! That’s all there is to it.

Figure 5.3: Free-Body Diagram of Modified Atwood’s Machine (Example for $\vec{v}$ constant, $\Sigma \vec{F} = m\vec{a} = 0.0 \text{ N}$).
**Experiment 5: Newton’s Second Law**

**Advance Reading**

*Text:* Newton’s Second Law, Atwood’s Machine, acceleration, velocity, displacement, vectors.

*Lab Manual: Appendix B*

**Objective**

The objective of this lab is to explore and analyze the relationship between force, mass, and acceleration.

**Theory**

According to Newton’s Second Law, the acceleration, $\vec{a}$, of a body is directly proportional to the vector sum of the forces, $\Sigma \vec{F}$, applied to the body:

$$\Sigma \vec{F} = m \vec{a} \quad (5.1)$$

where $m$ is the mass of the body.

The experimental configuration for this experiment is a variation of Atwood’s machine (Fig. 5.2, Fig. 5.3).

A force $T$ (tension) will be applied to the cart, $m_A$, by means of a string with an attached mass, $m_B$. If one can ignore the force of friction acting on the cart, then Eq. 5.1 in the direction of motion simplifies to:

$$\Sigma F_A_x = m_A a_{A_x} = T \quad (5.2)$$

Thus, if the mass of the cart is doubled while $T$ is held constant, the acceleration of the cart is halved (Part 1). Correspondingly, if $T$ is doubled while $m_A$ is held constant, the acceleration of the cart is doubled (Part 2).

This analysis assumes a frictionless environment. For simplicity, $F_f$ will be counterbalanced by a small mass, $m_f$, hanged from one end of the system. When the weight of $m_f$ is equal to the force of friction ($m_f g = F_f$), the system will be in equilibrium.

$$\Sigma F = 0 \text{ N} \quad a = 0 \text{ m/s}^2$$

In equilibrium, the cart set in motion continues moving with constant velocity. A new mass $m_f$ must be found each time the cart’s mass is adjusted in Part 1, as $F_f$ will have changed.

Once $F_f$ has been counteracted, any additional mass will be directly related to the acceleration of the system as in Eq. 5.1. The added mass, $m_B$, exerts a force equal to its weight on the cart/mass system.

When the cart is accelerating, deriving $T$ and $a$ of the cart is more involved. The relevant equations for this experiment are provided:

$$a = \frac{m_B \cdot g}{m_A + m_B + m_f} \quad (5.3)$$

$$T = \frac{m_A \cdot m_B \cdot g}{m_A + m_B + m_f} + m_f \cdot g \quad (5.4)$$

We will compare the acceleration from Eq. 5.3 to the acceleration obtained from the **kinematic equations for constant acceleration** (Page 22, Experiment 4).

In this experiment, acceleration will be found experimentally tracking the cart’s motion across a set distance. The time of travel will be carefully measured using a stopwatch.

Note that since the cart and $m_B$ are connected, their acceleration, velocity, and distance traveled are equal at all times. Thus, the horizontal distance the cart travels, $\Delta x$, is equal to the vertical drop of the attached mass, $\Delta y$.

$$x = x_0 + v_0 \cdot t + \frac{1}{2} a_x t^2 \quad (5.5)$$

We now define calculated and measured accelerations as $a_{\text{Theo.}}$ and $a_{\text{Meas.}}$:

- $a_{\text{Theo.}}$ is determined by Eq. 5.3;
- $a_{\text{Meas.}}$ is determined by Eq. 5.5.
1. State Newton’s First and Second Laws (qualitative explanations). (20 pts)

2. How is tension applied to the cart in this experiment? (15 pts)

3. How is friction compensated for in this experiment? (15 pts)

4. Should the acceleration determined in Step 5 (or any part of this experiment) be greater than, less than, or equal to the acceleration of gravity? Explain. Assume friction is negligible; refer to Eq. 5.3. (20 pts)

5. Complete the free-body diagrams for the two situations shown below. Draw to scale (i.e., your diagrams should delineate between $\Sigma \vec{F} = m\vec{a} = 0.0 \text{ N}$ and $\Sigma \vec{F} = m\vec{a} > 0.0 \text{ N}$). (Refer to the lab manual Theory) (30 pts)

\begin{align*}
\text{Situation 1:} \quad &\Sigma \vec{F} = m\vec{a} = 0.0 \text{ N} \\
\text{Situation 2:} \quad &\Sigma \vec{F} = m\vec{a} > 0.0 \text{ N}
\end{align*}
**PROCEDURE**

**PART 1: Vary the Mass of the Cart, \( m_a \)**

Trial #1
1. Measure the mass of the cart, \( m_A \), using the triple beam balance. Record it in the table provided.
2. Measure the frictional force acting on the cart: Add small masses to the string until the cart maintains a constant velocity when tapped. The weight of \( m_f \) is equal to \( F_f \). Record \( m_f \) in the table provided.
3. Attach \( m_B \) (0.05 kg) to the end of the string. [Measure its actual mass and record it]
4. Use a stopwatch to measure the time it takes for the cart to travel a distance \( \Delta x \) (or for \( m_B \) to fall the same distance \( \Delta y \)). Your data will be more accurate if \( \Delta x \) is as large as possible. Do not allow the cart to hit the pulley.
5. Determine theoretical and measured acceleration for the cart.

Trial #2
6. Remove \( m_B \) from the system. Add 0.5 kg to the cart and measure its total mass.
7. Determine the new force of friction on the cart and adjust \( m_f \).
8. Replace \( m_B \) and measure time and distance of the cart’s motion.
9. Determine acceleration for this heavier cart.

Trial #3
10. Remove \( m_B \) and add 0.5 kg to the cart. Measure the force of friction and adjust \( m_f \) for this case.
11. Replace \( m_B \) and determine acceleration for this case.

**PART 2: Vary the Applied Force**

12. Keep the weight of the cart (approx. 1.5 kg) and keep the same \( m_f \) as in the previous step.
13. Trial #1:
    Increase \( m_B \) to 0.07 kg, find \( a_{\text{Theo.}} \) and \( a_{\text{Meas.}} \).
14. Trial #2:
    Increase \( m_B \) to 0.09 kg, find \( a_{\text{Theo.}} \) and \( a_{\text{Meas.}} \).
15. Trial #3:
    Increase \( m_B \) to 0.11 kg, find \( a_{\text{Theo.}} \) and \( a_{\text{Meas.}} \).

**PART 3: Graphing**

The entire cart/hanging mass system follows the same law, \( \Sigma F = ma \). This means that plotting force vs. acceleration yields a linear relationship (of the form \( y = mx \)).

16. Open Graphical Analysis. Graph force \( (m_B g) \) vs. acceleration \( (a_{\text{Meas.}}) \) for the Varying Force trials. Apply a linear fit to the three data points. Print this graph.
17. Create a similar graph of \( m_B g \) vs. \( a_{\text{Theo.}} \) and print it.

**QUESTIONS**

**Reminder: Use the method specified on Page 28 for Question 2**

1. Assume a modified Atwood’s Machine arrangement similar to today’s experiment and a frictionless cart that continues moving after the hanging mass has reached the floor, thus no longer exerting a force on the cart. Qualitatively sketch \( v \) vs. \( t \) for this arrangement, starting from rest at \( t = 0.0 \) s, and include times after the hanging mass has reached the floor (long table!).

2. Draw a free-body diagram for the cart/hanging mass system shown in Fig. 5.2, ignoring friction. Use this diagram to derive an equation of force, \( T \), that has only masses and acceleration due to gravity, \( g \) (i.e., an equation similar to Eq. 5.4).
Experiment 6: Coefficients of Friction

Figure 6.1: Inclined Plane

**EQUIPMENT**

Inclined Plane
Wood Block
Triple-Beam Balance
Digital Balance
Lab Pro and Connections
Dual-Range Force Sensor
Masses
Mass Hanger
10 cm length of string
**Advance Reading**

**Text:** Newton’s Laws, maximum static friction, kinetic friction, coefficients of friction.

**Lab Manual:** Appendix B (Logger Pro)

**Objective**

To measure and analyze the coefficients of friction $\mu_s$ and $\mu_k$ between a wood block and wood plane.

**Theory**

Friction is the force that resists the relative motion of one surface in contact with another surface. We consider two types of friction: static and kinetic. Usually, kinetic friction is less than the maximum value of static friction.

The maximum static friction is given by:

$$F_{f_{\text{max}}} = \mu_s F_N$$  \hfill (6.1)

and the kinetic friction is given by:

$$F_{f_k} = \mu_k F_N$$  \hfill (6.2)

where $\mu_s$ is the coefficient of static friction, $\mu_k$ is the coefficient of kinetic friction, and $F_N$ is the normal force.

The angle of repose is defined as the maximum angle at which an object on an inclined plane will retain its position without tending to slide. It can be shown that the tangent of this angle equals $\mu_s$:

$$\tan \theta = \mu_s$$  \hfill (6.3)

Similarly, it can also be shown that when an object slides down an incline at constant velocity:

$$\tan \theta = \mu_k$$  \hfill (6.4)

In this experiment, the frictional force between a wooden block and the wooden surface of a horizontal and inclined plane will be derived and measured. By graphing these data, coefficients of static and kinetic friction will be obtained.

As you perform this experiment, theoretical quantities will be determined prior to measuring for Part 2 through Part 5. These calculations will require the free-body diagram solution method (refer to Page 28).

**Experiment 6: Coefficients of Friction**

When $a = 0.0 \text{ m/s}^2$, the force probe measures the force necessary to counteract friction and thus is equal to $F_f$.

If the block is pulled at constant velocity, starting from rest, there is a “bump” at the beginning of the graph, and the remaining graph is, on average, horizontal.

The bump at the beginning of the graph is a result of overcoming the maximum static friction, $F_{f_{\text{max}}}$, which is usually greater than kinetic friction, $F_{f_k}$. The maximum value of this bump allows us to determine $\mu_s$. The horizontal portion of the graph, $F_{f_{\text{avg}}}$, allows us to determine $\mu_k$. A sketch of how your graph should look is shown in Fig. 6.2. Note that the force begins at zero newtons, which will require you to leave slack in the string until data is being collected.

![Figure 6.2: Sample Force vs. Time graph](image-url)
1. Define the angle of repose. (20 pts)

2. Using the free-body diagram solution method (Page 28), derive the equation for $T$ for a block of mass $m$ being pulled horizontally ($\theta = 0^\circ$) at a constant velocity. (Draw the forces to scale and include friction.) Note: It is unlikely that you will be able to finish the experiment if you are not able to solve force problems in a timely manner; refer to Prelab Questions 2, 3, and 4. (30 pts)

3. Using the free-body diagram solution method, derive the equation for $T$ for a block of mass $m$ being pulled up an incline ($\theta > 0^\circ$) at a constant velocity (tension parallel to the plane). Set the coordinate system such that the $x$-axis is parallel to the incline. (Draw the forces to scale and include friction.) (30 pts)

4. Using the free-body diagram solution method, calculate $T$ for a block of mass $m = 150$ g being pulled up a $15^\circ$ incline at constant velocity. The coefficient of kinetic friction, $\mu_k$, is 0.3. (20 pts)
**Experiment 6: Coefficients of Friction**

13. A few trials using small and large diameter masses prior to collecting data will yield more accurate results.

14. You will need to determine for each trial $f_{max}$, $f_k$, and $F_N$ (details follow):
   - To determine the graph value of $f_{max}$, use the Examine button, then position the cursor at the highest peak.
   - To determine $f_k$, click-and-drag the mouse to select the constant force section of the graph, then use the Stats button.
   - To determine $F_N$, refer to your free-body diagram solutions.

15. Measure all masses!

16. Define $g = (9.80 \pm 0.01)$ m/s$^2$

17. Measure the mass $m = (\text{block} + 0.5 \text{ kg})$.

18. Attach the string from the block to the force sensor. Click Collect. Pull $m$ (block + 0.5 kg) across the plane at a constant velocity.

19. Determine $f_{max}$, $f_k$, and $F_N$.

20. Increase $m$ for each trial, in increments of 0.5 kg, until a total of 2.5 kg has been added.

**PART 3: Block on an Inclined Plane: $\theta = 30^\circ$**

21. Derive $T$ (algebraically) for an object of mass $m$ being pulled up an inclined (angle $\theta$) at constant velocity (quantities as before).

22. Use the force probe to measure $T$ for this situation ($\theta = 30^\circ$, $m = (\text{block} + 0.5 \text{ kg})$)

**PART 4: Coefficient of Static Friction**

23. Measure the angle of repose by slowly raising the inclined plane until the (block + 0.5 kg) just begins to slide.

24. Calculate $\mu_s$ from the measured angle (Eq. 6.3).

**PART 5: Coefficient of Kinetic Friction**

25. Measure the angle that the (block + 0.5 kg), when tapped, slides without acceleration.
Experiment 6: Coefficients of Friction

26. It can be shown that when an object slides without acceleration after being tapped, then $\tan \theta = \mu_k$. Calculate $\mu_k$ from the measured angle.

PART 6: Graphing

27. Plot $f_{max}$ vs. $F_N$ and $f_k$ vs. $F_N$ on the same graph.
28. What do the slopes represent?

PART 7: Analysis

29. Derive the angle of repose equation for an object of mass $m$ (Eq. 6.3).
30. It can be shown that when an object is lightly tapped, then slides without acceleration, $\mu_k = \tan \theta$. Derive this equation.
31. Calculate $T$ from Part 3 using your graph value of $\mu_k$.
32. Compare $\mu_s$ values obtained from Part 4 and Part 6.
33. Compare $\mu_k$ values obtained from Part 5 and Part 6.

QUESTIONS

1. Show that $\mu_s = \tan \theta$ for the angle of repose. Refer to Page 28.
2. If the mass of the block is tripled, does the angle of repose change?
3. Why was it necessary to tap the block to get it started in Part 5?
4. Why can anti-lock brakes stop a car in a shorter distance than regular brakes? (Comment on the difference between static and kinetic friction.)
5. Compare the graph and experimental values for the coefficients of friction. Is $\mu_s > \mu_k$ for each method?
Experiment 6: Coefficients of Friction
Experiment 7: Conservation of Energy and Linear Momentum

Figure 7.1: Ballistic Pendulum and Equipment

EQUIPMENT
Ballistic Pendulum
30 cm Ruler
Digital Balance
Triple-Beam Balance
Advance Reading


Objective

To determine the velocity of a ball as it leaves the ballistic pendulum using conservation of linear momentum and conservation of energy considerations; to perform error analysis.

Theory

Energy is always conserved. There are different forms of energy, and one form of energy may be transformed into another form of energy. Mechanical energy, a form of energy being investigated today, is not always conserved.

The mechanical energies being investigated today are kinetic energy, KE, and gravitational potential energy, PE grav:

\[ KE = \frac{1}{2}mv^2 \]  
where a mass, \( m \), has energy due to its speed, \( v \), and

\[ PE_{grav} = mgh \]  
where a mass \( m \) has energy as a result of its position (height, \( h \)), and \( g \) is acceleration due to gravity.

Linear momentum, \( \vec{p} \), is always conserved in an isolated system. An isolated system is a system in which all forces acting on the system are considered.

Linear momentum \( p \) is given by:

\[ \vec{p} = mv \]  
where a mass, \( m \), has a velocity, \( v \). There are three distinct categories of collisions: elastic, inelastic, and completely inelastic.

Elastic collisions result in conservation of both linear momentum and mechanical energy. Billiard balls are often used as examples when discussing elastic collisions.

Inelastic collisions result in deformation of one or more of the objects involved in the collision. Although linear momentum is conserved, mechanical energy is not. Car wrecks are examples of inelastic collisions.

Completely inelastic collisions refer to collisions that result in objects becoming attached to each other after the collision (i.e., stuck together). The objects thus move, after collision, with the same velocity. These collisions, like inelastic collisions, conserve linear momentum but not mechanical energy.

This experiment investigates completely inelastic collisions to determine the initial velocity of the ball. When the ballistic pendulum is fired, the ball is caught and held by the catcher; the two objects, ball and catcher, move together as one object after collision. We assume that the linear momentum of the system before and after the collision is conserved, and that no energy is lost during the ball’s flight.

First Process:

Conservation of linear momentum between state 1 and state 2 (just before and just after a collision) is:

\[ \vec{p}_1 = m \vec{v}_1 = \vec{p}_2 = m \vec{V}_2 \]  

Relevant to this experiment, we consider a collision between a ball of mass \( m \) and a catcher of mass \( M \). Before the collision, state 1, the velocity of the catcher is 0.0 m/s. Just after the collision, state 2, the velocity of the ball and the catcher are equal.

\[ \vec{p}_1 = m \vec{v}_1 = \vec{p}_2 = (m + M) \vec{V}_2 \]  

Second Process:

While mechanical energy is not conserved during an inelastic collision. However, after the collision, the ball-catcher system has KE due to its motion. We assume that mechanical energy is conserved (i.e., ignore rotational energy and energy losses due to friction). KE is transformed into \( PE_{grav} \) as the pendulum arm swings up to a height \( h \).

The initial mechanical energy is all \( KE \), as the pendulum arm is at the lowest possible position. The final energy is all \( PE_{grav} \) when the pendulum rises and stops, state 3. Therefore, conservation of mechanical energy is:

\[ KE = \frac{1}{2} (m + M) V_f^2 = PE_{grav} = (m + M) gh \]  

When a pendulum rotates \( \theta^\circ \), the center of mass, \( cm \), rises an amount \( h = h_f - h_i \). Refer to Fig. 7.2.

By measuring the change in height, \( h \), of the center of mass of the pendulum arm, \( PE_{grav} \) can be determined. A mark is scribed on the pendulum arm, just above the catcher. Once \( h \) is determined, one can calculate \( V_f \) using Eq. 7.6, which, in turn, allows calculation of \( v_1 \). Note the use of capital \( V \) for \( V_f \); this is to distinguish it from the notation used for the initial velocity of the ball when fired from detent 2, \( v_2 \).
Figure 7.2: The change in height of the center of mass is $h \equiv h_f - h_i$. 
1. State the Law of Conservation of Linear Momentum. (10 pts)

2. State the Principle of Conservation of Mechanical Energy. (10 pts)

3. Solve the following equations (2 equations with 2 unknowns) for $x$ in terms of: $m, g, h$. Refer to Appendix D: Math Review if necessary. (10 pts)

   \[6x = 9y\]
   \[5y^2 = mgh\]

4. Solve the following equations (2 equations with 2 unknowns) for $x$ in terms of: $m, M, g, h$. (20 pts)

   \[mx = (m + M)y\]
   \[\frac{1}{2}(m + M)y^2 = (m + M)gh\]
5. Solve Eq. 7.5 and Eq. 7.6 (2 equations with 2 unknowns) for \( v_1 \) in terms of: \( m \), \( M \), \( g \), \( h \). (20 pts)

\[
mv_1 = (m + M)V_2 \quad \text{(Eq. 7.5)}
\]

\[
\frac{1}{2}(m + M)V_2^2 = (m + M)gh \quad \text{(Eq. 7.6)}
\]

6. You shoot a ball, \( m = 50.0 \, \text{g} \), into a catcher, \( M = 200.0 \, \text{g} \); the center of mass rises 15.0 cm. Calculate \( v_1 \). Refer to your answer for Question 5. (20 pts)

7. You will fire the spring gun 3 times from the first detent and measure the change in height of the (pendulum + ball) for each shot. Write the equation for the change in height of the first shot. (10 pts)
Experiment 7: Conservation of Energy and Linear Momentum

**PROCEDURE**

1. Measure the mass of the ball (m) and catcher arm (M). [Unscrew the knurled nut to remove the arm; screw it back when finished.]

2. Measure the radius to the center of mass of the ball/pendulum system. The center of mass is the top of the opening of the ball catcher.

3. Fire the ball into the catcher and measure the change in height of the ball/pendulum system. \( \Delta h = r - r \cos \Theta \)

4. Record \( \Delta h \) for 3 trials at the first detent (short range). Calculate the average \( \Delta h \) for this detent. Record it in the table provided.

5. Conservation of energy dictates that the total energy remains constant throughout the motion of the pendulum [the force of friction is ignored]. Use this knowledge of energy conservation to determine the velocity of the ball + pendulum just after collision.

6. Write an equation relating the value of the conserved quantity (or quantities) before and after the collision. Calculate the initial velocity of the ball before collision, \( \vec{v}_i \).

7. Repeat Step 3 through Step 4 for detent 2 (medium range) to determine \( \vec{v}_2 \).

8. Repeat Step 3 through Step 4 for detent 3 (long range) to determine \( \vec{v}_3 \).

**QUESTIONS**

1. For each detent, calculate the momentum of the ball just after it is fired. (Assume an isolated system.)

2. Assume that the ballistic pendulum was not held firmly to the table when it was fired. What effect would this have on \( v_1 \)?

3. Write the algebraic equation for mechanical energy just before the collision between the ball and the pendulum arm.

4. Write the algebraic equation for mechanical energy just after the collision between the ball and the pendulum arm.

5. For each detent:
   - Calculate the mechanical energy of the system just before the collision.
   - Calculate the mechanical energy of the system just after the collision.
   - What percent of the initial mechanical energy remains after the collision?

6. What happened to the “lost” mechanical energy?
Experiment 8: Torques and Rotational Motion

Figure 8.1: The wood block provides necessary height for the hangers to not touch the table.

Figure 8.2: Clamp - The arrow indicates the correct edge for position measurement.

**EQUIPMENT**
- Fulcrum
- Meter Stick
- Vernier Caliper
- (3) Mass Hangers
- Masses
- (3) Hanger Clamps (Clamps)
- (1) Knife-Edge Clamp
- Digital Balance
- Triple-Beam Balance
- Block of Wood
- Unknown Mass (Marble or “Silver” Cube)

**TA’s Table:**
- (1) Dial-O-Gram Balance
**Advance Reading**

**Text:** Torque, center of mass, stable and unstable equilibrium, lever arm

**Lab Manual:** Appendix A

**Objective**

To measure torques on a rigid body, to determine the conditions necessary for equilibrium to occur, to perform error analysis.

**Theory**

When a force \( F \) is applied to a rigid body at any point away from the center of mass, a torque is produced. Torque, \( \tau \) (Greek letter, \( \tau \)), can be defined as the tendency to cause rotation. The magnitude of the vector is:

\[
\tau = rF \sin \theta \quad (8.1)
\]

where \( r \) is the distance from the point of rotation to the point at which the force is being applied (i.e., lever arm), and \( F \sin \theta \) is the component of the force perpendicular to \( r \). Note that the unit for torque is mN (meter × newton).

In this experiment, all forces will be acting normal (perpendicular) to the meter stick: \( \theta = 90^\circ \); therefore, \( \sin \theta = 1 \). The equation for torque is simplified:

\[
\tau = rF \quad (8.2)
\]

**Equilibrium**, Latin for equal forces or balance, is reached when the net force and net torque on an object are zero. The first condition is that the vector sum of the forces must equal zero:

\[
\Sigma \vec{F} = \Sigma F_x = \Sigma F_y = \Sigma F_z = 0.0 \text{ N} \quad (8.3)
\]

The second condition that must be met is that the net torques about any axis of rotation must equal zero. We will use the standard convention for summing torques. Torques that tend to cause counterclockwise rotation, \( \tau_{cc} \), will be positive torques, while torques that tend to cause clockwise rotation, \( \tau_c \), will be negative torques.

\[
\Sigma \tau = \Sigma \tau_{cc} - \Sigma \tau_c = 0.0 \text{ mN} \quad (8.4)
\]

The system under consideration for this experiment will need to not only attain equilibrium, but also remain in equilibrium. This will require that the object be in stable equilibrium, meaning if a slight displacement of the system occurs, the system will return to its original position (e.g., a pendulum). If the system were to move farther from its original position when given a slight displacement, it would be in unstable equilibrium (e.g., a ball on a hill).

Once stable equilibrium has been attained for each experimental arrangement, measure the mass at each position using the appropriate balance.

![Figure 8.3: Required sketch for each experimental arrangement](image)

Once stable equilibrium is attained, sketch each set-up:

- \( x \): position
- \( F \): magnitude of force
- \( Arrow \): direction of force
- \( r \): lever arm
- \( cc \): counterclockwise
- \( c \): clockwise
- \( cm \): center of mass
- \( f \): fulcrum
1. Define torque, and state the conditions necessary for stable equilibrium. (20 pts)

2. Why are the following equations equivalent for this experiment? (20 pts) \[ \tau = rF \sin \theta \quad \tau = rF \]

3. Refer to the procedure, Part 1, 1st arrangement. Assume \( x_{cm} = 50.0 \) cm, 150.0 g is suspended from a hanger clamp at the position \( x_{cc} = 15.0 \) cm, and a hanger clamp is at position \( x_c = 75.0 \) cm. If each hanger clamp has a mass \( m = 16.5 \) g, what mass must be added to \( x_c \) in order to attain stable equilibrium? Sketch a diagram of the situation (refer to Fig. 8.3). (30 pts)

4. Consider Part 2 of the procedure. Determine the additional mass required for stable equilibrium. Meter stick: \( x_{cm} = 50.0 \) cm, \( m = 150.0 \) g. Hanger clamp: \( x_{cc} = 0.0 \) cm, \( m = 16.5 \) g. (30 pts)
Experiment 8: Torques and Rotational Motion

PROCEDURE

PART 1: Quantitative Analysis of Torque
1. Place the knife-edge clamp at the 50 cm position of the meter stick with the screw pointing down. Adjust the knife-edge clamp until the meter stick is balanced and horizontal (stable equilibrium). Record this position as $x_{cm}$.
2. Place a clamp at the $x_{cc} = 15$ cm position and hang 200 g from it.
3. Place another clamp at the $x_{c} = 75.0$ cm position. Add enough mass to attain equilibrium. If small fractional masses are not available to you, it may be necessary to adjust the position of the 75 cm clamp in order to balance the system.
4. Measure the mass at each position; recall that the digital balance has a limit of 0.2 kg.
5. Determine the force, radius, and torque at each position.
6. Calculate the sum of the torques.

PART 2: One-Person See-Saw
7. Remove all clamps from the meter stick. Measure and record the mass of the meter stick.
8. Place the fulcrum at 20 cm on the meter stick.
9. Place a clamp as close to the zero end as possible. Add mass incrementally to attain static equilibrium. Measure this mass.
10. Make a torque-balance sketch similar to the one in Step 6; fill it in with the appropriate values. Calculate the net torque about the fulcrum. Note that the meter stick behaves as though all of its mass is concentrated at its center of mass.

PART 3: Unknown Mass
11. Determine the mass of a metal cube experimentally, using the torque apparatus however you choose.
12. Make a torque-balance sketch of your experimental setup.
13. Determine the density of the metal cube by measuring its dimensions.
14. Identify the metal using the density chart provided in Table 1.1 (Page 9). Compare your value of density to the accepted $\rho$ value for that material.

QUESTIONS
1. Consider the Dial-O-Gram balance and the triple-beam balance. The Dial-O-Gram balance has a spring, calibrated for Earth, behind the dial. This spring exerts a force that allows accurate measurement of mass. The triple-beam balance uses only the principles of torque, which you investigated in this experiment. Will either balance allow us to accurately measure the mass of an object on the moon?
2. Calculate $\delta \Sigma \tau$ for each $\Sigma \tau$ you determined in Part 1, Part 2, and Part 3. Is each $\Sigma \tau$ within experimental uncertainty?
Experiment 9: Moments of Inertia

EQUIPMENT
Beck’s Inertia Thing
Vernier Caliper
30cm Ruler
Paper Clips
Mass Hanger
50g Mass
Meter Stick
Stopwatch

Figure 9.1: Beck’s Inertia Thing with masses
**Advance Reading**

Text: Torque, Rotational Motion, Moment of Inertia.

**Objective**

To determine the moment of inertia of a rotating system, alter the system, and accurately predict the new moment of inertia.

**Theory**

Moment of Inertia ($I$) can be understood as the rotational analog of mass. Torque ($\tau$) and angular acceleration ($\alpha$) are the rotational analogs of force and acceleration, respectively. Thus, in rotational motion, Newton’s Second Law:

$$F = ma$$  \hspace{1cm} (9.1)

becomes:

$$\tau = I\alpha.$$  \hspace{1cm} (9.2)

An object experiencing constant angular acceleration must be under the influence of a constant torque (much like constant linear acceleration implies constant force). By applying a known torque to a rigid body, measuring the angular acceleration, and using the relationship $\tau = I\alpha$, the moment of inertia can be determined.

In this experiment, a torque is applied to the rotational apparatus by a string which is wrapped around the axle of the apparatus. The tension $T$ is supplied by a hanging mass and found using Newton’s second law.

$$\Sigma F = T - mg = ma$$  \hspace{1cm} (9.3)

so the tension is

$$T = m(g - a)$$  \hspace{1cm} (9.4)

The rotational apparatus has an original moment of inertia $I_0$ with no additional masses added. When additional masses are added, it has a new moment of inertia $I_{\text{new}}$. The added masses effectively behave as point masses. The Moment of Inertia for a point mass is $I_p = MR^2$, where $M$ is the mass and $R$ is the radius from the point about which the mass rotates. Thus, the relationship between $I_0$ and $I_{\text{new}}$ is given by

$$I_{\text{new}} = I_0 + I_{p1} + I_{p2} + ... = I_0 + M_1R_1^2 + M_2R_2^2 + ...$$  \hspace{1cm} (9.5)

where $M$ is an added mass and $R$ is the distance of this mass from the center of the wheel (i.e. from the axis of rotation). So, if multiple masses are added at the same radius, we have

$$I_{\text{new}} = I_0 + \Sigma I_p = I_0 + (\Sigma M)R^2$$  \hspace{1cm} (9.6)

In comparing this to Eq. 9.1, we consider that all masses, along with the disk, experience the same angular acceleration. If we were looking for the Force on a system of connected masses all experiencing the same acceleration, we would simply sum the masses and multiply by acceleration (i.e. a stack of boxes being pushed from the bottom). Similarly, when looking for the Torque on a system, we must sum the moments of inertia and multiply by angular acceleration.
Prelab 9: Moments of Inertia

Name: ________________________________

1. What is Moment of Inertia? Explain using words only. (20 pts)

2. Draw two force diagrams. One for the mass hanging from the sting, and a second for the disk as viewed from above. (20 pts)

3. The rotational apparatus pictured below begins at rest. Upon release, the hanging mass falls 75cm in 10 seconds. The apparatus experiences no friction torque. Calculate the following: (50 pts)

   ![Diagram](image)

   Figure 9.3

   a) The acceleration of the falling mass.

   b) Angular acceleration of the rotational apparatus.

   c) The tension in the string.

   d) The torque applied by the falling mass.

   e) The net torque applied.
PROCEDURE

PART 1: Moment of Inertia of apparatus with no additional masses.
1. Using the vernier caliper, measure the diameter of the axle around which the string wraps. Calculate the radius of the axle.
2. Wrap the string around the axle and attach enough mass to the string to cause the apparatus to rotate very slowly. The angular acceleration of the disk should be nearly zero. Record this mass and use it to calculate the frictional torque.
3. Holding the disk, place an additional 50 grams (mass hangers are 50 grams) on the string. Measure the distance from the bottom of the mass hanger to the floor.
4. Release the disk, be sure not to impart an initial angular velocity. Using the stopwatch, measure the time until the mass hanger reaches the floor.
5. Repeat Step 4 five times. Record the times in a table and calculate the average time.
6. Using the average time, calculate the linear acceleration of the masses.
7. Calculate the angular acceleration of the disk using \( \alpha = \frac{a}{r} \). Refer to Step 1 for \( r \).
8. Calculate the tension on the string, Eq. 9.4. Be sure to use the total hanging mass.
9. The applied torque on the spinning disk is provided by the tension of the string. Use the values from Step 2 and Step 8 to calculate the net torque, which is applied torque minus friction torque.
10. Repeat Step 3 through Step 9 for 100 grams on the mass hanger in addition to the mass from Step 2.
11. Using Graphical Analysis, plot the net torque vs. angular acceleration. Be sure to enter the origin as a data point. Determine the moment of inertia \( I_0 \).

PART 2: Moment of Inertia of apparatus with additional masses.
12. Measure the distance from the center of the disk to the outer set of tapped holes (Where you will attach the three large masses).
13. Attach the three masses to the disk. Using the mass stamped on the top/side of the masses, calculate the new moment of inertia, \( I_{new} \), for the system.
14. Repeat Step 2 through Step 11 for the altered system. Calculate the percent difference between the experimental value and the calculated value.

Questions
1. What are the units for Torque, Moment of Inertia, and Angular Acceleration? Show all work.
2. If the Torque applied to a rigid body is doubled, what happens to the Moment of Inertia?
3. Compare friction compensation in this experiment to friction compensation in Newton’s Second Law.
Experiment 10: Archimedes’ Principle

**EQUIPMENT**

- Triple-Beam Balance with string
- Graduated Cylinder
- Pipette
- Cylinders: (2) Metal, (1) Wood
  *(Note: The cylinders have sharp hooks)*
- Overflow Container
- Spouted Can
- Digital Balance
- (2) 123-Blocks
- Wood Board/Block
- Rod & Clamp
- Paper Towels
- Water

Figure 10.1
**Advance Reading**

*Text: Archimedes’ principle, buoyant force, density*

**Objective**

The objective of this lab is to investigate the buoyant force acting on a variety of objects, the density of the objects, and the density of our tap water.

**Theory**

Archimedes’ principle states that a body wholly or partially submerged in a fluid is buoyed up by a force equal in magnitude to the weight of the fluid displaced by the body.

It is the buoyant force that keeps ships afloat (object partially submerged in liquid) and hot air balloons aloft (object wholly submerged in gas). We will investigate the buoyant force using the following methods:

- Direct Measurement of Mass
- Displacement Method

When an object is submerged in water, its weight decreases by an amount equal to the buoyant force. The **direct measurement of mass** will measure the weight of an object first in air, then while it is submerged in water. The buoyant force, $F_B$, is equal to the weight in air ($F_g$) minus the weight in water, $F'_g = m'g$:  

$$F_B = F_g - F'_g$$  \hspace{1cm} (10.1)

The **displacement method** requires measurement of the volume of fluid displaced by the object. The weight of the fluid displaced is equal to the buoyant force exerted on the object. Thus, the buoyant force is given by:

$$F_B = \rho g V$$  \hspace{1cm} (10.2)

where $\rho$ (Greek letter, rho) is the density of the fluid displaced, $V$ is the volume of fluid displaced by the object, and $g$ is the acceleration due to gravity.

**Experiment 10: Archimedes’ Principle**

The following exercises will be informative, as both floating and sinking objects are used in this experiment.

- Sketch a free-body diagram for an object that is *floating* in water. How much water does it displace? Does it displace its volume in water? Does it displace its weight in water?

- Sketch a free-body diagram for an object that is *submerged* in water. How much water does it displace? Does it displace its volume in water? Does it displace its weight in water?

The accepted value for the density of pure water at 4°C and 1 atm is $\rho_{water} = (1000 \pm 1) \text{ kg/m}^3$. We will use this value for the density of water for *Part 2* through *Part 5*. That is, we assume a temperature in the lab of 4°C!

We will then experimentally determine the density of the tap water we used (*Part 6*) and compare it to the density of water at 20°C. The density of pure water at 20°C is:

$$\rho_{water} = (998.21 \pm 0.01) \text{ kg/m}^3$$  \hspace{1cm} (10.3)

When comparing the experimental densities of your objects or tap water, please use Table 1.1 provided at the end of *Experiment 1: Measurement & Analysis* on Page 9.
Name: ________________________________

1. State Archimedes’ principle. (20 pts)

2. Explain the relationship $F_B = \rho V g$. (25 pts)

3. Briefly explain the methods used in Part 1 through Part 3 of this experiment to determine buoyant force. (25 pts)

4. Draw a free-body diagram for an object of mass $M$, for the following two situations:
   a. a submerged object suspended by a string; b. a floating object. Draw to scale. (30 pts)
PROCEDURE

PART 1: Overflow Method

1. Measure the mass of the brass cylinder. Determine its weight, \( F_g \).
2. Place the overflow container on the digital balance.
3. Fill the spouted can with water. Position it so that its spigot pours into the overflow container.
4. Submerge the brass cylinder in the water, allowing displaced water to collect in the overflow container.
5. Measure the mass of the displaced water; calculate its weight. This is the buoyant force, \( F_B \).
6. Calculate \( \rho_{\text{obj}} \) (density of the object):
   \[
   \rho_{\text{obj}} = \frac{\rho_w F_g}{F_B} \tag{10.4}
   \]

PART 2: Direct Measurement - Mass

7. Calibrate the triple beam balance.
8. Suspend the object (brass cylinder) from a string attached to the balance.
9. Partially fill the overflow container with water, then submerge the object. Do not allow the object to touch the container. Measure the apparent mass of the object in water, \( m' \). Calculate \( F'_g \).
10. Determine \( F_B \) for the object. How much less does it weigh in water than in air? (Eq. 10.1)
11. Calculate \( \rho_{\text{obj}} \) using Eq. 10.4.

PART 3: Displacement Method - Volume

12. Partially fill the graduated cylinder with water; take note of the water level. Use the pipette to fine-tune the meniscus.
13. Carefully submerge the object in water and determine its volume.
14. Remove and dry the object, then empty the graduated cylinder and invert it on a paper towel to dry.
15. Determine \( F_B \) on the object with Eq. 10.2.
16. Calculate \( \rho_{\text{obj}} \) using Eq. 10.5:
   \[
   \rho_{\text{obj}} = \frac{m}{V} \tag{10.5}
   \]
   Use the volume determined from the displacement method and \( m \), not \( m' \).

PART 4: Aluminum Cylinder

17. Repeat Part 1 through Part 3 for the next object (aluminum cylinder).
18. Draw a free-body diagram for this object submerged in water.

PART 5: Buoyant Force - Floating Object

19. Although you need to modify or omit certain steps, repeat Part 1 through Part 3 for the wood cylinder:
   - Omit Step 6, Step 11, and Step 16.
   - Modify Step 9 and Step 13: Allow the wood object to float.
20. Draw a free-body-diagram for the wood object floating in water.

PART 6: Density of Tap Water

21. For each metal object: Use Eq. 10.6 and the graduated cylinder volume from Part 3 to determine the density of our tap water.
   \[
   m - m' = \rho_w V \tag{10.6}
   \]

QUESTIONS

1. Consider situations A and B, below. Does one tub weigh more than the other, or do they weigh the same? Draw a free body diagram for each case.
   A. A tub filled to the brim with water.
   B. A tub filled to the brim with water, with a boat floating in it.
2. A 827 cm\(^3\) gold nugget and a 827 cm\(^3\) aluminum block are immersed in water. Which object experiences the greater buoyant force?
3. A ship made of steel (\( \rho_{\text{steel}} = 7.8 \times 10^3 \text{ kg/m}^3 \)) will float in water. Explain, in terms of densities, how this is possible.
4. Verify that Eq. 10.4 is true for an object submerged in water. (Start by writing the equations for each force.)
5. Consider a pirate boat in a pond with the anchor on the boat. When the pirates throw the anchor overboard, what happens to the water level at the shore? Does the water at the shore rise, fall, or stay at the same level? Explain, in terms of Archimedes’ principle (density, volume, or weight), why this happens.
Experiment 11: Simple Harmonic Motion

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure11.1}
\caption{Figure 11.1}
\end{figure}

**EQUIPMENT**

- Spring
- Metal Ball
- Wood Ball
  \textit{(Note: sharp hooks)}
- Meter Stick
- Digital Balance
- Stopwatch
- Pendulum Clamp and Rod
- String
- Masses: (2) 100g, (1) 50g
- Mass Hanger
- Table Clamp
- Protractor
**Advance Reading**

**Text:** Simple harmonic motion, oscillations, wavelength, frequency, period, Hooke’s Law.

**Lab Manual:** Appendix B

**Objective**

To investigate simple harmonic motion using a simple pendulum and an oscillating spring; to determine the spring constant of a spring.

**Theory**

Periodic motion is “motion of an object that regularly returns to a given position after a fixed time interval.” Simple harmonic motion is a special kind of periodic motion in which the object oscillates sinusoidally, smoothly. Simple harmonic motion arises whenever an object is returned to the equilibrium position by a restorative force proportional to the object’s displacement.

$$F = -kx$$  \hspace{1cm} (11.1)

The illustrative example above is Hooke’s Law, which describes the restorative force of an oscillating spring of stiffness $k$ (spring constant).

For an ideal, massless spring that obeys Hooke’s Law, the time required to complete an oscillation (period, $T$, seconds) depends on the spring constant and the mass, $m$, of an object suspended at one end:

$$T = 2\pi \sqrt{\frac{m}{k}}$$  \hspace{1cm} (11.2)

The inverse of period is the frequency of oscillation. Recall that frequency, $f$, is the number of oscillations completed by a system every second. The standard unit for frequency is hertz, Hz (inverse second, $s^{-1}$).

The period of oscillation of an ideal, simple pendulum depends on the length, $L$, of the pendulum and the acceleration due to gravity, $g$:

$$T = 2\pi \sqrt{\frac{L}{g}}$$  \hspace{1cm} (11.3)

When setting the pendulum in motion, small displacements are required to ensure simple harmonic motion. Large displacements exhibit more complex, sometimes chaotic, motion. Simple harmonic motion governs where the small angle approximation is valid:

The arc length, $s$, of a circle of radius $r$ is:

$$s = r\phi$$  \hspace{1cm} (11.4)

When $\phi$ is small, the arc length is approximately equal to a straight line segment that joins the two points. Therefore, the following approximations are valid:

$$\phi \approx \sin \phi \approx \tan \phi$$  \hspace{1cm} (11.5)

**Figure 11.2: Small Angle Approximation**

The arc length, $s$, of a circle of radius $r$ is:

$$s = r\phi$$  \hspace{1cm} (11.4)

When $\phi$ is small, the arc length is approximately equal to a straight line segment that joins the two points. Therefore, the following approximations are valid:

$$\phi \approx \sin \phi \approx \tan \phi$$  \hspace{1cm} (11.5)
Prelab 11: Simple Harmonic Motion

Name: ________________________________

1. Define simple harmonic motion. What conditions must be met? (20 pts)

2. What physical phenomenon does the relationship $T = 2\pi \sqrt{\frac{m}{k}}$ describe? (20 pts)

3. What physical phenomenon does the relationship $T = 2\pi \sqrt{\frac{L}{g}}$ describe? (20 pts)

4. The following data were collected for Part 1 of the lab procedure. Complete the table. The force is due to the gravitational force. All distances are measured from the bottom of the hanger to the top of the stool. You should ignore the initial weight of the hanger. Note that $\Delta x$ is the change from initial position, $x_f - x_0$, not the change from the previous position, $x_2 - x_1$. (40 pts)

<table>
<thead>
<tr>
<th>Mass (g)</th>
<th>Height (cm)</th>
<th>$\Delta x$ (m)</th>
<th>Force (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>57.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>100</td>
<td>46.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>36.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>25.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>15.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>4.5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
PROCEDURE

PART 1: Spring Constant - Hooke’s Law

1. Hang the spring from the pendulum clamp and hang
   the mass hanger from the spring. Place a stool un-
   der the hanger and measure the initial height \( x_0 \)
   above the stool.
2. Add 50 g to the mass hanger and determine the
   change in position caused by this added weight.
3. Add 50 g masses incrementally until 250 g has been
   added to the mass hanger. Determine the total dis-
   placement and the total added weight with each ad-
   dition.
4. Generate a graph of \( F \) vs. \( \Delta x \) using Graphical Anal-
   ysis. Analyze the graph with a linear fit; print a
   copy for each partner.

PART 2: Spring Constant - Oscillation

5. Measure the mass of the spring, mass hanger, and
   100 g mass.
6. Hang the spring from the pendulum clamp. Hang
   the mass hanger + 100 g from the spring (refer to
   Fig. 11.1).
7. Pull the mass hanger down slightly and release it
   to create small oscillations. Measure the time re-
   quired for 20 oscillations. (This is like measuring
   one period twenty times over.)
8. Calculate the period for the oscillating spring.
9. Calculate the spring constant of the spring using
   your knowledge of the object’s mass and period of
   oscillation.

Note: The spring used for this experiment is not ideal;
its mass affects the period of oscillation. Account for
this by adding 1/3 the mass of the spring to the value
of suspended mass, \( m \), in your calculations.

PART 3: Simple Pendulum

10. Measure the mass of the metal ball.
11. Construct a simple pendulum 100.0 cm in length us-
    ing the metal ball and some string. (\( L \) is measured
    from the center of mass of the ball.)
12. Move the pendulum from equilibrium (about 10°-
    20°) and release it. Measure the time required for
    20 oscillations.
13. Determine the period. Record it in the table pro-
    vided.
    \[
    T = \frac{\text{time required}}{20 \, \text{cycles}}
    \]  
    (11.6)
14. Shorten \( L \) in increments of 20.0 cm and measure \( T \)
    for each length.
15. Repeat the procedure using the wood ball.
16. Produce graphs of \( T^2 \) vs. \( L \) for each ball. Apply a
    linear fit; print a copy for each partner.

QUESTIONS

1. Solve Eq. 11.3 for \( g \).
2. Does the period of a simple pendulum depend on
   the mass?
3. How long must a simple pendulum be to have a
   period of 1.5 s?
4. Assume you are safely located on the moon and have
   access to a simple pendulum, stopwatch, and meter
   stick. Is it possible to determine the acceleration
due to gravity of the moon, \( g_{moon} \), using only these
   three items? (Hint: \( g_{moon} \neq 0 \, \text{m/s}^2 \))
5. Consider your \( T^2 \) vs. \( L \) graph. What are the slope
    values? Show that the slope should be equal to
    \( 4\pi^2/g \). Compare each graph value to the accepted
    value.
Experiment 12: Speed of Sound in Air

Figure 12.1: Sound Tube

**EQUIPMENT**

Sound Tube
(3) Tuning Forks \( (f \geq 300 \text{ Hz}) \)
Mallet
Water Jug
Rubber Hose
Clamps and Rod
Paper Towels

Figure 12.2: Displacement of Air
**Advance Reading**

*Text*: Speed of sound, longitudinal waves, wavelength, frequency, standing wave, resonance.

**Objective**

The objective of this experiment is to measure the speed of sound in air.

**Theory**

There are a variety of wave types.

Sound is a longitudinal wave requiring a medium in which to propagate. A longitudinal wave is one in which objects oscillate in the same direction the wave propagates. The speed of sound depends on properties of the medium such as bulk modulus, density, and temperature. The speed of sound is not a constant value.

To calculate today’s speed of sound, $v$, we will determine the wavelength, $\lambda$ (lambda), of the sound produced by a tuning fork of known frequency, $f$:

$$v = \lambda f$$  \hspace{1cm} (12.1)

A vibrating tuning fork generates a sound wave that travels outward in all directions. When held above a sound tube, a portion of the wave will travel down the tube, reflect off the water’s surface, then return to the top. If the rising pressure wave reaches the top of the tube as the next wave is produced, the wave is reinforced, and the sound will resonate. A standing wave is generated in the tube, and the sound will be distinctly louder. This occurs when the column of air in the tube has an appropriate length (height) for a given tuning fork.

**Experiment 12: Speed of Sound in Air**

When considering the displacement of air for resonance (constructive interference), notice that there is an antinode near the open end of the tube, and a node at the water’s surface from which the sound is reflected (refer to Fig. 12.2). To locate multiple resonances for a particular tuning fork, one must be able to change the length of the air column in the tube. This will be accomplished by adjusting the water level in the tube: raise or lower the water reservoir, and the water level in the tube will change accordingly.

The distance between one resonance and the next is $\frac{1}{2}\lambda$. This experiment will attempt to locate at least three resonances to reduce uncertainty of the results.

$$\frac{\lambda}{2} = |x_1 - x_2|$$  \hspace{1cm} (12.2)

Having calculated $\lambda$ and being given $f$ of a tuning fork, the speed of sound can be calculated with Eq. 12.1.
Name: ______________________________________

1. What is a standing wave? (20 pts)

2. What is a resonant frequency? (20 pts)

3. Explain the relationship $v = \lambda f$. (20 pts)

4. Refer to Eq. 12.1 and Eq. 12.3 (Step 10 of the procedure). You measure $\frac{1}{2} \lambda$ to be 40 cm. The temperature in the lab is 22°C. What is the frequency of this resonance? (40 pts)
Experiment 12: Speed of Sound in Air

PROCEDURE

1. Raise the bottle of water until the tube is filled.
2. Hold a vibrating tuning fork above the tube and lower the water level gradually until the sound becomes loudest (resonates). Raise the water level as necessary.
3. Mark the water level with a rubber band.
4. Continue to lower the water level until all resonant positions have been marked in this manner. Record the positions in the data table provided.
5. Calculate the average distance \( \frac{1}{4} \lambda \) between resonance positions. Record it in the table provided.
6. Calculate the fundamental wavelength \( \lambda = 2l \) and record it.
7. Calculate the measured speed of sound \( v_e = f \lambda \) and record it.
8. Repeat this process for two other frequencies (3 tuning forks, total).
9. Average your three values of \( v_e \). Record them in the data table on the board.
10. Calculate a theoretical value for the speed of sound, \( v_T \), using:

\[
v_T = (331.5 + 0.6T) \text{m/s}
\]

(12.3)

where \( T \) is the temperature in degrees Celsius.
11. Calculate your percent error. [Eq. A.1, Page 141]

\[
\begin{align*}
\frac{1}{4} \lambda & = d_1 + d_2 \\
\frac{1}{4} \lambda & = \text{distance from position of water’s surface when 1}\text{" resonance is heard to anti-node.} \\
d_1 & : \text{Refer to Step 2.} \\
d_2 & \equiv \text{distance from the top of the tube to the anti-node.}
\end{align*}
\]

\begin{itemize}
    \item Figure 12.3: Proper position for tuning fork
    \item Figure 12.4: Determination of anti-node location.
\end{itemize}

QUESTIONS

1. Define ultrasonic, supersonic, and infrasonic.
2. As noted in the theory section, our sound tubes do not have an anti-node at the open end of the tube. Analyze your data and determine where, relative to the open end of the tube, the anti-node is located for each frequency used. State your answer in terms of \( \lambda \). Refer to Fig. 12.
3. Assume you are in a particular location (e.g., at the beach, in the mountains, in the lab). Two sounds, one a high frequency, one a low frequency, are generated. Does a high frequency sound travel faster than a low frequency sound in a particular location?
4. Does sound travel at the same speed in different materials? Specify the speed of sound in 3 media.
Experiment 13: Electrostatics

Figure 13.1

EQUIPMENT
Ebonite Rod (Hard Rubber)
Glass Rod (or Lucite Rod)
Rabbit Fur
Plastic Film
Silk
Electroscope
Matches

Front Table
Hair Dryer
Rubbing Alcohol
Paper Towels
Advance Reading

Text: Law of conservation of electric charge, electrostatic charge, electron, proton, neutron, atomic model, free electrons, ions, polarization, conductor, insulator, conduction, induction, plasma.

Objective

The objective of this lab is to qualitatively study conducting and insulating materials, electric charges, and charge transfer.

Theory

There are two kinds of charges in nature: positive charge carried by protons and negative charge carried by electrons. An object that has an excess of either is said to be charged. Like charges repel each other, and unlike charges attract.

Charge transfer is the exchange of charges between objects. In this experiment, only electrons are exchanged while protons remain stationary. These electrons may move around within materials or move between materials, but they can never be created or destroyed. This is known as the law of conservation of charge. The law of conservation of electric charge states that the net amount of electric charge produced in any process is zero.

A conductor is a material in which some loosely bound electrons can move freely (free electrons) while protons remain stationary. An insulator is a material in which both electrons and protons are tightly bound. Conductors and insulators have the following properties:

Conductors

- Conductors are objects that allow the free flow of electrons throughout the object.
- Charges are easily transferred between conductors.
- Charge can collect at one end of an object in the presence of other charged objects.

Insulators

- An insulator is a material in which electrons are tightly bound to the nucleus.
- Transferring charge between insulators requires a force, e.g. friction, and direct contact.
- Insulators brought near other charged objects experience polarization, a shifting of electrons to one side of an atom. (Fig. 13.2)

In this experiment, a glass rod or an ebonite rod (insulator) will be electrically charged by rubbing against another insulating material. Whether the rod gains or loses electrons will depend on the combination of materials used (refer to the electrostatic series provided in Table 13.1 on Page 67). The charged rod will be used to charge an electroscope (a conductor that indicates whether it is charged) by means of conduction and by means of induction.

To charge by conduction: Bring a charged rod close to, then touch, the electroscope. As the rod nears the electroscope, the free electrons in the electroscope are either attracted to or repelled by the charged rod (induction). When you touch the rod to the electroscope, the electroscope becomes charged as electrons transfer to (or from) the electroscope (charge transfer).

To charge by induction: Bring a charged rod close to, but do not touch, the electroscope. While holding the rod near the electroscope (induction), touch the opposite side of the electroscope with the tip of your finger (charge transfer). Your body will act as a reservoir of charge (ground), either giving or receiving electrons to the electroscope. Remove your finger before moving the rod from the proximity of the electroscope.

In Part 3, the effects of plasma on a charged electroscope will be observed. Plasma is an ionized gas containing positive ions and free electrons. Plasma is found in our upper atmosphere, inside fluorescent lights, and surrounding the flame of a match.

![Atom charge distribution, normal](image1)

Atom charge distribution, normal

![Atom charge distribution, polarized](image2)

Atom charge distribution, polarized

Figure 13.2: Polarization
Table 13.1: Electrostatic Series

Materials tend to receive electrons and become **NEGATIVELY CHARGED**

- Plastic Film
- Hard Rubber
- Celluloid
- Sulfur
- Rubber Balloon
- Polyethylene
- Polystyrene
- Amber
- Sealing Wax
- Lucite
- Wood
- Cotton
- Paper
- Silk
- Cat Fur
- Wool
- Nylon
- Mica
- Glass
- Rabbit Fur

Materials tend to lose electrons and become **POSITIVELY CHARGED**
Name: 

1. What are the objectives of this experiment? (5 pts.)

2. There are only two types of electric charge. Electrons carry a ________________ charge, and protons carry a ________________ charge. (5 pts.)

3. What is conservation of charge? (20 pts.)

4. Assuming both objects are initially neutral, use the concept of conservation of charge to explain what happens when an ebonite rod is rubbed with rabbit fur. (20 pts.)

5. Define the following: (10 pts. each)
   - conductor
   - insulator
   - conduction
   - induction
   - polarize
Experiment 13: Electrostatics

PROCEDURE

PART 1: Charging by Conduction

Negative by Conduction
1. Charge a rod negatively by rubbing it with a material that will give it extra electrons.
2. Bring the negatively charged rod close to the electroscope bulb without touching it. Observe how the leaves of the electroscope repel each other.
3. Touch the charged rod to the electroscope. Observe the behavior or the electroscope during conduction and as you remove the rod.
4. Draw a series of sketches showing the movement of charges during this process.

Positive by Conduction
5. Charge the electroscope positively by conduction using a glass rod.
6. Sketch the various stages of this process.

PART 2: Charging by Induction

Positive by Induction
7. Bring a negatively charged rod close to the electroscope, but do not touch it.
8. While the electroscope’s leaves are separated, touch the electroscope bulb with your finger to ground it. Remove your finger.
9. Remove the charged rod from the vicinity of the electroscope; observe its final state.
10. Draw a series of sketches showing the movement of charges during this process.

Negative by Induction
11. Charge the electroscope negatively by induction. Which rod will be required?
12. Sketch the various stages of this process.

PART 3: Electroscope and Plasma

13. Apply a charge to the electroscope by a method of your choosing.
14. Light a match and hold it near the electroscope, observing its behavior.
15. Sketch this process.

QUESTIONS

1. What is meant by conservation of charge?
2. Why do charged balloons cling to a wall, which is also an insulator and has a neutral charge?
3. Explain the difference between an insulator and a conductor. Give 3 examples of each.
4. Is the fur positively or negatively charged after charging an ebonite rod?
Experiment 14: Electric Fields and Potentials

*Power Supply:* Always connect the red lead to the red post and the black lead to the black post. Turn the power supply off and get a TA to check the circuit prior to plugging in the power supply.

*Digital Multi-Meter (DMM) as a Voltmeter:* Connect the red lead to the V/Ω jack and the black lead to the COM jack. Turn the dial to 20V DCV and turn on. You will need to adjust the voltmeter scale (turn the dial) as you perform experiment. Adjust the scale so that you obtain the most significant figures possible without incurring an overflow symbol ("1.").

**EQUIPMENT**

- Conductive/Resistive Paper (2)
- Electric Fields Circuit Board
- (2) Point Charge Connectors
- (2) Parallel Plate Connectors
- (4) Posts (2 Red, 2 Black)
- Tip Holder
- Digital Multi-Meter (DMM)
- Power Supply
- Grease Pencil (or white colored pencil)
- (4) Wire Leads
Advance Reading

Text: Electric field, electric potential energy, equipotential, voltage.

Lab Manual: Appendix B - iMac
Appendix C: Equipment - DMM

Objective

To map equipotential lines and electric field lines of two charge arrangements; to measure the electric field strength of each arrangement.

Theory

Electric potential (voltage) at a point is defined as the amount of potential energy per coulomb of charge placed at that point. Potential is only defined as a difference in voltage between two points. This change in voltage, $\Delta V$, is equal to the negative of the work done by the electric force to move a charge from one point to another:

\[ \Delta V = V_b - V_a = -\frac{W}{q} \] (14.1)

An equipotential surface is defined as a surface where all points on the surface have the same electric potential. To move a charge around on such a surface requires no work. In two dimensions, the equipotential surfaces are equipotential lines. How close the lines are to each other is an indication of the strength of the corresponding electric field.

An electric field, $E$, at a point is defined as the force per coulomb exerted on a charge at the point:

\[ \overline{E} = \overline{F}/q \] (14.2)

Electric fields push positive charges toward a lower state of potential energy, or towards a lower equipotential. Thus, electric field lines are always perpendicular to equipotential lines.

Electric field can also be measured by how quickly voltage is changing at that point, in volts/meter. A stronger electric field indicates electric potential is varying more rapidly over a particular distance.

The two conductive patterns we investigate for this experiment, point charges and parallel plates, are on conductive/resistive paper.

Figure 14.2: Point Charge Arrangement
Figure 14.3: Parallel Plate Arrangement
Figure 14.4: Tip Holder

Note the 3 holes for positioning tips of wire leads.
Prelab 14: Electric Fields and Potentials

1. Define electric potential. State the units. (15 pts)

2. What is an equipotential surface? (20 pts)

3. Define electric field. State the units (both that are listed in the text). (20 pts)

4. Complete the statement: Electric field lines are always __________________________ to equipotential surfaces. (10 pts)

5. Calculate the electric field strength of the following arrangement. Assume the leads are 1.00 cm apart and the electric potential difference measured with the voltmeter is 0.673 V. Refer to the procedure. (20 pts)

Figure 14.5: Electric Field Strength Arrangement
**Experiment 14: Electric Fields and Potentials**

**PROCEDURE**

**Digital Multi-Meter as Voltmeter**
1. Connect a black lead to the COM jack and a red lead to the V/Ω jack.
2. Turn the dial to 20V DCV and turn on the DMM (it is now acting as a voltmeter).

**PART 1: Setup and Connections**
3. Place the conductive/resistive paper on the circuit board. Poke two holes in the paper matching the holes in the circuit board as shown in Fig. 14.2.
4. Place a point charge connector over each hole; affix them with one red post and one black post.
5. Connect the power supply to the point charge posts, black-to-black (ground) and red-to-red (positive).
6. Connect the voltmeter to the point charges, on top of the power supply leads.
7. Ask your TA to approve your circuit. Then, plug in the power supply and set the voltage to 6.0 V.
8. Label the point charges with a grease pencil (the ground lead is at 0.0 V, the positive lead is at 6.0 V).

**PART 2: Point Charges**

*Equipotential Lines*
9. Remove the voltmeter positive lead from the point charge and drag it across the conductive paper. Bring it closer to the ground lead until the voltmeter reads 1.00 V. (Adjust the voltmeter scale to give the most significant figures without getting an overload symbol, “1.”)
10. Mark a dot at this location with the grease pencil. Move the voltmeter lead around the paper until you locate eight points where the voltmeter reads 1.00 V and mark each of them.
11. Connect the dots with the grease pencil to create an equipotential line. Label this line with its voltage.
12. Have your partner repeat this process to locate the 2.0 V equipotential.
13. Take turns with your partner to locate the 3.0 V, 4.0 V, and 5.0 V equipotentials.

**Electric Field Strength**
14. Measure the electric field strength ($\Delta V/\Delta x$) at the 3.0 V equipotential, at the center of the paper:
   (a) Measure the change in voltage between two points placed closely together, on either side of the 3.0 V line. Use the clear plastic tip holder to maintain a measurable separation of the leads (the holes are about 1 cm apart).
   (b) Calculate the field strength, in V/m, at this location. Record it in the table provided.
   (c) Mark the location on the conductive paper and label the strength of the field.

**Electric Field Lines**
15. Place the DMM leads in each end of the tip holder. Place the positive lead immediately next to the 6.0 V point charge, slightly off-center of a line that would connect the two point charges. Hold the positive lead steady and pivot the ground lead around it, noticing the $\Delta V$ readings on the voltmeter changing.
16. Pivot the black lead until you find the maximum reading on the voltmeter. Mark the location of the black lead by pressing the tip into the paper to make an indentation.
17. Move the red lead to the indentation made by the black lead. Pivot the black lead around this location as before, finding the direction of greatest voltage change.
18. Continue to “walk” the leads across the paper until you reach the 0.0 V point charge. Connect the dots with the grease pencil. The resulting line is an electric field line.
19. Have your partner repeat this process, Step 15 - Step 18, beginning from a different position on the 6.0 V point charge. Remove the point charge posts and conductive paper. Keep the paper as data.

**PART 3: Parallel Plates**
20. Affix a new sheet of conductive paper to the circuit board using four posts as shown in Fig. 14.3. Use red posts for one parallel plate and black posts for the other.
21. Outline the parallel plates with the grease pencil.
**Experiment 14: Electric Fields and Potentials**

*Equipotentials*

22. Locate the 2.0 V equipotential (as in Part 2). Connect the dots and label its voltage. Extend it at least three points past the ends of the plates.

23. Have your partner repeat this process for the 4.0 V equipotential.

*Electric Field Strength*

24. Measure the field strength at the center of the paper, between the plates (as in Part 2). Mark the position and field strength on the paper.

25. How does this compare to the average electric field between the two plates? Calculate the average electric field \((\Delta V/\Delta x)\) between the plates, then find the percent difference of your measured value.

*Electric Field Lines*

26. It is known that the electric field between two plates is a series of parallel lines going straight from one plate to the other. Locate and draw two of the field lines outside of the ends of the parallel plate configuration.

27. Unplug and organize the equipment on your table. Keep the conductive paper as data. Staple one of the charge arrangements to the back of your datasheet; staple the other to your lab partner’s datasheet.

**QUESTIONS**

1. Refer to Table 14.1. Show that the electric field unit of N/C equals V/m.

<table>
<thead>
<tr>
<th>Current</th>
<th>Amp [A]</th>
<th>Coulomb/second</th>
</tr>
</thead>
<tbody>
<tr>
<td>Potential Difference</td>
<td>Volt [V]</td>
<td>Joule/Coulomb</td>
</tr>
<tr>
<td>Power</td>
<td>Watt [W]</td>
<td>Joule/second</td>
</tr>
<tr>
<td>Resistance</td>
<td>Ohm [Ω]</td>
<td>Volt/Amp</td>
</tr>
</tbody>
</table>

Table 14.1: Electric Quantities and Units
Experiment 15: Ohm’s Law

Figure 15.1: Simple Series Circuit

**EQUIPMENT**

Universal Circuit Board
Power Supply
(2) DMM’s
150 Ω Resistor \( (R_1) \)
330 Ω Resistor \( (R_2) \)
560 Ω Resistor \( (R_3) \)
Miniature Light Bulb and Socket \( (R_4) \)
(1) Jumper
(6) Wire Leads

Figure 15.2: Schematic: Simple Series Circuit
Advance Reading

Text: Ohm’s Law, voltage, resistance, current.

Lab Manual: Appendix B, Appendix C - DMM

Objective

The objective of this lab is to determine the resistance of several resistors by applying Ohm’s Law. Students will also be introduced to the resistor color code and refresh their graphing skills.

Theory

Ohm’s Law states that the current, \( I \), that flows in a circuit is directly proportional to the voltage, \( V \), across the circuit and inversely proportional to the resistance, \( R \), of the circuit:

\[
I = \frac{V}{R} \quad (15.1)
\]

In this experiment, the current flowing through a resistor will be measured as the voltage across the resistor is varied. From the graph of this data, the resistance is determined for Ohmic resistors \( (R_i, \ i = 1, 2, 3) \). Non-Ohmic resistors \( (R_4, \ \text{light bulb}) \) do not obey Ohm’s Law.

Ammeters are connected in series so that the current flows through them. The ideal ammeter has a resistance of zero so that it has no effect on the circuit. Real ammeters have some internal resistance.

Voltmeters are connected in parallel to resistive elements in the circuit so that they measure the potential difference across (on each side of) the element. The ideal voltmeter has infinite internal resistance. Our voltmeters have approximately 10 MΩ \( (10 \times 10^6 \ \Omega) \) internal resistance so that only a minuscule amount of current can flow through the voltmeter. This keeps the voltmeter from becoming a significant path for current around the element being measured.

Resistors are labeled with color-coded bands that indicate resistance and tolerance. The first two color bands give the first two digits of the value (Fig. 15.3). The third band gives the multiplier for the first two, in powers of 10. The last band is the tolerance (Fig. 15.3), meaning the true value should be \( \pm x\% \) of the color code value. Refer to Table 15.1 for standard color values.

There is no need to memorize the color codes for lab.

For example, a resistor that has two red bands and a black multiplier band has a resistance of 22 Ω.

<table>
<thead>
<tr>
<th>Color</th>
<th>Number</th>
<th>Multiplier</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>10^0</td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>10^1</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>10^2</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>10^3</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>10^4</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>10^5</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>10^6</td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>10^7</td>
</tr>
<tr>
<td>Grey</td>
<td>8</td>
<td>10^8</td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>10^9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Tolerance</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gold</td>
</tr>
<tr>
<td>Silver</td>
</tr>
<tr>
<td>(No Band)</td>
</tr>
</tbody>
</table>

Table 15.1: Resistor Color Code Values

![Color Code Schematic](image)
1. Write the equation and a qualitative statement for Ohm’s Law. (20 pts)

2. What are “ohmic” and “non-ohmic” devices? (20 pts)

3. Complete the following statement: An ideal ammeter has an internal resistance of ____________, while an ideal voltmeter has an internal resistance of ____________. Explain why these are desirable attributes for the respective measuring instruments. (20 pts)

4. If $I \text{ vs. } V$ is plotted, what value is obtained from the slope? Note that we are investigating the function $I = V/R$ and fitting our data to the slope-intercept equation of a line. (40 pts)
80  

**PROCEDURE**

**PART 1: Measures of Resistance**

1. Determine the nominal resistance for the three resistors: interpret the color codes according to the color code chart in Table 15.1.

2. Measure the actual resistance, $R$, of the three resistors using the ohmmeter and record them in the table provided.

3. An ideal ammeter has no resistance; this ammeter does have a small resistance. Measure the resistance of the ammeter (200 mA DCA).

**PART 2: Ohm’s Law Applied**

4. Build a simple series circuit using $R_1$, an ohmmeter, an ammeter, and a jumper (This will look similar to Fig. 15.1, but without the power supply).

5. Measure the equivalent resistance of the circuit using the ohmmeter and record this value in the table provided. Include units and uncertainty.

6. Remove the ohmmeter and connect the unplugged power supply to the circuit. Connect a voltmeter to the circuit, across the power supply leads (in parallel).

7. Have your TA check your circuit. Plug in the power supply and turn it on.

8. Test Ohm’s Law ($V = IR$) by verifying that the current increases linearly with applied voltage. Apply 1 V, 2 V, 3 V, and 4 V to the circuit. Measure current and voltage and record them in the table provided. Include units and uncertainty.

9. Repeat the Part 2 procedure for $R_2$ and $R_3$.

**PART 3: Non-Ohmic Device**

10. Build a series circuit using $R_4$, the light bulb (Fig. 15.4).

11. Measure the current and voltage as you increase the applied voltage in 0.2 V increments up to 2.0 V, then continue in 1.0 V increments up to 4.0 V. Adjust the voltmeter scale to obtain the most significant figures possible.

12. Turn off and unplug the power supply; turn off the DMM’s.

**PART 4: Graphing**

13. Open Graphical Analysis. Enter all of your voltage and current data as four separate data sets (one for each resistor). Include the point (0,0) in each set. [Other graphing software may be used, provided the graphs include all requisite elements.]

14. Plot $I$ vs. $V$ for the three Ohmic resistors on one graph. Apply a linear fit to each one.

15. Calculate the resistance of each circuit using the slope of your $I$ vs. $V$ graphs. Compare these $R_{\text{graph}}$ values to the measured $R_{\text{eq}}$ values using the percent difference formula (Eq. A.2, Page 141).

16. Plot a separate $I$ vs. $V$ graph for the light bulb.

17. Print a copy of both graphs.
Experiment 15: Ohm’s Law

QUESTIONS

1. Read the information in the next column. How much current would it take to cause pain? What was the maximum current you measured for this experiment?

2. Why was there no danger to you while you performed this experiment? The current required for this experiment is as high as 30 mA. Some experiments will require current as high as 5.0 A. Explain why there will be no danger to you. Read the information in the next column again, more carefully if necessary.

3. Is the graph of $I$ vs. $V$ for the light bulb linear? What does this tell you about the resistance of a light bulb as the filament gets hotter?

4. Compare the experimental (DMM, graph) values for each ohmic resistor.

5. Do the experimental values fall within the tolerance of the resistors? What might cause the values to exceed the tolerance?

6. The power output of a circuit is given by:

$$P = I^2R = \frac{V^2}{R} = IV \quad (15.2)$$

The resistors used in this experiment are 2-watt resistors. What is the maximum power output of $R_1$ when 9.0 V is applied across it (use your graph value)?

7. Calculate the power output of each ohmic resistor (use your graph value) when a potential of 7.00 V is applied.

8. Verify, using only the units provided in Table 14.1, that each part of Eq. 15.2 is equal to J/s. What is the unit of power output?

Can voltage kill you?

It’s actually current that kills. So why are “Hazardous Voltage” signs so prevalent? Paul Hewitt\(^1\) explains it very nicely:

Consider Ohm’s Law: $V = IR$. What is the resistance of your skin? That depends on the state of your skin: dry or wet. If it’s wet, is it water or sweat? Sweat, of course, contains salt; salt water is a good conductor.

The resistance will be dramatically different for different situations! Very dry skin has a resistance of about 500,000 Ω, while skin wet with salt water has a resistance of about 100 Ω. Once the voltage of a device and your skin’s resistance are known, we can estimate the current that will flow through your body.

<table>
<thead>
<tr>
<th>Current</th>
<th>Effect</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001 A</td>
<td>Can be felt</td>
</tr>
<tr>
<td>0.005 A</td>
<td>Is painful</td>
</tr>
<tr>
<td>0.010 A</td>
<td>Causes involuntary muscle contractions (spasms)</td>
</tr>
<tr>
<td>0.015 A</td>
<td>Causes loss of muscle control</td>
</tr>
<tr>
<td>0.070 A</td>
<td>If through the heart, causes serious disruption; probably fatal if current lasts for more than 1 second</td>
</tr>
</tbody>
</table>

Table 15.2: Effect of Electric Current on the Body

Note that the effect caused by these currents are approximate values. It is quite difficult to get volunteers for this area of research!

---

Experiment 16: Series and Parallel Circuits

Figure 16.1: Series Circuit

Figure 16.2: Parallel Circuit
Figure 16.3: Combination Circuit

EQUIPMENT

Universal Circuit Board
(2) 100-Ω Resistors
(2) 200-Ω Resistors
(2) 300-Ω Resistors
(2) Digital Multi-Meters
Power Supply
(5) Jumpers
(6) Wire Leads
Experiment 16: Series and Parallel Circuits

Advance Reading

Text: Resistors in series, parallel, combination.

Lab Manual:
Appendix B
Appendix C - DMM

Objective

The objective of this lab is to study circuits with resistors connected in series, parallel, and combination and to determine the internal resistance of an ammeter.

Theory

In the previous experiment, you constructed 4 circuits, each circuit built with one resistive element. In this experiment, you will construct circuits using multiple resistors.

The first type of circuit you will construct is a series circuit (Fig. 16.1 and Fig. 16.4). In a series circuit, the resistors are connected end-to-end such that the current is the same through each resistor: The current has only one path available. The voltage drop across each resistor depends on the resistor value.

For a series circuit, the total equivalent resistance, $R_{eq}$, is:

$$R_{eq} = R_1 + R_2 + R_3 + \cdots + R_N = \sum_{i=1}^{N} R_i \quad (16.1)$$

(Resistors in Series)

The second type of circuit you will construct is a parallel circuit (Fig. 16.2 and Fig. 16.5). Resistors are said to be in parallel when they are connected end-to-end such that the current is the same through each resistor. The current through each resistor depends on the resistor value. The current has more than one path available and takes all available paths.

For a parallel circuit, the total equivalent resistance, $R_{eq}$, is:

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N} = \sum_{i=1}^{N} \frac{1}{R_i} \quad (16.2)$$

(Resistors in Parallel)

The third type of circuit you will construct is a combination circuit (Fig. 16.3 and Fig. 16.6). Resistive elements are not connected in series or parallel. To calculate the total equivalent resistance of a combination circuit, it should first be simplified (reduced to an equivalent resistor, $R_{eq}$). This is done by choosing resistors that are connected in either series or parallel, one step at a time, adding those elements by use of Eq. 16.1 or Eq. 16.2, then proceeding to the next set of elements.

Note that it is not correct to, for example, calculate the resistance of the 3 resistors across the top of the circuit using Eq. 16.1, and then calculate the resistance of $R_4$, $R_5$, and $R_6$ using Eq. 16.2. You must identify which resistors are either in parallel or in series, then apply the appropriate equation one step at a time.
Name: ____________________________________________

1. What is a series circuit? (10 pts)

2. What is a parallel circuit? (10 pts)

3. Is the equivalent resistance, \( R_{eq} \), of a series circuit greater than or less than any individual resistor? (10 pts)

4. Is the equivalent resistance, \( R_{eq} \), of a parallel circuit greater than or less than any individual resistor? (10 pts)

5. Calculate \( R_{eq} \) for each of the first three circuits shown in Fig. 16.4 - Fig. 16.6 using the stated nominal values for resistance. (Show all work on back of this sheet.) (25 pts)

6. You will plot \( I \ vs. \ V \) for each of the three circuits on one graph. What value should each slope have (use the stated values for resistance)? (25 pts)

7. Create **Data Tables** in your lab notebook for all parts of this experiment. Sketch the column headings on the back of this sheet. (10 pts)
Experiment 16: Series and Parallel Circuits

PROCEDURE

PART 1: Series Circuit

Record all data in table format.

Recall that \( i = 1, 2, \ldots, n \)

1. Measure \( R_i \), then construct a series circuit (Fig. 16.4) with 100-Ω, 200-Ω, and 300-Ω resistors and ammeter (200 mA DCA); do not connect the power supply yet.

2. Draw the schematic using measured \( R_i \)’s.

3. Calculate \( R_{eq} \).

4. Measure \( R_{eq} \).

5. Connect the unplugged power supply and the voltmeter (DCV) to your circuit.

Get instructor approval of your circuit

6. Always be sure the power supply is turned off before you plug it into an outlet. Plug in the power supply, and set the voltage to 1.00 V. Measure the current and voltage.

7. Record the current (A) and the voltage (V) as you increase the voltage in 1.0 V increments up to 4.0 V.

8. Leave the voltage at 4.0 V; disconnect the voltmeter from the power supply. Maintaining the same orientation of the leads (if clockwise, black follows red), measure \( V_i \).

9. Add these potential differences (\( \sum_{i=1} V_i \)).

10. Does \( \sum_{i=1} V_i \) equal \(-4.0\) V? If not, ask your TA for guidance.

PART 2: Internal Resistance of an Ammeter

11. Turn off, then unplug and disconnect the power supply and the ammeter from the circuit. (Note that you will need to insert a jumper when the ammeter is disconnected in order to complete the circuit.)

12. Measure \( R_{eq} \) of the circuit.

13. Insert the ammeter (scale: 200 µA DCA) in the circuit.

14. Measure \( R_{eq} \).

15. As you change the scale of the ammeter, measure \( R_{eq} \) for each of the ammeter scales (e.g., 2 mA, 20 mA). You will have a total of 8 \( R_{eq} \)’s.

PART 3: Parallel Circuit

16. Repeat Part 1, Step 1 - Step 8, for the parallel circuit (Fig. 16.5).

17. Does \( V_2 = V_1 + V_A \), or does \( V_2 = V_A = V_3 \)? Are each of these values negative or positive? Yes, it matters!

18. Does \( V = |V_1 + V_A| \)?

PART 4: Combination Circuit

19. Repeat Part 1, Step 1 - Step 8, for the combination circuit (Fig. 16.6).

PART 5: Graphing

20. Graph \( I \) vs. \( V \) for each of the first three circuits on one graph (Part 1, Part 3, and Part 4).

21. Part 5 of this experiment may also be on an exam. Be certain you know how to produce a complete graph. Ask for help if needed.

QUESTIONS

1. Why should the voltage drops (electric potential differences) across the resistors connected in parallel be the same? Were your values equal?

2. Calculate the equivalent resistance of each of the first three circuits you constructed for this experiment using your measured values. Show each step in this process (math and schematic). Remember to include \( R_A \) in your calculations.

3. Consider your data from Part 2. Create a table similar to the one shown below. Why does \( R_{eq} \) change when you change the scale of the ammeter?

<table>
<thead>
<tr>
<th>Ammeter Scale</th>
<th>( R_{eq} )</th>
<th>( R_A )</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Ammeter</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 µA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 µA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 mA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 mA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200 mA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 A</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20 A</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Experiment 17: Kirchhoff’s Laws for Circuits

Figure 17.1: Kirchhoff’s Law Circuit Board

Figure 17.2: Schematic for Kirchhoff’s Circuit

**EQUIPMENT**

1. Universal Circuit Board
2. D-Cell Batteries (1.5 V)
3. Battery Holders
4. Alligator Clips
5. DMM

- Resistors:
  - $R_1 = 10 \, \Omega$ Resistor
  - $R_2 = 12 \, \Omega$ Resistor
  - $R_3 = 15 \, \Omega$ Resistor
  - $R_4 = 18 \, \Omega$ Resistor
  - $R_5 = 22 \, \Omega$ Resistor
Advance Reading

Text: Kirchhoff’s Voltage Law, Kirchhoff’s Current Law

Lab Manual: Appendix D: Math Review (solving 3 equations with 3 unknowns)

Objective

The objective of this experiment is to apply Kirchhoff’s rules for circuits to a two-loop circuit to determine the three currents in the circuit and the electric potential differences around each loop.

Theory

The two basic laws of electricity that are most useful in analyzing circuits are Kirchhoff’s laws for current and voltage.

Kirchhoff’s Current Law (KCL) states that at any junction (node) of a circuit, the algebraic sum of all the currents is zero (sum of the currents entering the junction equals the sum of the currents leaving the junction). In other words, electric charge is conserved.

\[ \Sigma I_{in} = \Sigma I_{out} \]  

(17.1)

Kirchhoff’s Voltage Law (KVL) states that around any closed loop or path in a circuit, the algebraic sum of all electric potential differences is equal to zero.

\[ \Sigma V_i = 0 \]  

(17.2)

To calculate magnitudes of current and voltage in a circuit like Fig. 17.2, you will need to write two equations, making use of KVL and Ohm’s Law. This results in two equations with two unknowns. For this experiment, you will measure \( \varepsilon_1 \) and \( R_1 \), then solve for the two currents, \( I_1 \).

One might be able to guess the direction of current flow in a circuit, given a circuit such as the one in this experiment. However, the current direction can be safely ignored when using the loop method. For the purposes of this experiment, all currents will be assumed to be clockwise. If any current is measured or calculated to be negative, that current actually flows counterclockwise in the circuit.

Apply the following rules when writing a KVL equation for a loop:

- If a seat of emf is traversed from \(-\) to \(+\), the change in potential is \(+\varepsilon\); if it is traversed from \(+\) to \(-\), the change in potential is \(-\varepsilon\). *(Note that you must maintain orientation as you progress around each loop. \( \varepsilon_1 \) is traversed from \(-\) to \(+\), while \( \varepsilon_2 \) is traversed from \(+\) to \(-\).)*

- Current flows from high potential to low potential. Crossing a resistance with the current constitutes a negative potential difference. Measuring against the current yields a positive potential difference.

![Positive ΔV and Negative ΔV](Figure 17.3: Potential Difference Sign Convention)

Write an equation for each loop in terms of electric potential difference. The electromotive force, emf (\( \varepsilon \), voltage source), is provided by a D-cell battery.

For example, the equation for the left loop in Fig. 17.2 is:

\[ \varepsilon_1 + V_1 + V_2 + V_3 = 0 \]  

(17.3)

Rewrite this equation using Ohm’s Law, then simplify:

\[ \varepsilon_1 - I_1(R_1 + R_2 + R_3) + I_2R_2 = 0 \]  

(17.4)

A similar equation is written for the right loop. Taken together, the two loop equations can be solved for \( I_1 \) and \( I_2 \) by means of substitution. Theoretical potential differences can then be calculated using Ohm’s law.
1. Write the equation, then briefly explain: (20 pts ea.)

(a) **Kirchhoff’s Voltage Law (KVL)**

(b) **Kirchhoff’s Current Law (KCL)**

2. Consider the circuit shown in Fig. 17.2 and the *Equipment* list on Page 89. Use Kirchhoff’s Voltage Law to solve for the theoretical currents in each of the three branches of the circuit. Let us define two distinct loops of the circuit with currents $I_1$ and $I_2$.

Voltage sums for the left loop and for the right loop are written using KVL:

\[
\begin{align*}
\varepsilon_1 - V_1 - V_2 - V_3 &= 0 \quad \text{(KVL - left loop)} \\
-\varepsilon_2 - V_4 - V_5 + V_2 &= 0 \quad \text{(KVL - right loop)}
\end{align*}
\]

In performing experiment, measured values will be used for $\varepsilon_i$ and $R_i$. For the pre-lab, use the nominal values as stated in the *Equipment* list. Recall that the electric potential difference, or voltage, across a resistor has the same subscript as the resistor. For example, the voltage across resistor $R_6$ is $V_6$. Use the back of this sheet for *Question 2*.

(a) Rewrite the KVL equations using Ohm’s Law (e.g., $V_1 = I_1 R_1$). (30 pts)

(b) Solve for $I_1$ and $I_2$ using substitution. Refer to *Appendix D* as necessary. (30 pts)
**PROCEEDURE**

**PART 1: Loop Method - Calculations**

1. Determine the nominal resistance and tolerance of each resistor by reading its color code (Table 15.1, Page 78). They should have the following approximate resistances:
   \[
   \begin{align*}
   R_1 &= 10 \, \Omega \\
   R_2 &= 12 \, \Omega \\
   R_3 &= 15 \, \Omega \\
   R_4 &= 18 \, \Omega \\
   R_5 &= 22 \, \Omega 
   \end{align*}
   \]

2. Measure the resistance of each resistor using an ohmmeter.

3. Construct the circuit shown in Fig. 17.2. Do not connect the ammeter.

4. Measure \( \varepsilon \) of the two batteries using a voltmeter. They should each be at least 1.1 V. Turn off the DMM and disconnect the batteries so they do not drain.

5. Using your knowledge of Kirchhoff’s Voltage Law, write two equations relating the potential differences across each element in the two loops. Remember that both currents flow through the central branch of the circuit.

6. Solve these equations by substitution to find the theoretical currents in each loop. A negative value simply indicates the current flows in the other direction.

**PART 2: Current & Voltage Laws Applied**

7. Connect the batteries to the circuit.

8. Measure the current in each of the three branches of the circuit. Refer to Fig. 17.4 for proper ammeter connection technique. Disconnect the batteries and turn off the DMM after measurement.

9. Compare the measured values of current with the calculated values. If they are not approximately equal, check your calculations or retest the circuit.

10. Reconnect the batteries and measure the electric potential across each element of the circuit. Sign and direction are crucial; measure clockwise, leading with the red lead and following with black.

**PART 3: Non-Ideal Voltmeter**

At the front of the room, your TA has set up two series circuits. One circuit has two 100 \( \Omega \) resistors, the other circuit has two 10.0 M\( \Omega \) resistors. Take your voltmeter to this table. Adjust the power supply on each circuit to 10.0 V.

11. Measure the potential differences across each of the resistors in the 100 \( \Omega \) circuit. Is the magnitude of their sum equal to the potential difference across the power supply? Show work.

12. Measure the potential difference across each of the resistors in the 10.0 M\( \Omega \) circuit. Is the magnitude of their sum equal to the potential difference across the power supply? Show work.
QUESTIONS

1. Explain what effect the DMM will have on the circuit when inserted to measure current.

2. Do the values from Part 2 verify Kirchhoff’s Current Law?

3. Do the values from Part 2 verify Kirchhoff’s Voltage Law?

4. Would disconnecting the power supply on the left loop of the circuit, $\varepsilon_1$, affect the current $I_2$ (no jumper inserted)? Calculate what $I_2$ is for this case. Comment on $I_3$.

5. A voltmeter is connected in parallel to a resistor when measuring $V_i$. Remember that the internal resistance of the DMM, when used as a voltmeter, is approximately 10 MΩ. Calculate $R_{eq}$ of the voltmeter connected to a resistor in each circuit of Part 3.

6. Use the above information to discuss why the voltage measured across the 10 MΩ resistors did not equal the voltage measured across the power supply.
Experiment 18: Earth’s Magnetic Field

Figure 18.1: Earth’s Magnetic Field - Note that each of the 3 elements of the circuit are connected in series. Note the large power supply: large power supply → large current. Use the 20A jack and scale of the ammeter.

Figure 18.2: Earth’s B-Field Schematic

**EQUIPMENT**
- Tangent Galvanometer
- Ammeter (20A jack, 20A DCA)
- Dip Needle
- Large Power Supply
- (2) 12” Wire Leads
- (2) 36” Wire Leads


**Advance Reading**

*Text:* Magnetic field, vectors, right-hand rule for a wire loop, resistivity.

**Objective**

The objective of this lab is to measure the magnitude of Earth’s magnetic field in the lab.

**Theory**

The magnetic field of Earth resembles the field of a bar magnet. All magnetic field lines form a closed loop: a field line originates at the north pole of a magnet, enters the south pole, then moves through the magnet itself back to the north pole. Although we usually think of this field as two-dimensional (north, south, east, west), remember that it is, in fact, a three-dimensional vector field.

The horizontal component of the magnetic field of Earth is typically measured using a compass. The needle of a compass is a small magnet, which aligns with an external magnetic field. Recall that opposite poles attract, and like poles repel. Thus, the north pole of the compass needle points to the south magnetic pole of Earth, which is sometimes close to the geographic north pole.

We will measure the horizontal component of Earth’s magnetic field, $B_e$, then use this information to determine the magnitude of the total magnetic field of Earth, $B_t$.

Determining the magnitude of an unknown magnetic field can be accomplished by creating an additional, known magnetic field, then analyzing the net field. The magnetic fields will add (vector math) to a net magnetic field (resultant vector).

$$B_{\text{net}} = B_{\text{known}} + B_{\text{unknown}} \quad (18.1)$$

The known magnetic field, $B_{\text{galv}}$, will be produced by use of a *tangent galvanometer*. A tangent galvanometer is constructed of wire loops with current flowing through the loops. The current produces a magnetic field. The magnitude of this magnetic field depends on the current, the number of loops, and the radius of each loop:

$$B_{\text{galv}} = \frac{\mu_0 I N}{2r} \quad (18.2)$$

where $\mu_0 = 4\pi \times 10^{-7}$ Tm/A is the permeability constant, $I$ is the current, $N$ is the number of loops, and $r$ is the radius of the loop.

The direction of the magnetic field of a current-carrying wire is given by the right-hand rule. When the thumb of the right hand points in the direction of the current (positive current; conventional current), the fingers will curl around the wire in the direction of the magnetic field. Refer to Fig. 18.3.

Figure 18.3

The coil of the tangent galvanometer is first aligned with the direction of an unknown field, $B_e$, or north. The compass inside the tangent galvanometer allows accurate alignment. Once current begins flowing, the two magnetic fields will add (vector addition) to yield a resultant magnetic field. The compass needle then rotates to align with the net field. The deflection angle $\alpha$ is the number of degrees the compass needle moves. $\alpha$ is measured, and $B_e$ is calculated from:

$$\frac{B_{\text{galv}}}{B_e} = \tan \alpha \quad (18.3)$$
Experiment 18: Earth’s Magnetic Field

A typical compass is constrained to 2 dimensions and rotates to point to Earth’s magnetic south pole, which is (approximately) geographic north. Earth’s magnetic field, however, is a 3 dimensional phenomenon. It has components that point into and out of the earth, not just along the surface. We need to measure at our location the direction of the total magnetic field of Earth (the angle $\theta$).

To determine field declination, $\theta$, we will use a dip needle. A dip needle (Fig. 18.5 and Fig. 18.6) is a compass that rotates. It measures both horizontal and vertical angles.

First, arrange the dip needle in a horizontal position, compass needle and bracket aligned, pointing north (normal compass). Refer to Fig. 18.5, below, for clarification. The needle should align with $270^\circ$.

Now rotate the compass $90^\circ$ (Fig. 18.6) to a vertical position. The needle rotates to a new angle; the difference between the initial angle and the final angle is the angle $\theta$.

From Fig. 18.6, we see that the dip needle points in the direction of Earth’s total magnetic field at our location.

By determining the magnitude of the horizontal component of Earth’s magnetic field, $B_e$, using $\alpha$, and measuring the direction of Earth’s total magnetic field, $B_t$, using $\theta$, the magnitude of $B_t$ can be determined. (Refer to Fig. 18.7.)
Prelab 18: Earth’s Magnetic Field

Name: _________________________________

1. What physical phenomenon does the relationship $B_{galv} = \frac{\mu_0 NI}{2r}$ describe? (10 pts)

2. Explain the right-hand rule for current. (10 pts)

3. Consider Fig. 18.4. Determine the following in terms of $B$’s ($B_e$, $B_{galv}$, and $B_{net}$). (10 pts)
   \[
   \sin \theta = \\
   \cos \theta = \\
   \tan \theta =
   \]

4. Consider Fig. 18.7. Determine the following in terms of $B$’s ($B_e$, $B_z$, and $B_t$). (10 pts)
   \[
   \sin \alpha = \\
   \cos \alpha = \\
   \tan \alpha =
   \]

5. Given $B_e$ of $45 \times 10^{-6}$ T and a dip angle of $55^\circ$, calculate $B_z$. See Fig. 18.7. (30 pts)
6. Consider the top-view diagram of the tangent galvanometer, Fig. 18.11. Given the galvanometer’s alignment with North, as shown, indicate the direction that current flows through the top of the wire loops. (30 pts)
**PROCEDURE**

**PART 1: Horizontal Component**

1. Connect the galvanometer ($N = 5$), ammeter (20A DCA), and power supply in series.

2. Align the galvanometer such that it creates a magnetic field perpendicular to that of Earth’s field (the compass needle should be parallel to the wire loop). Do not move the galvanometer while taking data.

3. Turn on the power supply to flow current through the galvanometer. Adjust the current until the compass needle on the galvanometer reaches 30°, 40°, and 50°, recording the current required for each position in the data table provided.

4. Repeat this process for $N = 10$ and $N = 15$ (a total of nine trials).

5. Calculate the average horizontal field, $B_e$, for each of the nine trials using Eq. 18.2 and Eq. 18.3. The diameter of the coils is approximately 20 cm. $[\mu_0 = 4\pi \times 10^{-7} \text{ Tm/A}]$

6. Find the average value of $B_e$ from your nine trials.

**PART 2: Field Declination**

7. Use the dip needle to determine the direction of magnetic north. Align the dip needle’s supporting arm with its compass needle (pointing north).

8. Roll the compass arm 90° until the bracket is vertical (and still pointing north/south).

9. Record the declination of Earth’s magnetic field, the angle $\theta$ from the horizontal.

10. Use this declination to calculate the magnitude of the total magnetic field of the Earth, $B_t$, in the lab. Refer to Fig. 18.7.

**QUESTIONS**

1. Calculate the total resistance of 10 loops of copper wire of the galvanometer if the wire is 1 mm in diameter and the loops are 20 cm in diameter: $R = \rho L/A$. $L$ is the length of the wire, $A$ is the cross-sectional area of the wire, $\rho$ is the resistivity of copper; look it up in your text, a CRC text, or online.

2. Compare your measured $B_t$ from this experiment to a sample value of 43 $\mu$T. This is the magnitude of the magnetic field in Tucson, Arizona.
Experiment 19: The Current Balance

From Left to Right: Power Supply, Current Balance Assembly, Ammeter (20A DCA scale, 20A jack). North pole of magnet is red; south pole of magnet is white (standard). The magnetic field is from north to south (red to white).

Equipment

*Pasco* Current Balance Apparatus
Digital Balance
Large Power Supply
Current Limiting Resistor
(If new power supply, no resistor)
Ammeter (20A jack, 20A DCA)
(6) Wire Leads
Rod
Clamp
**Advance Reading**

*Text:* Magnetic force, magnetic field.

**Objective**

The objective of this experiment is to measure the effects of a magnetic field on a current carrying conductor.

**Theory**

A magnetic field exerts a force, \( \vec{F}_B \), on a moving charge. The magnitude of \( \vec{F}_B \) is:

\[
F_B = qvB \sin \theta
\]  

(19.1)

where \( q \) is the charge, \( v \) is the magnitude of the velocity (speed) of the charge, \( B \) is the magnitude of the magnetic field strength, and \( \theta \) is the angle between the direction of the magnetic field and the direction of the charge velocity.

Current is a collection of charges in motion; thus, a magnetic field also exerts a force on a current carrying conductor.

The magnitude and direction of this force is dependent on four parameters:

1. Magnitude of the current, \( I \)
2. Length of the wire, \( L \)
3. Strength of the magnetic field, \( B \)
4. Angle between the field and the current, \( \theta \)

The magnitude of the magnetic force in this case is given by:

\[
F_B = ILB \sin \theta
\]  

(19.2)

**Experiment 19: The Current Balance**

In this experiment, a current-carrying wire will be placed between the two poles of a magnet. When current is flowed perpendicular to the magnetic field, the wire and the magnet will either push or pull on each other in the vertical direction. We quantify the force on the wire using Eq. 19.2; by Newton’s Third Law, the force exerted on the magnet will be equal and opposite.

The force on the magnet can be determined by measuring differences in the magnet’s apparent weight on a scale as the current-carrying wire pushes it up or down. While the scale gives readings of “mass,” the associated “weight” or force, \( F_B \), can be calculated:

\[
F_B = mg
\]  

(19.3)

Current, wire length, and angle will be varied one at a time, and magnetic force will be measured in order to determine the magnetic field strength, \( B \), of each magnet. Current and length will be varied for one magnet (\( B_1 \)), then \( \theta \) will be varied for a second magnet (\( B_2 \)).

Consider Eq. 19.2. At what angle is \( F_B \) at a maximum value? At what angle is \( F_B \) at a minimum value?

When we investigate the force due to either a change in current or a change in the length of wire, we will want the maximum force. This allows us to compare 2 values for the magnetic field strength, \( B \). We must, therefore, be certain that the current carrying wire and the magnetic field of the magnet are perpendicular to each other (90°). Refer to Fig. 19.1.

When we investigate the force due to a change in angle, we must find a way to be sure of our initial angle, 0°. What will \( F_B \) equal if the wire and the magnetic field are parallel? With this arrangement, a non-zero current will result in a force of 0.0 N. If the balance has a non-zero mass reading, we know that the wires and the field are not parallel.
1. What physical phenomenon does the relationship \( F = qvB \sin \theta \) describe? (15 pts)

2. What physical phenomenon does the relationship \( F = ILB \sin \theta \) describe? (15 pts)

3. Magnetic force is dependent upon what four factors for this experiment? (15 pts)

4. In this experiment you will plot four sets of data (refer to Part 19 of the procedure). Sketch a sample plot for each data set on the back of this sheet. You will not need numerical values. Label the axes. State the slope in algebraic terms. (35 pts)

5. Consider the apparatus below. It represents a magnet sitting on the mass pan of a balance. Refer to Fig. 19.1 and Fig. 19.3 (Step 3 of the experiment). Given the following setup, does the balance read heavier or lighter? Explain using the right-hand rule and Newton’s Third Law. (20 pts)
PROCEDURE

PART 1: Force vs. Current ($\Delta I$)

1. Build the circuit as shown in Fig. 19.2 using the current loop numbered SF 42.

2. Locate the magnet for the wire loops, $B_1$ (Fig. 19.3), and center the magnet on the balance pan. On the digital balance, there is a “Zero” button. Push this button once to “tare” the balance (zero it).

3. Lower the balance arm so the wire loop passes through the pole region of the magnet (i.e., the horizontal section of the wire is just below the top edge of the magnet, Fig. 19.3). Note that this will result in the wire loop and the slot of the magnet being parallel.

4. Refer to Fig. 19.1 and Fig. 19.2 to verify that you have the circuit built correctly. Get instructor approval of your circuit. Be sure the power supply is turned off before you plug it in.

5. Plug in the power supply. Adjust the power until the current is approximately 0.5 A. When the current is on, the magnet should be repelled (deflected downward; positive mass reading). If it is attracted (pulled upwards; negative mass reading), reverse the wires connected to the power supply, turn off the balance, and start again from Step 1.

6. Measure $m$ and $I$.

7. Repeat this procedure as you increase the current in 0.5 A increments through 5.0 A.

8. When data collection is complete, turn off the power supply and DMM.

9. Unplug the power supply.

10. Calculate $F_B$ for each current value (Eq. 19.3).

PART 2: Force vs. Length of Wire ($\Delta L$)

Note: For this part of the experiment, you will need to know the effective length of the wire loops, which is listed in Table 19.1, below:

<table>
<thead>
<tr>
<th>Wire Loop</th>
<th>Length (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SF 40</td>
<td>1.2</td>
</tr>
<tr>
<td>SF 37</td>
<td>2.2</td>
</tr>
<tr>
<td>SF 39</td>
<td>3.2</td>
</tr>
<tr>
<td>SF 38</td>
<td>4.2</td>
</tr>
<tr>
<td>SF 41</td>
<td>6.4</td>
</tr>
<tr>
<td>SF 42</td>
<td>8.4</td>
</tr>
</tbody>
</table>

Table 19.1: Wire Lengths

11. Insert the shortest wire segment (i.e., SF 40) into the balance arm. Always verify that the wire loop is in the appropriate position before recording measurements.

12. Set the current to 2.5 A. Measure $m$.

13. Repeat for each of the wire loops.

14. Turn off the power supply and the DMM.

15. Unplug the power supply.


17. Return the wire loops and magnet to their box. The balance arm is used for Part 3 with the Current Balance Accessory (CBA, Fig. 19.4).
**Experiment 19: The Current Balance**

**PART 3: Force vs. Angle (Δθ)**

18. Center the variable-angle magnet, $B_2$ (Fig. 19.6), on the balance pan and tare the balance.

19. Plug the CBA into the balance arm. Adjust the height of the CBA as needed and lower it into the magnet.

20. The horizontal section of the wire loops must be just inside of, but not touching, the top edge of the magnet (Fig. 19.6).

21. Turn the CBA to 0° (Fig. 19.5).

22. Initially, the horizontal section of the wire loops of the CBA must be parallel to the magnetic field of the permanent magnet. If they are parallel and current is flowing, $F_B = 0.0 \, N$, and the mass measurement will not change. If the mass changes when the current is turned on, adjust the alignment by carefully rotating the balance.

23. When you have the correct arrangement, turn the CBA from 0° through 180°, checking to see that it will not hit the magnet at any point.

24. Return the CBA to 0°.

25. Set the current to 2.5 A.

26. Measure $m$ as you increase $θ$ in 10° increments through 180°.

27. Turn off the power supply and DMM.

28. Unplug the power supply.

29. Calculate $F_B$ for each $θ$. 

---

**Figure 19.4:** Current Balance Accessory (CBA) Arrangement for varying $θ$

**Figure 19.5:** Top view of CBA

**Figure 19.6:** Close-up of CBA: Wire Loops Inside Magnet; Assume $L = 11.5 \, cm$ for CBA
PART 4: Analysis

30. Plot separate graphs of the following data; one data table with 4 data sets. Determine if (0,0) is a valid data point for each data set. When analyzing your graphs, for $B_2$ assume that $L = 11.50$ cm.

- $F_B$ vs. $I$
- $F_B$ vs. $L$
- $F_B$ vs. $\theta$ (File $\Rightarrow$ Settings: Change radians to degrees). Apply a curve fit under Analyze $\Rightarrow$ Curve Fit (consider Eq. 19.2 when selecting an appropriate fit)
- $F_B$ vs. $\sin \theta$

QUESTIONS

1. Show that Eq. 19.1 and Eq. 19.2 have equivalent units.
2. Is the relationship between current and magnetic force linear?
3. What is the relationship between the length of a wire and magnetic force?
4. What is the shape of the curve for $F_B$ vs. $\theta$? How does it relate to Eq. 19.2?
5. Calculate and compare $B_1$ and $B_2$ from their respective graphs.
Experiment 20: Exponentials and Oscilloscopes

**EQUIPMENT**

Universal Circuit Board
(1) 680 kΩ Resistor
(4) Jumpers
(1) 47 µF Capacitor
(4) Wire Leads
Digital Multi-Meter (DMM)
Power Supply
Stopwatch

Figure 20.1: Part 2: RC Circuit

Figure 20.2: Exponential growth general equation is \( N = N_0 e^\lambda t \)

Exponential decay general equation is \( N = N_0 e^{-\lambda t} \)
Objective
The objective of Part 2 is to investigate an exponential curve by analysis of an RC circuit. Our RC circuit will be a resistor $R$ and a capacitor $C$ connected in parallel.

Theory
When the rate of change of a quantity is proportional to the initial quantity, there is an exponential relationship.

An exponential equation is one in which $e$ is raised to a power. $e$ is the irrational number 2.7182818\ldots.

An exponential equation has the form:

$$A(t) = A_0e^{Bt} + C \quad (20.1)$$

where $A$ is the amount or number after a time $t$ and $A_0$ is the initial amount or number.

Exponential rates are found everywhere in nature. Some examples include exponential growth (e.g., population of Earth) and exponential decay (e.g., decay of radioactive elements), as well as heating and cooling rates.

Analysis of exponential curves often involves the doubling-time or half-life ($T_{1/2}$). Doubling-time is the time required for an initial amount to double in quantity (what is $t$ when $A = 2A_0$?). The half-life is the time required for 1/2 of the initial amount to be gone (what is $t$ when $A = \frac{1}{2}A_0$?).

Analyzing these curves is often simple. For doubling-time or half-life, begin with the appropriate value on the $y$-axis. Then draw a horizontal line that intersects the plot. Next, drop a perpendicular line to the $x$-axis. Read the value of the time directly from the $x$-axis.

The data for our exponential curve will be obtained by measuring the potential difference across the resistor of an RC circuit as a function of time.

A capacitor is connected in parallel with a resistor, then charged to some initial voltage. When the power supply is disconnected, the potential difference across the capacitor will decrease exponentially. The voltage, $V$, across the capacitor as it discharges is given by:

$$V_t = V_0 e^{-t/RC} \quad (20.2)$$

where $V_0$ is the initial potential difference across the capacitor at time $t_0$, $R$ is resistance, and $C$ is capacitance.

Consider Eq. 20.2. When $t = RC$,

$$V_t = V_0 e^{-1} = V_0 \frac{1}{e} \quad (20.3)$$

We define $\tau$ (Greek letter, tau) to be the time it takes for the voltage to drop to $(1/e) \cdot (V_0)$ its original value (about 37\%). This is the value when $\tau = RC$.

For our discharging capacitor:

A graph of Voltage vs. Time is an exponential decay curve. Analysis of this curve provides the time constant by locating the point at which $V$ has dropped to $1/e$ its original value of $V_0$.
Part 1 EQUIPMENT

Circuit Gear Digital Oscilloscope
Oscillator
BNC Cable
DMM
BCN - Coaxial Adapter
Advance Reading (Oscilloscopes)

Text: Wavelength ($\lambda$), frequency ($f$), and period ($T$), oscilloscope, EKG.

Objective

The objective of Part 1 is to become familiar with oscilloscopes by measuring various quantities using an analog oscilloscope and a digital oscilloscope.

Theory

An oscilloscope is a measuring device that plots Voltage vs. Time. When a signal is received by the oscilloscope, a trace (line) appears on the screen. The signal could be generated from an oscillator, a tuning fork, or a heart monitor at the doctor’s office.

The screen of an analog oscilloscope has a grid, or gradecule, superimposed on it (Fig. 20.4). The gradecule is used as a reference to read information from the screen of an oscilloscope. The controls of an oscilloscope will adjust the view of an incoming signal; they will not adjust the signal itself.

Voltage

We will generate a periodic function (a sine wave; AC voltage) for Part 1 using an oscillator. Note that a sine wave has an amplitude of equal magnitude both above and below zero, resulting in an average value of zero. To obtain a meaningful value for this AC voltage, we calculate the root-mean-square, rms, of the voltage.

Refer to Fig. 20.5. We will determine, from the trace, the number of DIVISIONS for both the peak-to-peak voltage ($y$-axis) and the time required for one wavelength ($x$-axis).

$V_{max}$ is the amplitude of the wave (half of the peak-to-peak voltage).

The rms voltage, $V_{rms}$, is given by:

$$V_{rms} = \sqrt{\frac{(V_{max})^2}{2}} = 0.707V_{max} \quad (20.4)$$

Recall that once the period $T$ of a cyclic function is known, the frequency $f$ is given by:

$$T = \frac{1}{f} \quad (20.5)$$

One DIVISION

![One DIVISION](image)

Figure 20.4: Gradecule

![Gradecule Sample](image)

Figure 20.5: Gradecule Sample
Exponential Curve

1. Refer to Fig. 20.2. Consider the equations:

\[ y = Ae^{(C \cdot x)} + B \]
\[ N = N_0 e^{(\lambda t)} \]

What is \( N_0 \) [i.e. \( N_0 = 0 \)]? (You may read the information directly from the auto-fit box.) (10 pts)

2. Refer to Fig. 20.6, below. The doubling-time is the time it takes for the initial number or amount to double. What is the doubling-time \( (N = 2N_0) \)? (Find the value on the y-axis, then move horizontally until you intersect the plot. Drop a perpendicular line to the x-axis and read the time.) (20 pts)
Radioactive Decay - Carbon Dating (30 pts)

Assume Fig. 20.7 represents 100% of the carbon found in all living matter. Carbon 14 ($^{14}$C) has a half-life ($T_{1/2}$) of 5,730 years. The square represents a sample of $^{14}$C. $^{14}$C emits $\beta^-$ particles.

3. Divide the square in half with a vertical line and write “5,700 yrs” (rounded for simplification) on the left side, to represent the amount of $^{14}$C decayed after its half-life of 5,700 years ($T_{1/2} = 5,700$ yrs).

4. Now divide the right side in half with a horizontal line and write “11,400 yrs” to represent the amount of $^{14}$C decayed after an additional half-life.

5. Continue to divide the remaining sample in this manner to show the amount of $^{14}$C decayed after 17,100 yrs; 22,800 yrs; 28,500 yrs; 34,900 yrs; and 45,600 yrs.

Figure 20.7: Carbon Dating Square
6. Define frequency, wavelength, and period. (10 pts)

7. What does an oscilloscope plot? (10 pts)

8. Determine the period and peak-to-peak voltage of Fig. 20.8, below (refer to Fig. 20.4). (20 pts)
   Assume each major box is \( \frac{1}{2} \) V on the vertical scale and \( \frac{1}{2} \) ms on the horizontal scale.
PROCEDURE

PART 1: Oscilloscopes

1. Connect the oscilloscope to the computer. Open the Syscomp Circuitgear software on the desktop.

2. Plug in the oscillator and connect the output to the oscilloscope, as in Fig. 20.3. The signal has been set for you. You may shift the waveform by dragging the A, B, or X cursors at the edge of the gradecule. The horizontal and vertical scales can be adjusted by using the Timebase Controls and Vertical Controls, respectively.

3. Measure the peak-to-peak voltage of your trace (refer to Fig. 20.9). Note the VOLTS/DIV scale on-screen under Vertical Controls. You can use the measurement tool under View ⇒ Toggle Channel A Cursors to facilitate more accurate measurement. Record this value in the table provided.

4. Calculate $V_{\text{max}}$ (do not measure).

5. Calculate $V_{\text{rms}}$ (Eq. 20.4).

6. Measure the period, $T$, of your trace. Note the SEC/DIV scale on-screen under Timebase Controls. You can use the measurement tool under View ⇒ Toggle Time Cursors to facilitate measurement.

7. Use your measurement of period to calculate the frequency of the signal (Eq. 20.5).

8. Connect the oscillator to the DMM using a BNC - Coaxial adapter. Set the DMM to 20V ACV (instead of DCV) to measure root-mean-square voltage.

9. Record the frequency, $f$, of the signal by reading the oscillator display.

10. Reconnect the BNC cable to the oscilloscope. Turn off the oscilloscope, oscillator, and DMM.

Figure 20.9: Gradecule Sample

Figure 20.10: BNC - Coaxial Adapter
PART 2: RC Circuit

The capacitors are electrolytic. They must be connected with the negative end of the capacitor on the negative side of the power supply; failure to do so will damage the capacitor.

Refer to Fig. 20.11

11. Determine the nominal resistance of $R_1$. Then measure $R_1$ with the DMM. Record these in the table provided.

12. Measure the capacitance, $C$, of the capacitor using the DMM at the TA’s table. Take care not to plug it in backwards - this will damage the capacitor.

13. Construct the circuit shown in Fig. 20.11. Connect, but do not plug in, the power supply.

14. Measuring voltage with a voltmeter changes the equivalent resistance of the circuit. To account for this, calculate $R_{eq}$ of the resistor in parallel with the 10 MΩ voltmeter. Use the measured value of $R_1$.

15. Calculate the theoretical value of the time constant $\tau$ using $R_{eq}$ and $C$.

16. Ask your TA to approve the circuit. Plug in and turn on the power supply. Charge the capacitor by applying 10.0 V across the circuit.

17. Disconnect one power supply lead from the circuit. Take voltage measurements every five seconds for three minutes. [The voltage should drop to the mV range, requiring you to adjust the voltmeter scale twice. If voltage drops to zero, check your circuit.]

18. Disassemble the circuit. Turn off the DMM.

PART 3: Graphing

19. Graph $V$ vs. $t$ using Graphical Analysis. Analyze the curve using a natural exponent fit. Determine the time constant from your curve fit. Print the graph.

Figure 20.11: RC Circuit Schematic

QUESTIONS

Part 1

1. Compare the two $f$ values (oscilloscope and oscillator).

2. Compare the two $V_{rms}$ values (oscilloscope and DMM).

3. Given $V_{peak−to−peak} = 18.54$ V, calculate $V_{rms}$.

Part 2

4. Show that the product of $RC$ has units of seconds.

5. If an RC circuit has $\tau = 15$ seconds, how long would it take for the circuit to discharge to $1/e^2$ its original value?

6. Calculate $\tau$ for an RC circuit consisting of a 3 $\mu$F capacitor and a 1.5 MΩ resistor.
Experiment 21: Geometric Optics

Figure 21.1: Geometric Optics Equipment

Figure 21.2: Mirror Placement: The “Plexi-Ray Kit” contains a small piece of cork or plastic to stabilize the mirror during your experiment. The mirror must sit on the paper, not on the stabilizer.

EQUIPMENT

Plexi-Ray Kit
Corkboard
Paper (9 sheets per 2-member team)
(2) Protractors
(2) 30-cm Rulers
(2) White Pins
(2) Color Pins
Lens Cleaning Towelettes (TA’s Table) - optical elements may need to be cleaned
Advance Reading

Text: Geometric optics, law of reflection, law of refraction, index of refraction, total internal reflection, critical angle, parallax, real image, virtual image, speed of light.

Objective

The objective of this experiment is to study the behavior of light using the ray model.

Theory

We will investigate light reflecting from objects as well as light that is transmitted through objects.

An image viewed in mirror is a virtual image as are images for some lenses. Light rays do not actually pass through a virtual image. A virtual image cannot be projected on a screen. The image seen in a mirror does not appear to be at the surface of the mirror, but rather to be located some distance behind the surface. The image appears to be located the same distance behind a plane (flat) mirror as the object is in front of the mirror. We investigate virtual images for plane mirrors in Part 1 and Part 2 of this experiment; we will investigate virtual images from a lens in Experiment 22.

The Law of Reflection states that when light reflects from a smooth, flat surface (e.g., a plane mirror), the angle of incidence equals the angle of reflection:

\[ \theta_1 = \theta_r \] (21.1)

In geometric optics, all angles are measured with respect to the normal (the perpendicular). We investigate this law in Part 3 of this experiment.

When light passes from a vacuum into a transparent medium, it slows down. The ratio of the speed of light in a vacuum, \( c \), to the speed of light through a transparent medium, \( v \), is the index of refraction, \( n \), of that medium. \( n \geq 1 \) at all times, since:

\[ n = \frac{c}{v} \] (21.2)

The speed of light in a vacuum, \( c \), has been defined: \( 299,792,458 \) m/s, or approximately \( 3 \times 10^8 \) m/s. Note that each material has its own index of refraction; \( n \) is a property of the medium.

The Law of Refraction (Snell’s Law) describes the behavior of a ray of light that passes from one medium into another:

\[ n_1 \sin \theta_1 = n_2 \sin \theta_2 \] (21.3)

where \( n_1 \) and \( n_2 \) are the indices of refraction for the two media, \( \theta_1 \) is the angle of incidence, and \( \theta_2 \) is the angle of refraction. We investigate the law of refraction in Part 4 of this experiment.

Total Internal Reflection is a special case in which light is unable to cross the boundary between the two media. Consider Eq. 21.3 and a specific combination of media, such as glass and air, with light passing through a medium of higher \( n_1 \) into a medium of lower \( n_2 \). As \( \theta_1 \) increases, \( \theta_2 \) increases more rapidly. \( \theta_1 \) can reach a value that results in \( \theta_2 \) being equal to 90°. When this happens, the light will not exit the medium. It will be totally internally reflected. This \( \theta_1 \) angle is called the critical angle, \( \theta_C \):

\[ \theta_1 = \theta_C \quad \text{when} \quad \theta_2 = 90^\circ \] (21.4)

Note that each combination or media can have a different \( \theta_C \). Note also that total internal reflection only happens when light passes from a medium of higher index of refraction into a medium of lower index of refraction. Total internal reflection is a part of our everyday world - fiber optics, for example, used for cable TV. We investigate total internal reflection in Part 5 of this experiment.

Experiment 21: Geometric Optics
Parallax is the effect whereby the position or direction of an object appears to differ when viewed from different positions. This means that the object and the two observations points are not collinear. Fig. 21.4 and Fig. 21.5 represent the effect.

The apparent change in direction can be useful for determining the distance from Earth to a star. The diagram below is, of course, not to scale.

Parallax is larger when the object is closer. While parallax can present problems when making measurements, it is very useful for determining the position of a virtual image.

Consider two objects that are aligned from your initial position (they appear to be one object). When you change your position, each object will appear to be displaced but by a different amount. The objects appear to separate (Fig. 21.5). They are no longer aligned with your position.

We make use of parallax (2 objects) in Part 1 and, if you so choose, Part 2 of this experiment.

Since a virtual image cannot be projected on a screen, it can be difficult to determine its location. Consider an object whose image is observed in a mirror. The image is a virtual image. To determine the position of a virtual object, one can align a real object (pin) with a virtual object (image of a different pin). If the observer changes their position and the two objects do not separate, the real and virtual objects are in the same location.
Name: _________________________________

1. State the law of reflection. Write the equation. (10 pts)

2. State the law of refraction. Write the equation. (10 pts)

3. What is the definition of a virtual image? (10 pts)

4. What is total internal reflection? (10 pts)

5. Define critical angle. (10 pts)

6. Define parallax. (10 pts)

7. Write Snell’s Law. Using Eq. 21.4, derive the equation for the critical angle: \( \theta_C = \ldots \). (40 pts)
Experiment 21: Geometric Optics

PROCEDURE

1. Read through Step 6 before beginning. All arrangements (paper) should be aligned with the edge of the table.

2. Measure all normal lines with a protractor to ensure they are $90^\circ$; do not estimate. Draw the lines long enough to allow accurate measurements using the protractor. Do not fold the paper!

3. When placing more than one pin, separate them as much as possible (e.g., one pin close to the mirror, the other pin close to the edge of the paper). This will increase accuracy and improve your results. When aligning objects and images, close one eye.

PART 1: Parallax

4. For Part 1, you will use a virtual image (the mirror image of a pin); use the image of the white pin as the first object and a real object (a color pin) as a second object. If the second object is placed at a location other than the location of the first object, note that when you change your position there is a shift in the apparent position of the two objects. You will see two images (Fig. 21.5); they will not be aligned.

5. When parallax occurs, there are two possibilities:
   - The pin is in front of the virtual image. Parallax is larger for the pin.
   - The pin is behind the virtual image. Parallax is smaller for the pin.

6. If the pin is placed at the same location as the virtual image and you change your position, there will be no change in the apparent position of the two objects (i.e., there will be no separation of the objects). You will see only one image, thus eliminating parallax.

7. Refer to Fig. 21.6. Draw a line across the center of a sheet of paper (baseline) and a line normal to the baseline down the center of the paper.

8. Place the paper on the corkboard and place the back of the mirror on the baseline (Fig. 21.2). Place a white pin midway on the normal in front of the mirror. The image of this pin will be object 1.

9. Place a color pin on the normal behind the mirror. This pin will be object 2.

10. Observe from an orientation to the right or left of the normal. If object 1 and object 2 are not aligned, move object 2 towards or away from you, along the normal, until they are aligned. (Your partner will help you stay on the normal.)

11. Measure and record the distance from the mirror to the white pin, $d_0$, and the distance from the mirror to the second pin, $d_i$. 
PART 2: Image from a Mirror

12. Team performs once; take turns for each vertex.

13. Refer to Fig. 21.7. Draw a baseline across the center of a sheet of paper. Place the triangular plexiglass in the center of the front half of the paper, trace around it, then return it to the kit. Label the vertices of the triangle $A$, $B$, and $C$. Place the paper on the corkboard and the mirror on the baseline.

14. You may use either the parallax method or the following method to determine the position and dimensions of the virtual image of the triangle.

15. Consider Fig. 21.7. Two points are required to designate a particular line (e.g., two $a'$ or two $a''$). (Always separate the pins as much as possible to improve your results.) Two intersecting lines designate a particular point (e.g., $A'$).

16. Place a white pin at vertex $A$; the image of the white pin is the object. Observe from the left and align two color pins (sighting pins) with the object.

17. Mark the positions, then remove the pins and label the positions of the sighting pins $a'$.

18. Repeat, observing from the right. Mark the sighting pin positions as $a''$.

19. Repeat for vertices $B$ and $C$ (obtaining two sight lines for each vertex).

20. Draw a line through the two $a'$ points, the length of the paper. Repeat for each pair of points. Mark the intersection of the $a'$ and $a''$ lines as $A'$. Repeat for $B'$ and $C'$.

21. Connect the points $A'$, $B'$, and $C'$. Measure and compare the dimensions of the object and image triangle.

PART 3: Reflection

22. Refer to Fig. 21.8. Draw a baseline across a sheet of paper, near the top. Draw a line normal to the baseline down the center of the paper. Draw a line to the left of the normal (incident line) with angle $\theta_i$ between $25^\circ$ and $35^\circ$.
Experiment 21: Geometric Optics

23. Place the two white pins on the incident line. Label these positions $P_1$ and $P_2$.

24. Look at the mirror from the right side of the normal so that you can see the image of the first two pins. Adjust your position so that the images of the white pins are aligned; align two color pins with them.

25. Label these two points $P_3$ and $P_4$.

26. Draw a line connecting point $P_3$ and $P_4$ to the baseline. Measure and record $\theta_i$ and $\theta_r$.

PART 4: Refraction

27. Place the plexiglass square at the center of a sheet of paper and trace around it.

28. In the upper-left corner of the traced square, draw a line normal to the square, about 2 cm from the corner (Fig. 21.9).

29. Draw an incident line ($\theta$ between 25° and 35°). Place the paper on the corkboard, the square on its traced outline, and two white pins on the incident line. Label their positions $P_1$ and $P_2$.

30. Look through the plexiglass square from the edge opposite the pins and close your left eye. (It will help if your partner holds a piece of paper behind the square to block images of other objects in the room.)

31. Adjust your position until the two white pins are aligned, then place two sighting pins (color pins) that align with the image of the two white pins. Label these points $P_3$ and $P_4$.

32. Draw a line connecting $P_3$ and $P_4$ to the edge of the square. Draw a line normal to the square through this intersection (refer to Fig. 21.10).

33. Draw a line inside the square to connect the normals (Fig. 21.10).

34. Measure and record each $\theta$. You have 2 sets of data, $A$ and $A'$ (top and bottom).

35. Consider the law of refraction and Fig. 21.10.

- **Situation A**: As light enters the plexiglass from the air, it bends towards the normal, since it enters a medium with an index of refraction lower than the index of refraction of the medium it is leaving.

- **Situation B**: As light enters the air from the plexiglass, it bends away from the normal, since it enters a medium with an index of refraction greater than the index of refraction of the medium it is leaving.

36. Therefore, $\theta_1 = \theta'_2$ and $\theta_2 = \theta'_1$. ($n_{air} = 1.0003$)

37. Calculate the index of refraction of the plexiglass for each situation (situation $A$ and $A'$).

38. Average your values of $n_{plexiglass}$ and calculate the speed of light through the plexiglass.
PART 5: Total Internal Reflection

39. Place the plexiglass triangle in the center of a sheet of paper and trace around it. Draw a line normal to the bottom edge (the long side), about 1 cm from the left corner (Fig. 21.11). Place the paper and triangle on the corkboard; place two white pins on the normal line with their positions labeled $P_1$ and $P_2$.

40. Look through the right side of the bottom edge of the triangle and adjust your position until the two white pins are aligned. (It will help if your partner holds a piece of paper behind the triangle to block images of other objects in the room.) Now align two color pins with the white pins as viewed through the plexiglass. Label these position $P_3$ and $P_4$.

41. Return the triangle and pins to the kit; place the paper on the table.

42. Draw the path of the light ray by:
   - Extending the normal ($P_1P_2$) to the far edge of the triangle
   - Connecting $P_3$ and $P_4$, extending the line to the far edge of the triangle
   - Connecting the line segments ($P_1P_2$ to $P_3P_4$) at the top (far edges of the triangle)

43. Draw a line normal to the edge of the triangle where each light ray ($P_1P_2$ and $P_3P_4$) intersects the far edge of the triangle.

44. Measure $\theta_1$ and $\theta_2$ at each location where the light reflects internally. You have two sets of data, $A$ and $A'$ (right and left).

QUESTIONS

1. Calculate $\theta_C$ for plexiglass in air (using your average value of “n” from Part 4).
2. What is the shortest height a plane mirror must be so that a person who is 1.6 meters tall is able to see his or her whole body? Draw a ray diagram to support your answer.
Experiment 22: Thin Lenses

Figure 22.1: Optical Bench Arrangement

**EQUIPMENT**

Optical Bench
(2) Lens Holders
(4) Optical Bench Clamps
Object Box (Light Source)
Small Screen
Large Screen (clipboard, paper)
Bi-Convex Lens (Converging Lens)
Bi-Concave Lens (Diverging Lens)
30-cm Ruler
Flashlight (1 per person)
Lens Cleaning Towelettes (TA’s Table)
**Advance Reading**

*Text:* Thin lenses, converging lens, diverging lens, lens equation, object distance, image distance, refraction, focal length, magnification, index of refraction, real image, virtual image.

**Objective**

The objective of this experiment is to measure the focal lengths of a converging lens and a diverging lens and investigate magnification.

**Theory**

Light refracts (bends) when passing through media with difference indices of refraction. This property can be very useful, especially when a thin lens is used. A thin lens’ thickness is much less than its diameter.

A converging (convex, positive) lens is thicker in the center than at the edges. It can be used to focus parallel light rays and form a *real image* as the light travels from air to glass and back to air \( (n_{\text{air}} \approx 1.0, n_{\text{glass}} \approx 1.5) \). A real image is formed by light actually passing through the image. A real image can be projected on a screen. The image exists regardless of whether or not a screen is in position to show it.

A diverging (concave, negative) lens normally forms a *virtual image*. Light rays do not actually pass through a virtual image. It cannot be projected on a screen. When you look at yourself in a mirror, you are looking at a virtual image. If the object is real, the image is virtual. However, when a diverging lens is used in combination with a converging lens, for instance, the object can be virtual, the image real. Parameters must be met for a real image to be formed; read the Part 2 procedure carefully.

An important property of a lens is its focal length, \( f \). The focal length of a thin lens is given by:

\[
\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}
\]  

(22.1)

where \( d_o \) is the object distance and \( d_i \) is the image distance. These distances are measured from the lens.

Consider Eq. 22.1. For an object that is infinitely far away \((d_o \rightarrow \infty)\).

Rays of light from an object very far away from a thin lens will be approximately parallel when they reach the lens. The light rays will then refract as they pass through the lens. For a converging lens, rays parallel
d to the optical axis refract towards the normal and focus at a point (small area) called the *focal point*, \( F \). The distance between the center line of the lens and the focal point is the *focal length*, \( f \). Refer to Fig. 22.2.

Fig. 22.2 through Fig. 22.6 are courtesy of Giancoli’s *Physics*.

**Figure 22.2:** Ray Tracing: \( d_o \rightarrow \infty, d_i \equiv f \)

![Figure 22.2: Ray Tracing: Focal Plane](image)

For a converging lens, rays of light that are parallel to each other but not parallel to the optical axis will still refract towards the normal, but will focus at the *focal plane*.

**Figure 22.3:** Ray Tracing: Focal Plane

---

Experiment 22: Thin Lenses

Rays of light from a nearby object will arrive at the lens at various angles. The light rays will then refract as they pass through the lens and, for a converging lens, form an image at a distance $d_i$ (refer to Fig. 22.4).

![Figure 22.4: Ray Tracing: Nearby Object](image)

As mentioned, a diverging lens will usually form a virtual image. The image can be seen but cannot be projected onto a screen.

![Figure 22.5: Ray Tracing: Diverging Lens](image)

To determine the focal length of a diverging lens in lab, we will need to use two lenses. The real image from the converging lens will become the virtual object for the diverging lens. Refer to Fig. 22.6; although our arrangement must be somewhat different than shown below, the figure has the same concept we require.

![Figure 22.6: Ray Tracing: Combination Lenses](image)

Lateral magnification, $M$, is defined as the ratio of the image height, $h_i$, to object height, $h_o$. The object height is assumed to be positive; the image height is positive if the image is upright and negative if the image is inverted.

$$M = \frac{h_i}{h_o} \quad (22.2)$$

Magnification is also proportional to the relative distances of object and image from the lens:

$$M = -\frac{d_i}{d_o} \quad (22.3)$$

The sign conventions for object distance and image distance remain the same. These are calculated as in Eq. 22.4 and Eq. 22.5.

The lab will be dark (lights off) for the remaining experiments this semester. It is important that your flashlight be pointed below horizontal at all times. This limits the bleaching of visual purple, which permits night vision. Please turn off the flashlight when it is not in use and before you leave lab.
Name: ________________________________

1. Define the terms of the relationship \( \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \) (Eq. 22.1) and state how each term is measured. (20 pts)

2. State the sign conventions for \( d_o \) and \( d_i \). (20 pts)

3. What is the difference between a real image and a virtual image? (20 pts)

4. Define the terms of the relationship \( M = \frac{h_i}{h_o} \) and state the sign conventions. (20 pts)

5. What does optical axis mean? (10 pts)

6. What two methods will be used to calculate the focal length of a converging lens? (10 pts)
Experiment 22: Thin Lenses

PROCEDURE

PART 1: Converging Lens

Method I - Use the lens equation (Eq. 22.1)

1. Refer to Fig. 22.1. Mount the lens, screen, and light source on the optical bench. Adjust the height of the object, lens(es), and screen so that the optical axis passes through the center of each element.

2. Adjust the position of the lens and the screen until a clear image of the object is projected onto the screen. Considering Eq. 22.1, how many combinations of \( d_i \) and \( d_o \) are possible?

3. Sketch the diagram shown in Fig. 22.7. Record the position of each device: \( O \), \( L \), \( i \). Positions are measured directly from the optics bench; a line is scribed on each holder for accuracy.

4. Calculate \( d_o \), \( d_i \), and \( f \) (Eq. 22.1).

\[
\begin{align*}
    d_o &= L - O \quad (22.4) \\
    d_i &= i - L \quad (22.5)
\end{align*}
\]

\[\text{O: } \quad \text{L: } \quad \text{i: } \]

\[\text{d}_o: \quad \text{d}_i: \]

\[f = \]

Figure 22.7: Sketch required for all arrangements in Part 1 and Part 3.

- \( O \) is the position of the object.
- \( L \) is the position of the lens.
- \( i \) is the position of the image.
- \( d_o \) is the calculated object distance (absolute value).
- \( d_i \) is the calculated image distance (absolute value).

Method II - Use a distant object \( (d_o \to \infty) \)

5. Hold lenses carefully by the edge. Project the image of a distant object on a screen. One way to achieve this is to take the lens and a ruler to a long hallway. Hold the lens such that light from a distant light source at the other end of the hallway passes through the lens and focuses on the wall.

6. Adjust the distance between the lens and the screen until a clear, distinct image of the distant light source is projected onto the screen.

7. Measure the distance from the lens to the screen. This distance is \( f \), as shown in Fig. 22.3. Eq. 22.1 shows that when \( d_o \) is large, \( d_i \to f \).

8. Is the image inverted? Magnified? Reversed?

9. Compare \( f \) (average) from the two methods.
PART 2: Diverging Lens

To determine the focal length of a diverging lens, the lens must create a measurable, real image as in Part 1. However, light cannot be focused through a diverging lens to form a real image unless that light was already converging. To accomplish this, a real image from a converging lens will be used as a virtual object.\(^2\)

10. Form a real image using a converging lens. Note: \(d_o\) should be greater than \(2f\).

11. Begin your required sketch (Fig. 22.9).

12. Place the diverging lens between the converging lens and its (real) image; refer to Fig. 22.8. The real image from the converging lens is now a virtual object for the diverging lens.

13. Determine the position of the diverging lens’ image by adjusting the screen’s position.

\[\begin{align*}
\text{Converging Lens:} \\
O: & \quad L: \quad i: \\
d_o: & \quad d_i: \\
\end{align*}\]

\[\begin{align*}
\text{Diverging Lens:} \\
L: & \quad O: \quad i: \\
d_o: & \quad d_i: \\
f = & \quad \\
\end{align*}\]

Figure 22.9: Sketch required for Part 2.

14. Complete your sketch (Fig. 22.9).

15. Determine \(d_o\) and \(d_i\) for the diverging lens.

16. Calculate \(f\) for the diverging lens.

\textbf{Compare }\(f\)\textbf{ of a Diverging Lens}

17. Recalculate \(f\) using the following equation:

\[f = \frac{VW}{V - W}\] (22.6)

where \(V\) and \(W\) are defined as:

\[V \equiv |d_o| \quad \text{and} \quad W \equiv |d_i|\]

18. Compare \(f\) values from Eq. 22.1 and Eq. 22.6 for the diverging lens. If you followed the sign conventions closely, the \(f\) values should be identical.

19. Remove the diverging lens; set it carefully aside.

---

\(^2\)For a diverging lens, either the object or the image can be real; the other must be virtual.
**Experiment 22: Thin Lenses**

**PART 3: Lateral Magnification, \( M \)**

Use only the converging lens to investigate lateral magnification. Remember to sketch all arrangements as in Fig. 22.7.

20. Set \( d_o > 2f \) by adjusting the distance between the object and the lens. Find the image using your large screen. Record the position of the image and mark on the screen the top and bottom of the image.

21. Measure the image height, \( h_i \), and the object height, \( h_o \). Be sure to measure the same dimension on both object and image.

22. Calculate \( M \):

\[
M = \frac{h_i}{h_o} \quad \text{(Eq. 22.2)}
\]

23. Verify the magnification \( M \) using the following equation:

\[
M = \frac{-d_i}{d_o} \quad \text{(Eq. 22.3)}
\]

24. Compare the two values of \( M \).

25. Set \( f < d_o < 2f \). Locate the image.

26. Measure \( h_i \) and \( h_o \).

27. Calculate \( M \) using Eq. 22.2.

28. Calculate \( M \) using Eq. 22.3.

29. Compare the two values of \( M \).

30. Set \( d_o = f \). Try to find \( d_i \). Consider Method II and Eq. 22.1; where should the image be?

31. Set \( d_o < f \). Look through the lens at the object. Note your observations.

**PART 4: Equipment**

32. Set the lens carefully aside where it will not be damaged.

33. Unplug the light source and lay the power cord across the optics bench.

**QUESTIONS**

1. Draw ray diagrams for Method I and Method II (Part 1, converging lens).

2. Consider a concave lens made out of air that is immersed in water (perhaps two watch glasses glued to each end of a piece of pipe, with air inside). Will it form a real image that can be focused on a screen? Draw a ray diagram to support your answer.

3. If a convex lens with \( n = 1.30 \) and \( f = 25 \) cm is immersed in a fluid with an index of refraction that is also 1.30, what is the new focal length of the lens? Draw a ray diagram.

4. What are the major sources of uncertainty in this experiment?
Experiment 23: Wave Optics

Figure 23.1: Diffraction and Interference

Not to scale: For optimal measuring, distance between
the slide holder and measuring surface should be at least 1 meter.

EQUIPMENT

Laser
Black Felt
Single-Slit Slide
Double-Slit Slide
Slide Holder
Meter Stick
Vernier Caliper
Clipboard, Paper (Screen)
(1) Flashlight per person
Note the central envelope of the diffraction pattern. It is about twice the width of the envelopes on either side. The pattern repeats with decreasing intensity (dimmer) as the angle increases left or right of $\theta = 0^\circ$.

“W” is measured from the center of the dark to the center of the dark on either side of the central envelope.
Note the interference pattern (bright/dark fringes) contained within the envelopes of the diffraction pattern. The central envelope is about twice the width of envelopes on either side. The pattern repeats with decreasing intensity as the angle increases left or right of $\theta = 0^\circ$.

"m" is the order number of each bright fringe. $m$ equals zero when $\theta = 0^\circ$; $m$ is symmetric about $\theta = 0^\circ$. You will make a mark through the center of each bright fringe in the central envelope. There is, of course, an odd number of fringes in the central envelope. Use Eq. 23.4 to determine $m$ for calculations of $\lambda$. 
Experiment 23: Wave Optics

Advance Reading

Text: Wave optics, wavelength, frequency, electromagnetic spectrum, diffraction, interference, principle of superposition.

Objective

The objective of this experiment is to study diffraction and interference and to determine $\lambda$, the wavelength of the laser light.

Theory

Single-Slit Diffraction

Light passing through a narrow slit (slit width, $D$) will produce a diffraction pattern. The diffraction pattern consists of a series of light and dark bands, or envelopes.

Fig. 23.2 shows a plot of the light intensity vs. angle in the diffraction pattern. The slit width, $D$, is provided on the slide; $\delta D = 0.005$ mm. We use the small angle approximation to derive Eq. 23.1; this method requires angles in radians. The central bright envelope of the diffraction pattern subtends an angle of $2\phi$, where:

$$\phi = \frac{\lambda}{D} \quad (23.1)$$

and $\lambda$ is the wavelength. The angle will not be measured directly. $\phi$ can be determined by first measuring the distance from the slit to the screen ($L$), then measuring the width of the central envelope, marked as $W$ in Fig. 23.2.

$$2 \tan \phi = \frac{W}{L} \quad (23.2)$$

Using substitution, Eq. 23.1 and Eq. 23.2 are solved for $\lambda$, in terms of quantities that can be determined in the lab: $W$, $D$, $L$. The distance $L$ should be at least 1 m for both diffraction and interference procedures.

Double-Slit Interference

When coherent light (e.g., laser light) passes through two narrow slits that are close together, an interference pattern will result. The two slits are treated as if they are two points sources of coherent light. This pattern results from the addition and cancellation of light waves from the two slits, known as constructive and destructive interference, respectively. The resulting bands of light (conversely, bands of darkness) are referred to as bright fringes (or dark fringes).

Figure 23.4: Double Slit - Path Length Difference

Constructive interference occurs when the path length difference, $\delta$, is equal to an whole number of wavelengths (i.e., even number of $\frac{1}{2}$-wavelengths). Destructive interference occurs when $\delta$ is an odd number of $\frac{1}{2}$-wavelengths.

Constructive interference occurs when $\delta = m\lambda$. Path length difference is necessarily: $\delta = d \sin \theta$, where $d$ is slit spacing.

$$\delta = d \sin \theta = m\lambda \quad (23.3)$$

where $m = 0, 1, 2, \ldots$, with $m = 0$ being the central maximum fringe. The fringe order number can be found by counting the number of fringes:

$$m = \frac{\# fringes - 1}{2} \quad (23.4)$$

For small angles measured in radians, the small angle approximation (refer to Experiment 11) is useful:

$$\sin \theta \approx \tan \theta = \frac{y}{L} \quad (23.5)$$

Using substitution, Eq. 23.3 and Eq. 23.5 are solved for $\lambda$, in terms of quantities that can be determined in the lab: $m$, $d$, $y$, $L$. 
Name: __________________________________________

1. Define diffraction. (10 pts)

2. Define interference. (10 pts)

3. What does the symbol $D$ in Eq. 23.1 represent? (10 pts)

4. What does the symbol $d$ in Eq. 23.3 represent? (10 pts)

5. Use substitution to solve the single-slit equations (Eq. 23.1 and Eq. 23.2) for $\lambda$, in terms of the quantities that we will determine in lab: $W, L, D$. (20 pts)

6. Use substitution to solve the double-slit equations (Eq. 23.5 and Eq. 23.3) for $\lambda$, in terms of the quantities that we will determine in lab: $m, L, d, y$. (20 pts)

7. Given $D_1 = .02$ mm and $\delta D = .005$ mm, calculate $\delta \% D_1$. Given $D_2 = .16$ mm, calculate $\delta \% D_2$. Why will it be important to begin with the widest slit? (20 pts)
**PROCEDURE**

**PART 1: Single-Slit Diffraction**

1. Place the single-slit slide on the slide holder. Align the slide holder in front of the laser aperture.

2. Set up the clipboard and paper in front of the laser, at least one meter away from the slide holder. Record this distance, $L$.

3. Turn on the laser to create a diffraction pattern on the clipboard. Begin with the widest slit. Slit width is provided on the slide; its uncertainty is $\delta D = \pm 0.005$ mm.

4. Mark the width, $W$, of the central band. Measure from the center of the dark bands on either side. Take measurements in a tidy manner, use extra paper as needed.

5. Close the shutter. Calculate $\lambda$; show your work. [The mathematical model uses radians.]

6. Repeat for the three remaining slits. Calculate average $\lambda_{\text{ss}}$ for the single-slit procedure.

**PART 2: Double-Slit Interference**

7. Place the double-slit slide on the holder. Turn on the laser. Begin with the widest spacing.

8. Mark a line through each of the bright fringes in the central bright envelope. The distance between the outermost fringes is $2y$.

9. Close the shutter. Determine $2y$ and $m$.

10. Calculate $\lambda$. Show your work.

11. Repeat for the remaining pairs of slits. Calculate average $\lambda_{\text{ds}}$ for the double-slit procedure.

12. Calculate the total average $\lambda$ from your measurements. Compare this to the laser’s theoretical wavelength: $\lambda = 6328 \, \text{Å}$ [$\text{Å}=10^{-10}$ m]. Show your work.

**QUESTIONS**

1. Explain and compare diffraction and interference.

2. What are the sources of uncertainty in this experiment?
Appendix A: Measurement and Uncertainty

GENERAL INFORMATION

Every measurement has some uncertainty. These uncertainties are called errors. “Error analysis is the study and evaluation of uncertainty in measurement.” Measurements are usually made against some standard to compare the object or quantity being measured with some known value. For instance, if the length of a table is measured with a meter stick, the table is being compared to the meter stick, but the meter stick is also referenced to some standard. It is important to keep in mind that any “known” value given as a standard has an uncertainty associated with it. Any measurement you make has an uncertainty associated with it as well.

Error analysis is an interesting and complex subject. As an introduction to this topic, certain experiments will focus on particular types of analysis: statistical analysis, uncertainty of measurements, and propagation of error (propagation of uncertainty). The details follow.

Significant Figures

There are rules for significant figures (lab manual: Experiment 1 and Giancoli: Chapter 1, Section 4). All data should be recorded with the proper number of significant figures in your lab notebook as well as in your lab reports. To prevent rounding errors, keep one more significant figure than is justified until you are finished with your calculations.

Accuracy and Precision

An important consideration in research (laboratory) is the accuracy and the precision of a measurement. Commonly, these two terms are used as synonyms, but they are quite different. The accuracy of a measurement is how close the measurement is to some “known” value (how small the percent error is; related to systematic and personal error). For instance, if an experiment is performed to measure the speed of light, and the experimental value is very close to the known (accepted) value, then it can be said that the value is accurate. On the other hand, the precision of an experiment is a measure of the reproducibility of an experiment (related to random error).

When performing an experiment, one needs to keep in mind that a measurement that is precise is not necessarily accurate and vice versa. For example, a vernier caliper is a precision instrument used to measure length (the resolution is 0.005 cm). However, if a damaged caliper is used that reads 0.25 cm too short, then all of the measurements would be incorrect. The values may be precise, but they would not be accurate.

---

Appendix A: Measurement and Uncertainty

EXPERIMENTAL ERRORS

Experimental errors are generally classified into three types: systematic, personal, and random. Systematic errors and personal errors are seldom valid sources of uncertainty when performing and experiment as simple steps can be taken to reduce or prevent them. You will need to discuss “sources of uncertainty” for each experiment when you write a full lab report. Assume you performed the experiment correctly using calibrated equipment. Useful questions to ask yourself for this section of the report are what is “wrong” with the method (errors inherent in the method) and what assumptions are made that are not valid? The specifics follow for types of experimental errors.

Systematic Error

Systematic errors are such that measurements are pushed in one direction. Examples include a clock that runs slowly, debris in a caliper (increases measurements), or a ruler with a rounded end that goes unnoticed (decreases measurements). To reduce this type of error, all equipment should be inspected and calibrated before use.

Personal Error

Carelessness, personal bias, and technique are sources of personal error. Care should be taken when entering values in your calculator and during each step of the procedure. Personal bias might include an assumption that the first measurement taken is the “right” one. Attention to detail and procedure will reduce errors due to technique.

Parallax, the apparent change in position of a distant object due to the position of an observer, could introduce personal error. To see a marked example of parallax, close your right eye and hold a ringer several inches from your face. Align your finger with a distant object. Now close your left eye and open your right eye. Notice that your finger appears to have jumped to a different position. To prevent errors due to parallax, always take readings from an eye-level, head-on perspective.

Random Error

Random errors are unpredictable and unknown variations in experimental data. Given the randomness of the errors, we assume that if enough measurements are made, the low values will cancel the high values. Although statistical analysis requires a large number of values, for our purposes we will make a minimum of six measurements for those experiments that focus on random error.
COMPARISON of EXPERIMENTAL VALUES

Percent Error and Percent Difference

We frequently compare experimental values (percent error) and experimental values with other experimental values (percent difference).

Accepted Values might be found in tables, determined experimentally (e.g., \( g = 9.80 \text{ m/s}^2 \)), or calculated from equations that assume ideal conditions.

Experimental Values: We often use more than one method to determine a particular quantity in lab, then compare the values, or compare a particular value with the class average.

\[
\text{Percent Error} = \left| \frac{\text{accepted} - \text{experimental}}{\text{accepted}} \right| \times 100 \tag{A.1}
\]

\[
\text{Percent Difference} = \left| \frac{\text{value}_1 - \text{value}_2}{\text{value}_1 + \text{value}_2} \right| \times 100 = \left| \frac{\text{value}_1 - \text{value}_2}{\text{value}_{\text{avg}}} \right| \times 100 \tag{A.2}
\]

These values are rounded to one or two significant figures, determined by the following:

- Keep only one significant figure unless the value begins with the number one.
- Keep two significant figures if the value begins with the number one.
Appendix A: Measurement and Uncertainty

ERROR ANALYSIS

Random Error

When analyzing random error, we will make a minimum of six measurements \((N = 6)\). From these measurements, we calculate an average value (mean value):

\[
\bar{x} = \frac{x_1 + x_2 + x_3 + \cdots + x_6}{6} = \frac{1}{N} \sum_{i=1}^{N} x_i = \frac{1}{6} \sum_{i=1}^{6} x_i \quad (A.3)
\]

To determine the uncertainty in this average, we first compute the deviation, \(d_i\):

\[
d_i = x_i - \bar{x} \quad (A.4)
\]

The average of \(d_i\) will equal zero. Therefore, we compute the standard deviation, \(\sigma\) (Greek letter, sigma). Analysis shows that approximately 68% of the measurements made will fall within one standard deviation, while approximately 95% of the measurements made will fall within two standard deviations.

\[
\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (d_i)^2} = \sqrt{\frac{1}{5} \sum_{i=1}^{6} (d_i)^2} = \sqrt{\frac{1}{5} \sum_{i=1}^{6} (x_i - \bar{x})^2} \quad (A.5)
\]

or

\[
\sigma = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + (x_3 - \bar{x})^2 + (x_4 - \bar{x})^2 + (x_5 - \bar{x})^2 + (x_6 - \bar{x})^2}{5}} \quad (A.6)
\]

Keep two (2) significant figures for standard deviation.
Appendix A: Measurement and Uncertainty

**UNCERTAINTY OF MEASUREMENTS: \( \delta \)**

In this lab, the uncertainty, \( \delta \) (Greek letter, delta), of a measurement is usually \( \frac{1}{2} \) the resolution (smallest division) of the measuring device. The resolution of a 30-cm ruler is one millimeter (0.1 cm), thus \( \delta x \) is

\[
\delta x = \pm \frac{1}{2}(0.1 \text{ cm}) = \pm 0.05 \text{ cm}
\]

For example, an object is measured to be \( x \pm \delta d = (23.25 \pm 0.05) \text{ cm} \). The 5 in 23.25 is estimated. The measurement \((23.25 \pm 0.05) \text{ cm}\), means that the true measurement is most likely between 23.20 cm and 23.30 cm.

Using multiple measurements of the same quantity, \( \delta x \) can be represented by the standard deviation, \( \sigma \).

The fractional uncertainty, \( \frac{\delta x}{x} \), is used to compute the percent uncertainty, \( \delta \% x \), of a measured value:

\[
\delta \% x = \pm \frac{\delta x}{x} \times 100
\]  
(A.7)

**PROPAGATION OF ERROR**

Once the uncertainties of measurements are known, they can be propagated in all mathematical manipulations (i.e., equations) that use measured quantities. This allows a reader to know the precision of your work. Methods required for propagating error vary, depending on the equation(s) used. Always begin by writing the algebraic equation for the quantity first. After determining the uncertainty of a calculated value, round off the uncertainty using the rule stated below. The order of magnitude of the uncertainty (e.g., tenths, thousandths) determines where the calculated quantity must be rounded off, thus limiting the number of significant figures in the calculated quantity.

Uncertainty is rounded to one or two significant figures, determined by the following:

- Keep only one significant figure unless the uncertainty begins with the number one.
- Keep two significant figures if the uncertainty begins with the number one.

**Added or Subtracted Quantities**

When measured values are added (or subtracted), the uncertainties are added in quadrature (i.e., combining two numbers by squaring them, adding the squares, then taking the square root). If there is a constant, multiply the calculated uncertainty by the constant. The uncertainty of a value \( A \), \( \delta A \), is added in quadrature in Eq. A.8:

\[
(x \pm \delta x) + (y \pm \delta y) \Rightarrow A = Bx + y \quad \text{or} \quad (x \pm \delta x) - (y \pm \delta y) \Rightarrow A = Bx - y
\]

\[
\delta A = \pm B \sqrt{(\delta x)^2 + (\delta y)^2}
\]  
(A.8)

Express this result as \( A \pm \delta A \). Uncertainties are added when measurements are added or subtracted.
Appendix A: Measurement and Uncertainty

Uncertainty Example: Added or Subtracted Quantities

\[ x \pm \delta x = (2.40 \pm 0.05) \text{ cm} \quad \text{and} \quad y \pm \delta y = (1.35 \pm 0.05) \text{ cm} \]

\[ A = x + y = 2.40 \text{ cm} + 1.35 \text{ cm} = 3.75 \text{ cm} \]

\[ \delta A = \pm \sqrt{(0.05 \text{ cm})^2 + (0.05 \text{ cm})^2} = \pm (\sqrt{2})0.05 \text{ cm} = \pm 0.070710678 \text{ cm} \]

Rounding appropriately for uncertainty, we know that \( \delta A = \pm 0.07 \text{ cm} \). Thus,

\[ A \pm \delta A = (3.75 \pm 0.07) \text{ cm} \]

Note that the measurement and uncertainty are inside the parentheses, the unit is outside the parentheses. All measured or calculated values and their uncertainties must be written this way.

If a quantity is calculated (addition or subtraction only) that involves a constant, multiply the uncertainty by that constant. Examples are equations such as the average of multiple measurements, or average reaction time.

Example

\[ t_{avg} = \frac{t_1 + t_2 + t_3}{3} \quad \delta t = \pm \frac{1}{3} \times \sqrt{(\delta t_1)^2 + (\delta t_2)^2 + (\delta t_3)^2} \]
### Appendix A: Measurement and Uncertainty

**Multiplied or Divided Quantities**

When measured values are multiplied (or divided), use the following rules. The fractional uncertainty of each measurement is squared and summed; the square root of this sum is then multiplied by the calculated quantity. If a measured value is raised to a power, multiply the fractional uncertainty by the exponent prior to squaring. These uncertainty rules may be best explained by use of examples:

Assume a volume \( V \) of a block, calculated from measured dimensions \( x, y, \) and \( z \):

\[
x \pm \delta x, \ y \pm \delta y, \ z \pm \delta z
\]

\[
V = xyz
\]

\[
\delta V = \pm V \times \sqrt{\left(\frac{\delta x}{x}\right)^2 + \left(\frac{\delta y}{y}\right)^2}
\]  \( \text{(A.9)} \)

Express this result as \( V \pm \delta V \).

\( \delta V \) can be expressed as a percent uncertainty, \( \delta \% V \), by using Eq. A.7. Express the result as \( V \pm \delta \% V \).

When a measured quantity is raised to a power, the uncertainty is determined by:

\[
V = x^n y \quad \rightarrow \quad \delta V = \pm V \times \sqrt{(\frac{\delta x}{x})^2 + (\frac{\delta y}{y})^2}
\]  \( \text{(A.10)} \)

Note that the exponent (power) is inside the parentheses.

A numerical example is provided on the next page, followed by an example involving many of the rules that have been outlined.

*Always:* First write the equation for the quantity you want to calculate in algebraic terms (no measured value). Then write the uncertainty equation algebraically. Every quantity that has uncertainty must be under the radical. If the equation for the quantity involves addition or subtraction, refer to Eq. A.8. If the equation for the quantity involves multiplication or division, refer to Eq. A.9 and Eq. A.10. If the equation has a mixture of addition/subtraction and multiplication/division, it is best to re-write the equation using only multiplication/division; define new terms as needed.

*Always* write your equations, algebraically, in terms of the quantities involved. For example, if the equation for the volume of a cylinder involves the diameter and height, do not write the equation for \( V \pm \delta V \) in terms of \( x, y, \) and \( z \).

*Every measurement used to calculate a quantity requires that the uncertainty of the measurement be included, under the radical, when calculating the quantities’ uncertainty.*
Uncertainty Example: Multiplied or Divided Quantities
An area, $A$, and its uncertainty, $\delta A$, are to be determined from the following measurements:

\[
\text{Length} = x \pm \delta x = (2.40 \pm 0.05) \text{ cm}; \quad \text{Width} = y \pm \delta y = (1.35 \pm 0.05) \text{ cm}
\]

Area is length times width: $A = xy = 2.40 \text{ cm} \times 1.35 \text{ cm} = 3.24 \text{ cm}^2$

The uncertainty in the area, $\delta A$, is given by:

\[
\delta A = \pm A \sqrt{ \left( \frac{\delta x}{x} \right)^2 + \left( \frac{\delta y}{y} \right)^2 } = \pm 3.24 \text{ cm}^2 \sqrt{ \left( \frac{0.05 \text{ cm}}{2.40 \text{ cm}} \right)^2 + \left( \frac{0.05 \text{ cm}}{1.35 \text{ cm}} \right)^2 } = \pm 0.13768 \text{ cm}^2 \Rightarrow \pm 0.14 \text{ cm}^2
\]

Express the area and its uncertainty as $A \pm \delta A = (3.24 \pm 0.14) \text{ cm}^2$. Please note that the quantity and its uncertainty are inside the parentheses, the unit is outside the parentheses!

$\delta \% A$ can be determined using Eq. A.7. For this example,

\[
\delta \% A = \frac{\delta A}{A} \times 100 = \frac{0.14 \text{ cm}^2}{3.24 \text{ cm}^2} \times 100 = 4.3\% \Rightarrow 4\%
\]

$A \pm \delta \% A$ is expressed as

\[
A \pm \delta \% A = 3.24 \text{ cm}^2 \pm 4\%
\]

Please note that % is a dimensionless quantity; no parentheses are in the above example.

A comprehensive example of propagating error is on the next page. Think through this example.
Appendix A: Measurement and Uncertainty

Comprehensive Example

In terms of propagating error, one of the most involved experiments we perform is *Experiment ??: Centripetal Force*, during the first semester of General Physics. It is hoped that you will be able to correctly interpret the uncertainty equations for a particular experiment if an explicit example is provided for a slightly complex situation.

For this experiment, the centripetal force (radial force, \( F_R \)) is given by:

\[
F_R = ma_R = m\frac{v^2}{r}
\]

where \( a_R \) is centripetal acceleration, \( m \) is the mass of the object, \( v \) is the speed of the object, and \( r \) is the radius of the circular path the object travels. The speed of the object is determined by:

\[
v = \frac{\Delta d}{\Delta t} = \frac{2\pi rn}{t}
\]

where \( d \) is the distance the object travels, \( r \) is the radius of the circular path, \( n \) is the number of times the object traveled this circular path, and \( t \) is the time elapsed to travel this distance.

For this experiment, \( F_R \) and \( \delta F_R \) must be determined using measured values.

\[
F_R = \frac{4\pi^2 mn^2}{t^2}
\]

This experiment requires that you measure multiple masses, then add those values to determine the total mass, as the digital balance has a limit of 200 g. Once the uncertainty of the mass, \( \delta m \), has been determined (Eq. A.8), it can be used in the following equation to determine \( \delta F_R \).

\[
\delta F_R = \pm F_R \times \sqrt{\left( \frac{\delta m}{m} \right)^2 + \left( \frac{\delta r}{r} \right)^2 + \left( \frac{2\delta n}{n} \right)^2 + \left( \frac{\delta t}{t} \right)^2}
\]

Note in the example how uncertainty is determined for quantities that are multiplied, divided, or raised to a power. Consider the equation for \( F_R \) and the quantities under the radical. Every measured quantity in the equation belongs under the radical.
Appendix A: Measurement and Uncertainty
Appendix B: Computers and Software

iMac OPERATING INSTRUCTIONS

To TURN ON your computer, push the power button; it’s on the left back side, near the bottom.
The Command (used for keyboard shortcuts) key is on each side of the space bar.
To Select something on a computer means to click on it once with the mouse.
To Open software, a file, or a folder, double-click on it with the mouse cursor.
To Close software: click on the Software Title ⇒ Quit [Command + Q]
To Close a document or folder: FILE ⇒ Close Window [Command + W]
To Re-size a document, click and drag the bottom right corner of the window.
To Collapse (minimize) a window, click on the software title, select Hide [Command + H]
To Switch between different programs, move your mouse cursor to the Dock (the dock is at the bottom of your
computer desktop). Icons for all open software will be displayed. Select the needed software.
To TURN OFF the computer: Apple Icon (top left of computer desktop) ⇒ Shut Down ⇒ Shut Down

When you have finished your experiment:

- ALWAYS close all software: Software Title ⇒ Quit ⇒ Don’t Save [Command + Q]
- Note that closing the software window is not sufficient when using an iMac.
- Close all folders; shut down the computer.

LOGGER PRO Software

Always be certain that Lab Pro is connected before you open Logger Pro software. Connect Lab Pro to the

- Computer using the USB cable; do not connect it to the keyboard
- Sensor(s) - motion detector: DIG/Sonic 2 port; force sensor: CH 1, ±50 N; thermistor: CH 1
- Power supply; plug the power supply into an electrical outlet

If you need to confirm the sensor you are using, a window will open automatically. On the right side of the window,
specify the “Interface & Channel” being used, then specify the sensor. Click on “Connect.”

To open Logger Pro software, open the following folders on your computer:
Software ⇒ Logger Pro 3 ⇒ Experiments ⇒ Physics with Vernier (Exp 14: Heat, open the “Physical
Science with Vernier” folder.)

Then open the specified experiment that is needed for the experiment you are performing.
If you print graphs from this software, File ⇒ Page Setup: Orientation: Landscape; File ⇒ Printing Options: Enter
team names as needed.

When finished with the experiment, close the software: Logger Pro ⇒ Quit ⇒ Don’t Save, then close the folders and
disconnect all Lab Pro equipment. Turn off the computer.

If software or computer problems develop, close the software, then open it again. If problems persist, close the
software, restart the computer, then open the software again. If this fails, ask your TA for help.
**GRAPHING**

Consider the slope-intercept equation of a line: \( y = mx + b \). If \( b = 0 \), the equation reduces to \( y = mx \). When you graph “\( y \) vs. \( x \)” and produce a linear (straight line) graph, the slope, \( m \), of the graph is the constant of proportionality. The \( y \)-axis is the ORDINATE (dependent variable). The \( x \)-axis is the ABSCISSA (independent variable).

Consider two proportional quantities, \( A \) and \( B \). Graphing \( A \) vs. \( B \) is equivalent to graphing \( y \) vs. \( x \). The resulting linear graph has slope \( m \), which represents the constant of proportionality. When investigating linear functions, if \( x = 0 \) and \( y = 0 \), then \( b = 0 \). When this is true, \((0,0)\) is a relevant and necessary data point. Always determine if the data point \((0,0)\) is necessary.

A nonlinear graph is a curve(s); it is not a straight line. Note that a curve does not have a single slope, it has a constantly changing slope (an infinite number of slopes). Always determine if \((0,0)\) is necessary.

*A properly drawn graph must have the following components:*

- Title
- Axes Labels with Units
- Appropriate Scale
- Appropriate Analysis (refer to Fig. B.3)
- Error Bars (not if graphs are produced by hand)
- Mathematical Analysis after the graph is printed
- Team Names and Date

**Graphing by Hand**

When plotting your data, you will need to determine the scale based on the range of values. Always choose easily divisible increments for the major divisions (*e.g.*, 2, 5, 25). Your plot should fill most of the graph (refer to Fig. B.1 and Fig. B.2). The same data is plotted on both graphs; only the scales have changed.

Use a *best fit* connecting line when analyzing your graph; do not “connect the dots.” A *best fit* line will have about as many data points above the curve as below. If the plot is linear, a ruler should be used.

For example, assume you are graphing distance vs. time. If your distance values range from 0 m to 40 m, your \( y \)-axis should span 0 to 40 meters, perhaps up to 50 m. If your time values range from 0 to 80 seconds, your \( x \)-axis should span from 0 to at least 80 s. Refer to Fig. B.1.
Appendix B: Computers and Software

Graphing with Graphical Analysis

Graphs for physics lab required Graphical Analysis software. The software is available at no charge for physics students to install on their home computer. The required username is “physics.” The password is “physicslabs.” Download and install the software on your home computer. To obtain the software, go to the link on your Lab Physicist’s website or:

http://www.phy.olemiss.edu/Courses/Software/

If you do not have your own computer, you may need to use public computers; you might need to “Run” the software from the website rather than “Download” the software. Weir Hall, Honors College, and UM Library have the software installed for your use on the student computers. Be certain you can print from the computer you are using! Printing is not available for students in the Department of Physics and Astronomy.

Analysis of Graphs

To analyze your graph, fit your data to a general equation, and match up the parts to a known function. A general equation has the main elements of a particular type of function (equation). The variables used in the general equation can represent many quantities, thus the name. The known function is easy to determine: What is the equation for the concept or theory you are investigating? To match up the parts of the two equations, write one below the other and connect the corresponding parts with a line (match up the parts). To determine the type of general equation you need, consider the following questions:

- Is one variable squared? If yes → it is a quadratic equation.
- Are there trigonometric functions in the equation? If yes → it is a sinusoidal equation (or tangential, or . . . ).
- Is there an exponential (i.e., the number “e,” not an exponent) in the equation? If yes → it is an exponential equation.
- If none of the above, is this equation linear? If yes → use the slope-intercept equation of a line: \( y = mx + b \).

General Equation \(\Leftrightarrow\) Known Function Examples

Consider the equation for the circumference of a circle, \( C = \pi D \). Plot “\( C \) vs. \( D \)” (\( C \) on the \( y \)-axis, \( D \) on the \( x \)-axis).

\[
\begin{align*}
C &= \pi D \\
y &= m x + b
\end{align*}
\]

If \( b = 0 \), the slope \( m \) equals everything else in the equation, in this case, the number \( \pi \).

Consider an equation for exponential decay: \( N(t) = N_0 \cdot e^{-\lambda t} \). \( N \) is the number of particles as time progresses; \( N_0 \) is the initial number of particles. \( e^{(-\lambda t)} \) can also be written “exp \(-\lambda t\)” Plot “\( N \) vs. \( t \)” and analyze the graph:

\[
\begin{align*}
N(t) &= N_0 \cdot \exp(-\lambda t) \\
f(x) &= A \cdot \exp(-Cx) + B
\end{align*}
\]

We want to know \( \lambda \) (lambda), a coefficient that represents the rate of decay.

\( B = 0 \).

The “Information Box” will provide a value for the coefficient “\( C \),” which is equal to \( \lambda \).

Most of the graphs we analyze are linear. When nonlinear graphs are analyzed, follow the same process: Match up parts of the two equations (general and known function).

Graphical Analysis has many built-in general equations but not all. You may need to enter (type) the general equation if it is not available; the general equation will be given by the TA or in the procedure if needed.
Appendix B: Computers and Software

The Data

Data ⇒ (Take care of data details before graph details.)

Data Set Options ⇒
Name the data set; keep it short.

Column Options ⇒
Select each data column and fill in the relevant information. Note that often tabs are available that open additional windows. The style of the point protector must always be “Empty” rather than “Filled.” When uncertainties need to be entered, refer to “Error Bar Calculations.”

Enter The Data ⇒
When entering data that have multipliers: e.g., 3.1 cm = $3.1 \times 10^{-2}$ m = 3.1 E-2 m, type: “3.1 e - 2.”
When entering angle measurements in degrees, Select File ⇒ Settings for Untitled ⇒ Change the unit to degrees.
If you have multiple sets of data, Select Data ⇒ New Data Set
The software will calculate a new data set for you: Data ⇒ New Calculated Column ⇒ Enter the necessary information.

The Graph

Options ⇒ (Click once on the graph to make it the active field.)

Graph Options ⇒
Fill in the relevant information. Except for Experiment 15: Ohm’s Law, never select “Connect Points” as we will fit data to known functions. When more than one data set is plotted on a single graph, you must enter the label and unit for the y-axis.
Specify the column(s) of data to be plotted on the y-axis as well as the column of data to be plotted on the x-axis. Do not use the “Right Y-Axis.”

The Analysis (1st Step)

Analyze ⇒ Curve Fit ⇒
Select the relevant data set ⇒ Select the general equation ⇒ Try Fit ⇒ OK; or
Select the relevant data set ⇒ Define Function ⇒ Type the general equation ⇒ Try Fit ⇒ OK.
We will use “Curve Fit” for all graphs, even if they are linear. The value of all quantities in the general equation is specified in the “Information Box” that will appear on your graph for each data set that you analyze.

The Printing

File ⇒
Page Setup ⇒ Always landscape.
Printing Options ⇒ Enter names, Date, Page Title, Page Number
File ⇒ Print Graph (Never use the print icon or File ⇒ Print)
File ⇒ Print Data Table (if needed)

The Analysis (2nd Step)

On the graph, in ink, write the general equation and, below it, the known function being investigated.
Match parts of the general equation to the corresponding parts of the known function.
Refer to the “Information Box” on the graph. The value of the unknown quantity can be determined, although it may require some algebra. Calculate the necessary quantities. Refer to Fig. B.3.
General Equation: \[ y = mx + b \quad b = 0 \]

Known Function: \[ I = \frac{V}{R} \]
\[ m = \frac{1}{R} \]
\[ R_1 = \frac{1}{m_1} = 502.01 \, \Omega; \quad R_2 = \frac{1}{m_2} = 1006.04 \, \Omega \]

**Mathematical analysis is required for every graph of known functions.**

The software performs the 1st step of analysis.

You complete the analysis after printing.

*Match parts of the general equation to corresponding parts of the known function.*

Current, \( I \), is plotted on the \( y \)-axis; electric potential difference, \( V \), is plotted on the \( x \)-axis (\( I \) vs. \( V \)).

The slope \( m \) equals everything else in the equation, in this case, inverse of the resistance, \( R \).

Note that 6 of the 7 required graph components are present in this figure.

When you Print Graph, the team names and date will print (7th graph component).

Note that the software does the first step in the analysis. After the graph is printed, you must complete the analysis.
You are usually required to print your data: *File ⇒ Print Data Table…*

Set the printing parameters prior to printing.

*Note:* Use *Column Options* to name each data set, name each column of data, enter the unit for each column of data, and the uncertainty of the measurement (*error bar calculation*) before you enter the data. Two (2) data sets are on this data table. Note that:

- Each data set is named
- Each column of data is named
- Unit symbols are entered
- Error Bars (uncertainty of measurement) have been entered; (uncertainties appear only on the graph but only if “empty” geometric figures, such as an empty circle, are chosen)
- If the uncertainty of measurements varies, enter the uncertainty for each measurement in a new column: *Data ⇒ New Manual Column.* Be sure to set all the parameters (name, unit) for the column of data. Now return to the measurement column of data, *Options tab, Select “Error Bar Calculations,”* and specify the appropriate column of data to use for the uncertainty of those measurements

<table>
<thead>
<tr>
<th>Voltage (V)</th>
<th>Current (A)</th>
<th>Voltage (V)</th>
<th>Current (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.000</td>
<td>0.0000</td>
<td>1.000</td>
<td>0.0000</td>
</tr>
<tr>
<td>2.010</td>
<td>0.0019</td>
<td>3.010</td>
<td>0.0059</td>
</tr>
<tr>
<td>3.990</td>
<td>0.0080</td>
<td>2.030</td>
<td>0.0039</td>
</tr>
</tbody>
</table>
Appendix C: Equipment

Figure C.1: Vernier Caliper - Closed

- Note the zero graduation marks (tick marks) on each side of the scales (vernier and main).
- We will always use metric units; ignore British units (inches) on each of the scales (the top of this caliper).
- In our lab, the uncertainty of a measurement, $\delta$, is defined as $\frac{1}{2}$ the resolution (smallest unit); the caliper is one of three exceptions. The uncertainty of the caliper is the resolution of the caliper. The vernier scale states the resolution: 0.05 mm (0.05 mm=0.005 cm). Thus, $\delta = \pm 0.005$ cm for the vernier caliper, allowing a measurement of $(a.bcd \pm 0.005)$ cm.
- Do not over-tighten the clamp. Tighten just enough that the vernier scale does not shift.
Appendix C: Equipment

Note that the zero tick mark of the vernier scale has passed the 2.2 position but not the 2.3 position of the main scale. You now have \( a.b \) (2.2) of the "\( a.bcd \) cm" measurement.

To obtain the remaining values, determine the first tick mark (from left to right) on the vernier scale that is aligned with a tick mark on the main scale. In this case, 5.5 is the first alignment and provides \( cd \) of your measurement.

The final measurement reading of the vernier caliper is \( (2.255 \pm 0.005) \) cm.

A tutorial is available on your lab computer: Items for Students ⇒ YP Vernier

Figure C.3: Vernier Caliper - End View

Note that as you open the caliper, a device to measure depth is available.
Always calibrate before use. To calibrate, slide all poises (sliding masses) to zero; rotate the dial to zero. When oscillation of the arm has subsided, the pointer on the right end of the arm should alight with zero. If necessary, adjust the \textit{knurled balance compensator knob} (knurled knob) at the left end of the balance.

Figure C.4: Dial-O-Gram Balance

Figure C.5: Dial zero aligns with vernier zero.

Figure C.6: Pointer aligns with zero.
A measurement of \((a.bcd \pm 0.005)\) grams can be obtained from a Dial-O-Gram balance. Above the main scale (dial) is a vernier scale. To determine the main scale value, read the nearest gram and tenth of a gram \((a.b)\) to the right of the zero vernier graduation \((a.b = 2.3 \text{ grams} = 2.3 \text{ g})\).

Add the vernier graduation value that aligns with the main scale graduation \((.06 \text{ g} \rightarrow c = 6)\).

The smallest unit for the Dial-O-Gram balance is 0.01 g: \(\delta = \frac{1}{2}(0.01) \text{ g} = 0.005 \text{ g}\).

We will assume a zero for the thousandths place \((d = 0)\) of the measurement to accommodate the order of magnitude (decimal place) of the uncertainty \((0.005)\). Given the width of the tick marks, we claim this is a valid assumption.

Thus, the measurement shown is \((a.bcd \pm 0.005 \rightarrow 2.360 \pm 0.005)\) g.
DIGITAL MULTI-METER

A Digital Multi-Meter (DMM) measures a variety of electrical quantities, depending on the model used. There are differences among the three models used in our lab; consider carefully (i.e., read and think) as you select the function, scale, and connections.

We will use a DMM to measure **resistance**, **current**, and **voltage**.

- The symbol for **resistance** is $R$; the unit is ohms, Ω (Greek letter, omega). Use an ohmmeter to measure $R$.
- They symbol for **current** is $I$; the unit is amperes, A (often shortened to amps). Use an ammeter to measure $I$.
- The symbol for **voltage** (electric potential difference) is $V$; the unit is volts, V. Use a voltmeter to measure $V$.

Directions are written for the *Kelvin 200* model {with notations for other models}.

To specify the required function (**ohmmeter**, **ammeter**, **voltmeter**), select:

1. **POWER SOURCE**: Turn the dial from OFF to the desired function. Set the switch to DC (Direct Current) or AC (Alternating Current). Assume DC unless specified otherwise, as most experiments make use of DC. {Metex: ON/OFF button.}

2. **DIAL**: Select the function the DMM will perform by turning the dial to the appropriate setting as well as the appropriate scale. Always choose the function with the leads unplugged from the device. The various functions on a DMM are denoted by the unit.

3. **JACKS**: The black lead should always plug into the COM jack. Although the color of the wire covering is irrelevant, it is standard to define the black wire as the common (ground). The red lead will be plugged into the VΩ jack when measuring voltage or resistance. Plug the red lead into the mA {Metex: A} jack when measuring current unless the procedure specifically states that you should use the 10A {Metex: 20A} jack.

4. **LEADS**: You must connect the leads to your circuit properly. Damage to the DMM can occur with improper use.

   - **An ohmmeter is used without an external power supply.**
     
     Although current must flow through a circuit or element to measure the resistance, the ohmmeter provides its own current. Therefore, never measure resistance with the power supply connected to the circuit.

   - **An ammeter is always connected in series.**
     
     Current will always flow through all available paths. Since we will want to measure the total current at a specific location, the ammeter must not be connected to an element. This may require the removal of a jumper so that the ammeter can be inserted into the circuit.

   - **A voltmeter is always connected in parallel.**
     
     The leads connect to each end of the same element, as we want to measure the voltage drop (potential difference) across an element.
Figure C.8: Digital Multi-Meters (DMM’s)

(Models used in General Physics Labs: Metex, Kelvin 150, Kelvin 200)

Note the multipliers $\mu$, m, K, and M on the different scales. This means that you would multiply the digital reading of the DMM by the appropriate multiplier. For example, the $200K$ scale for the ohmmeter means you should multiply the digital readout by $10^3$.

Note also that the DMM will only read values up to the scale value. For example, the $200mA$ scale for the ammeter means the DMM will measure current up to 20 mA if using the mA jack, but will measure up to 10 A if using the 10 A jack.
Appendix C: Equipment

Figure C.9: Ohmmeter - Ohmmeter Reading: $0.501 \times 10^3 \Omega = 501\Omega$

To use the DMM as an ohmmeter, select:

- **POWER**: DC; turn dial to $\Omega$-function {Metex: ON}.
- **DIAL**: 2k $\Omega$ {Metex: 2k OHM}.
- **JACKS**: Plug the black lead into COM and the red lead into $V\Omega$.
- **LEADS**: Place leads on each end of the same element.
Appendix C: Equipment

Figure C.10: Ohmmeter and Circuit - Ohmmeter Reading: 0.501 kΩ = 501Ω

Note: Power supply is disconnected from the circuit when measuring the resistance of the circuit, $R_{eq}$.

Note: To avoid confusion, the ammeter is not shown in this figure; we will usually insert the ammeter in our circuits prior to making any measurements of the circuit.

Figure C.11: Ohmmeter and Resistor - Ohmmeter Reading: 0.200 kΩ = 200Ω

Connect ohmmeter to resistor to measure the resistance of an element, $R_i$. If your resistor has a double-banana plug, you can simply plug it into the jacks instead of using wire leads.
To use the DMM as an ammeter, select:

- **POWER**: DC; turn dial to A-function (Metex: ON/ Kelvin 150: Turn dial to DCA).
- **DIAL**: Turn to the appropriate scale on A (Metex and Kelvin 150: DCA).
- **JACKS**: Plug the black lead into COM and the red lead into mA (Metex: A).
- **LEADS**: Place leads into circuit in series (current travels from one element, through the ammeter, into a different element).
Figure C.13: Ammeter and Circuit - Ammeter Reading: 9.9 mA = 0.0099 A

Note: The ammeter is connected in series. A jumper has been removed from the circuit so that the ammeter can be inserted, in series, to measure the current. When the current needs to be measured at a different location, remove the ammeter and replace the jumper. Power supply is connected, set to specified voltage.

Note: To avoid confusion, the voltmeter is not shown in this figure. We often need both devices connected when analyzing circuits.
To use the DMM as a voltmeter, select:

- **POWER:** DC, turn dial to V {Metex: ON; Kelvin 150: Turn dial to DCV}
- **DIAL:** 20V {Metex and Kelvin 150: 20 DCV}
- **JACKS:** Plug the black lead into COM and the red lead into VΩ
- **LEADS:** Place leads on each end of the same element
Note: The circuit is complete, power supply is connected to the circuit, plugged into the electrical outlet, and set to specified voltage. The voltmeter is connected in parallel to the power supply as it measures voltage across an element in a circuit. Connect the voltmeter at the circuit, not at the power supply. The ammeter should be connected in your circuit prior to setting the voltage of the circuit.

To set the voltage of the circuit: Connect the voltmeter to the circuit in parallel with the power supply, insert the ammeter at the location specified in the schematic, then adjust the power supply to the required voltage.
Appendix D: Math Review

A great deal of information can be obtained by first considering the type of equation being investigated.

- Is one variable squared? If yes → it is a quadratic equation.

- Are there trigonometric functions in the equation? If yes → it is a sinusoidal equation (or tangential, or . . . ).

- Is there an exponential (i.e., the number “e,” not an exponent) in the equation? If yes → it is an exponential equation.

- If none of the above, is this equation linear? If yes → use the slope-intercept equation of a line: \( y = mx + b \).

Consider the kinematic equations below (from Experiment 4: Projectile Motion).

\[
\begin{align*}
x &= x_0 + v_{0x} t + \frac{1}{2} a_x t^2 & \text{Quadratic in Time} \\
y &= y_0 + v_{0y} t + \frac{1}{2} a_y t^2 & \text{Quadratic in Time} \\
v_x &= v_{0x} + a_x t & \text{Linear} \\
v_y &= v_{0y} + a_y t & \text{Linear} \\
v_x^2 &= v_{0x}^2 + 2a_x (x - x_0) & \text{Quadratic in Velocity} \\
v_y^2 &= v_{0y}^2 + 2a_y (y - y_0) & \text{Quadratic in Velocity}
\end{align*}
\]

We will use other types of equations this year as well:

\[
\begin{align*}
F_B &= ILB \sin \theta & \text{Magnetic Force is Sinusoidal as the Angle Varies} \\
N &= N_0 e^{\lambda t} & \text{Exponential Growth (Your calculator has the function } e).\text{)}
\end{align*}
\]
General Physics requires mastery of college-level algebra and trigonometry.

If it has been some time since you used this level of math, it is important that you refresh your knowledge and skills. Your text for physics lecture may have a math appendix; if it does, please read through it to be certain you have the math knowledge required.

This appendix is a simple summary of math that is frequently used in General Physics Lab. Some experiments will require more advanced knowledge.

_Solve for x_ means that the variable $x$ is on one side of the equation, everything else in the equation is on the other side. If $x$ is the only variable, the equation has a unique solution (has only one correct answer).

If unique solutions are required, the number of unknowns _must_ equal the number of equations.

Consider the equation for the circumference of a circle: $C = \pi D$. There is 1 equation and 2 unknowns; there is no unique solution. Refer to Fig. D.1. _Any_ point on the line is a correct solution. There are an infinite number of correct solutions.

![Figure D.1: Linear Plot](image)
Solving 3 equations with 3 unknowns is required in lab. The variables are the unknowns. Four different methods are provided here to help you. The first method offered is the substitution method:

Choose one of the equations; if all 3 equations have all 3 unknowns, it does not matter which one you choose first. If this is not the case, remember you need to eliminate and unknown! Choose the equation that has all 3 unknowns. Designate this equation as Eq. 1. Solve Eq. 1 for one of the unknowns (it does not matter which one).

Substitute Eq. 1 into a second equation; designate this second equation as Eq. 2. Note that the substitution must eliminate an unknown. Designate the last equation as Eq. 3.

You now have 2 equations with 2 unknowns: Eq. 2 and Eq. 3. Choose either Eq. 2 or Eq. 3, then solve it for an unknown; substitute the value of this unknown into the remaining equation. Solve the equation; substitute this value(s) into the first equation.

Solve the first equation. Done. Check your answer!

Example for “Solving 3 Equations with 3 Unknowns”

Given the following 3 equations, solve for \( x \), \( y \), and \( z \):

\[
\begin{align*}
x &= y + z \\
1.5 - 3x - 4z &= 0 \\
-1.5 - 5y + 6z &= 0
\end{align*}
\]

Choose the equations that has all 3 unknowns. Designate this equation as Eq. 1. Solve Eq. 1 for one of the unknowns: \( z \).

Eq 1: \( x = y + z \) → \( z = x - y \)

Substitute Eq. 1 into a second equation; designate this second equation as Eq. 2.

Eq 2: \( 1.5 - 3x - 4z = 0 \) → \( 1.5 - 3x - 4(x - y) = 0 \) → \( 1.5 - 7x + 4y = 0 \)

Choose Eq. 2 or Eq. 3, then solve it for an unknown: Eq. 2.

Eq 2: \( 1.5 - 7x + 4y = 0 \) → \( x = (1.5 + 4y)/7 \)

Substitute the value of this unknown into the remaining equation:

Eq 3: \( -1.5 - 5y + 6x = 0 \) → \( -1.5 - 5y + 6(1.5 + 4y)/7 = 0 \)

Solve the equation:

\[
-1.5 - 5y + 9 + 24y = 0 \rightarrow 24y - 5y = 1.5 - 9/7 \rightarrow 15y = 3/7 \rightarrow y = (3/7)(1/11) = -0.136
\]

Substitute this value into the previous equation and solve the equation:

Eq 2: \( x = 1.5 + 4y \) → \( x = (1.5 + 4(-0.136))/7 \) → \( x = 0.137 \)

Substitute these value(s) into the first equation and solve the equation:

Eq 1: \( z = x - y \) → \( z = 0.137 - (-0.136) \rightarrow z = 0.273 \)

Done. Check your answer!
Appendix D: Math Review

A second method available to you is Cramer’s Rule. Consult a linear algebra text.

A third method is an online calculator that solves 3 equations with 3 unknowns: http://www.1728.com/unknown3.htm

A fourth method is to use Mathematica software; it is installed on your computer in lab. When you have finished measuring $\varepsilon_i$ and $R_i$ and disconnected the batteries (Experiment 17), simplify your equations. Open the Software folder, open Mathematica. Type the 3 equations using the format shown below; it is followed by an example. Note that the “s” is capitalized, there are no spaces, the “equals” symbol is entered twice, and square or curly brackets are not the same as parentheses. When you finish entering the information, use the “enter” key on the number pad, not the “enter” or “return” key on the main keyboard. Note that the same variables you used in the equations are specified at the end. The following makes use of spaces so that it is easier for you to read. (If you have a different version of this software, there may be small changes.)

\[
\text{Solve}\left\{\text{equation1, equation2, equation3}, \{I_1, I_2, I_3\}\right\}
\]

\[
\text{Solve}\left\{(25x + 12z - 1.5 == 0, -40y + 12z == 1.5, x == y + z), \{x, y, z\}\right\}
\]

The coefficients of the variables $x$, $y$, and $z$ ($I_1$, $I_2$, $I_3$) are the measured resistances of your circuit.

Solving a quadratic equation is easily accomplished by the use of the quadratic formula.

A quadratic means that one variable is squared.

To use the quadratic formula, the equation must first be in the correct form: $ax^2 + bx + c = 0$.

The coefficients are $a$, $b$, and $c$; $a \neq 0$.

The coefficient of $x^2$ is $a$, the coefficient of $x^1$ is $b$, the coefficient of $x^0$ is $c$.

The quadratic formula is $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

$7x^2 - 3x = 5$ is a quadratic equation. $7x^2 - 3x - 5 = 0$ is the required form to use the quadratic formula.

The coefficients for this equation are $a = 7$, $b = -3$, and $c = -5$.

To use the quadratic formula, substitute the coefficients (i.e., $a$, $b$, $c$) into the formula.

$x = \frac{-(-3) \pm \sqrt{(-3)^2 - (4)(7)(-5)}}{(2)(7)}$

Solve the equation. Note that a quadratic equation will always have 2 solutions. An equation involving a variable raised to any power $n$ has $n$ solutions. For example: $y = x^7$, $n = 7$. There are 7 solutions.
Appendix D: Math Review

Trigonometry

An angle is measured from the +x-axis, in a counterclockwise direction unless specified otherwise.

![ Cartesian Graph Diagram ]

The sum of the angles of any triangle equals 180°. \(180° = \pi\) (radians).

The Pythagorean Theorem states that \(a^2 + b^2 = c^2\); we often use other variables: \(x^2 + y^2 = r^2\) (refer to Fig. D.3).

![ Right Triangle Diagram ]

Note that the \(\cos \theta = \sin \alpha\), etc.

Note that to solve a trig function for an angle, you must take the inverse of the trig function.

When using a calculator to determine the inverse tangent of an angle, your calculator may give you an angle that is in the wrong quadrant [calculated angle will only be in quadrant I (+\(\theta\)) or quadrant IV (−\(\theta\))]. To determine the correct quadrant, consider the ± sign of the argument: If \(x\) is negative, the angle must be in quadrant II or III; if \(y\) is negative, the angle must be in quadrant III or IV. If both \(x\) and \(y\) are negative, the angle must be in quadrant III (refer to Fig. D.2).

If the calculated angle is in the wrong quadrant, simply add 180° to the calculated angle.
If an angle is measured in radians, the arc length $s$ equals the radius $r$ times the angle $\theta$ (refer to Fig. D.4).

$$s = r \theta$$

![Figure D.4: Arc Length](image1)

If an angle is small and measured in radians: $\sin \theta \approx \tan \theta \approx \theta$ (refer to Fig. D.5).

![Figure D.5: Small Angle Approximation](image2)

Consider a circle of radius $r$ (refer to Fig. D.6).
The circumference $C$ of the circle is $C = 2\pi r$.
The area $A$ of the circle is $A = \pi r^2$.

![Figure D.6: Circle of radius $r$](image3)

Consider a right-circular cylinder of radius $r$ and height $h$ (refer to Fig. D.7).
The volume $V$ of the cylinder is $V = \pi r^2 h$.

![Figure D.7: Right-Circular Cylinder](image4)
Appendix D: Math Review

Assume the density ($\rho$) and uncertainty of density ($\delta\rho$) are to be determined for a cylinder using a caliper to measure the dimensions (diameter, height) and a digital balance to measure mass.

The equations for volume and density are:

$$V = \frac{\pi D^2 h}{4}, \quad \rho = \frac{m}{V}$$

Once you have written the algebraic equation for the quantities, write the equations for the uncertainty of those quantities. Refer to Appendix A for details.

$$\delta V = \pm V \cdot \sqrt{\left(\frac{2\delta D}{D}\right)^2 + \left(\frac{\delta h}{h}\right)^2}$$

$$\delta \rho = \pm \rho \cdot \sqrt{\left(\frac{\delta m}{m}\right)^2 + \left(\frac{\delta V}{V}\right)^2}$$

The measured values are:

$$(D \pm \delta D) = (2.345 \pm 0.005) \text{ cm}$$

$$(h \pm \delta h) = (5.024 \pm 0.005) \text{ cm}$$

$$(m \pm \delta m) = (14.56 \pm 0.01) \text{ g}$$

To determine $V \pm \delta V$, insert the necessary values into the equation, then calculate:

$$V = \frac{\pi (2.345 \text{ cm})^2 (5.025 \text{ cm})}{4} = 21.702594 \text{ cm}^3 \rightarrow V = 21.703 \text{ cm}^3$$

(Keep at least one more significant figure...)

$$\delta V = \pm 21.703 \text{ cm}^3 \cdot \sqrt{\left(\frac{2 \cdot 0.005 \text{ cm}}{2.345 \text{ cm}}\right)^2 + \left(\frac{0.005 \text{ cm}}{5.025 \text{ cm}}\right)^2} = 0.0950 \text{ cm}^3$$

(Round to one significant figure...)

$$V \pm \delta V = (21.70 \pm 0.10) \text{ cm}^3$$

(Recall that the uncertainty dictates the significant figures in the quantity.)

$$\rho = \frac{14.5 \text{ g}}{21.70 \text{ cm}^3} = 0.67096 \text{ g/cm}^3$$

$$\delta \rho = \pm 0.67096 \text{ g/cm}^3 \cdot \sqrt{\left(\frac{0.01 \text{ g}}{14.56 \text{ g}}\right)^2 + \left(\frac{0.10 \text{ cm}^3}{21.70 \text{ cm}^3}\right)^2}$$

$$\rho \pm \delta \rho = (0.671 \pm 0.003) \text{ g/cm}^3$$

(Recall that the precision of the quantity and the precision of the uncertainty must always match.)
Appendix D: Take-Home Math Exam ~ Physics 223

Name: ___________________________________________  Section _______

Be careful to read the instructions, show your work, and include units as necessary. Use the symbols specified. No partial credit for math exams. One point (1 pt) per question.

Solve each equation for $x$:

1. $y = 7x - 6 + 2x$

2. $5x = 2x - 7 + y$

3. $3x^2 - 4 = x$

4. Solve for $x$ and $y$ (2 equations with 2 unknowns) using substitution:
   
   $2x + y = 3$
   
   $y - 2x = 9$

Consider a sphere of radius $r$.

5. $V =$ ___________ (volume)

6. $A =$ ___________ (surface area)

Consider a circle of diameter $D$.

7. $C =$ ___________ (circumference)

8. $A =$ ___________ (area)

Refer to Fig. D.8 for questions 9 through 11:

9. $V =$ ___________ (volume)

10. $C =$ ___________ (circumference)

11. $A =$ ___________ (surface area)

Figure D.8: Solid Right-Circular Cylinder height $h$, radius $r$. 
12. Consider a triangle. How many degrees and radians are there?

__________ degrees  ____________ radians

Refer to Fig. D.9 for questions 13 through 19. Answer in terms of x, y, and r.

13. Pythagorean Theorem: ________________

14. \( \cos \alpha = \) ________________

15. \( \sin \beta = \) ________________

16. \( \sin \alpha = \) ________________

17. \( \cos \beta = \) ________________

18. \( \tan \alpha = \) ________________

19. \( \tan \beta = \) ________________

Simplify the following expression:

20. \( 9z + 3 - 7x + 2z - 3x - 8 + 5z - 3y \) ________________

Convert the following:

21. 55 mph = ________________ m/s

22. 162.56 cm = ________________ ft \( \cdot \) in” (e.g., 6’1”)

23. 1 cm\(^3\) = ________________ m\(^3\)

Use the quadratic formula to solve the following equation:

24. \( 9x^2 + 7x = 2 \)

25. \( \sqrt{x} = x \) to the power ______